A note on the assumptions made while computing the postseismic lithospheric deformation

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The postseismic lithospheric deformation is usually explained as viscoelastic relaxation of the coseismic stresses. In general, for computing the postseismic deformation, the shear modulus (μ) is relaxed, keeping either the bulk modulus (k) or the Lamé parameter (λ) fixed. It is shown that the two assumptions yield significantly different results. The assumption $k =$ const. implies that the medium behaves like an elastic body for dilatational changes which can be justified on physical grounds, but such a justification cannot be given in the case of the assumption $\lambda =$ const.

1. Introduction

The elasticity theory of dislocations has been successfully applied in the recent past to model the coseismic lithospheric deformation. The postseismic deformation can be explained as viscoelastic relaxation of the coseismic stresses. The correspondence principle of linear viscoelasticity has been widely used for calculating the postseismic deformation. In the case of an isotropic material, there are only two elastic modulii. In the general case, both these elastic modulii should be relaxed for calculating the viscoelastic response of the medium. However, for simplicity, only the shear modulus (μ) is relaxed, keeping either the bulk modulus (k) or the Lamé parameter (λ) fixed. Rosenman and Singh (1973a,b), Singh and Rosenman (1974), Nur and Mavko (1974), Peltier (1974), Matsu'ura and Tanimoto (1980), Cohen (1980, 1982), Matsu'ura *et al* (1981), Iwasaki and Matsu'ura (1981, 1982), Malosh and Raefsky (1983), Iwasaki (1985, 1986), Bonafede *et al* (1986), Dragoni and Magnanensi (1989) and Pollitz (1992) assumed a constant value for the bulk modulus while computing the viscoelastic response. In contrast, Rundle and Jackson (1977), Rundle (1978, 1982), Thatcher and Rundle (1979) and Ma and Kusznir (1994a,b; 1995) assumed a constant value for the Lamé parameter λ when computing the viscoelastic response.

Singh and Singh (1990) gave a simple procedure for obtaining the quasi-static displacements, strains and stresses in a viscoelastic half-space under the assumption $k =$ const. In this note, we give the corresponding results for $\lambda =$ const. and show that the two assumptions yield significantly different results. The assumption $k =$ const. might be more appropriate since it implies that the medium behaves like an elastic body for dilatational changes and like a viscoelastic body for deviatoric changes. This type of physical interpretation cannot be given for the assumption $\lambda =$ const.

2. Auxiliary functions for fixed k

In the expressions for the displacements, strains and stresses due to a source in a homogeneous, isotropic, elastic half-space, the elastic modulii occur in the following combinations:

$$Q_1 = \frac{1}{3k+4\mu} = \frac{1}{3(\lambda+2\mu)},$$

$$Q_2 = \frac{2\mu}{3k+4\mu} = \frac{2\mu}{3(\lambda+2\mu)},$$

$$Q_3 = \frac{1}{3k+\mu} = \frac{1}{3(\lambda+\mu)},$$

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$$Q_4 = \frac{2\mu}{3k+\mu} = \frac{2\mu}{3(\lambda+\mu)},$$

$$Q_5 = \frac{2\mu^2}{3k+4\mu} = \frac{2\mu^2}{3(\lambda+2\mu)},$$

$$Q_6 = \frac{2\mu^2}{3k+\mu} = \frac{2\mu^2}{3(\lambda+\mu)}, \tag{1}$$

where λ, μ are the Lamé parameters and $k = \lambda + (2/3)\mu$ is the bulk modulus. For example, the displacements due to a centre of dilatation in a half-space can be expressed in terms of Q_1 and Q_3 (we need only Q_3 for surface displacements). The displacement field due to a shear dislocation in a half-space can be expressed in terms of Q_2 and Q_4 (only Q_4 for surface displacements). For stresses, we need Q_2 and Q_4 in the case of a centre of dilatation and Q_5 and Q_6 in the case of a shear dislocation.

Let the source-time function be the unit step function $H(t)$. Then, from the correspondence principle, the Laplace transform of the viscoelastic solution is obtained on replacing Q_1 by

$$\frac{1}{s[3\bar{k}(s)+4\bar{\mu}(s)]} = \frac{1}{3s[\bar{\lambda}(s)+2\bar{\mu}(s)]} \tag{2}$$

and similarly for the other functions. Here, s is the Laplace transform variable and $\bar{\lambda}(s)$, $\bar{\mu}(s)$ and $\bar{k}(s)$ are the transform elastic modulii. On inverting, we find that Q_1 in the elastic response is replaced by $\hat{Q}_1(t)$ in the viscoelastic response, where

$$\hat{Q}_1(t) = L^{-1}\left[\frac{1}{s\{3\bar{k}(s)+4\bar{\mu}(s)\}}\right]$$

$$= L^{-1}\left[\frac{1}{3s\{\lambda(\bar{s})+2\mu(\bar{s})\}}\right], \tag{3}$$

where L^{-1} denotes the inverse Laplace transform. Similarly, Q_i $(i = 2,3,4,5,6)$ are replaced by the auxiliary functions $\hat{Q}_i(t)$.

Singh and Singh (1990) derived the auxiliary functions for the Kelvin model, the Maxwell model and the SLS (Standard Linear Solid) when the medium behaves elastically for dilatational changes and viscoelastically for deviatoric changes. For this purpose, Singh and Singh (1990) relaxed the shear modulus, keeping the bulk modulus fixed so that

$$\bar{k}(s) = k, \quad \bar{\lambda}(s) = k - \tfrac{2}{3}\bar{\mu}(s).$$

Defining

$$q_i(t) = \hat{Q}_i(t)/Q_i \quad (i = 1,2,3,4,5,6),$$

the results of Singh and Singh (1990) for the Maxwell model and SLS can be summarized as under.

2.1 *Maxwell model*

The constitutive equation for a Maxwell material is

$$\frac{\partial \tau}{\partial t} + \frac{\tau}{t_2} = 2\mu\frac{\partial e}{\partial t}, \tag{4}$$

where τ is the shear stress, e is the shear strain and t_2 is the relaxation time. The expressions for the auxiliary functions for the Poisson case ($\lambda = \mu$) are

$$Q_1 = \tfrac{9}{5} - \tfrac{4}{5}\exp(-\tfrac{5}{9}T),$$
$$Q_2 = \exp(-\tfrac{5}{9}T),$$
$$Q_3 = \tfrac{6}{5} - \tfrac{1}{5}\exp(-\tfrac{5}{6}T),$$
$$Q_4 = \exp(-\tfrac{5}{6}T),$$
$$Q_5 = \tfrac{9}{4}\exp(-T) - \tfrac{5}{4}\exp(-\tfrac{5}{9}T),$$
$$Q_6 = 6\exp(-T) - 5\exp(-\tfrac{5}{6}T), \tag{5}$$

where $T = t/t_2 > 0$.

2.2 *Standard linear solid*

Assuming that the elastic contants of the two springs of the SLS model are equal, the constitutive equation for SLS can be written as

$$\tau + t_2\frac{\partial \tau}{\partial t} = \mu\left(e + 2t_2\frac{\partial e}{\partial t}\right). \tag{6}$$

Assuming $\lambda = \mu$, the expressions for the auxiliary functions are ($T > 0$)

$$q_1 = \tfrac{9}{7} - \tfrac{2}{7}\exp(-\tfrac{7}{9}T),$$
$$q_2 = \tfrac{9}{14} + \tfrac{5}{14}\exp(-\tfrac{7}{9}T),$$
$$q_3 = \tfrac{12}{11} - \tfrac{1}{11}\exp(-\tfrac{11}{12}T),$$
$$q_4 = \tfrac{6}{11} + \tfrac{5}{11}\exp(-\tfrac{11}{12}T),$$
$$q_5 = \tfrac{9}{28} + \tfrac{9}{8}\exp(-T) - \tfrac{25}{56}\exp(-\tfrac{7}{9}T),$$
$$q_6 = \tfrac{3}{11} + 3\exp(-T) - \tfrac{25}{11}\exp(-\tfrac{11}{12}T). \tag{7}$$

3. Auxiliary functions for fixed λ

The auxiliary functions when the Lamé parameter λ is kept fixed can be derived similarly. In this case

$$\bar{\lambda}(s) = \lambda, \quad \bar{k}(s) = \lambda + \tfrac{2}{3}\bar{\mu}(s), \tag{8}$$

Table 1. *Limiting values of $q_i(t)$ as $t \to \infty$.*

$q_i(t)$	Maxwell model		SLS	
	k fixed	λ fixed	k fixed	λ fixed
q_1	$\frac{9}{5}$	3	$\frac{9}{7}$	$\frac{3}{2}$
q_2	0	0	$\frac{9}{14}$	$\frac{3}{4}$
q_3	$\frac{6}{5}$	2	$\frac{12}{11}$	$\frac{4}{3}$
q_4	0	0	$\frac{6}{11}$	$\frac{2}{3}$
q_5	0	0	$\frac{9}{28}$	$\frac{3}{8}$
q_6	0	0	$\frac{3}{11}$	$\frac{1}{3}$

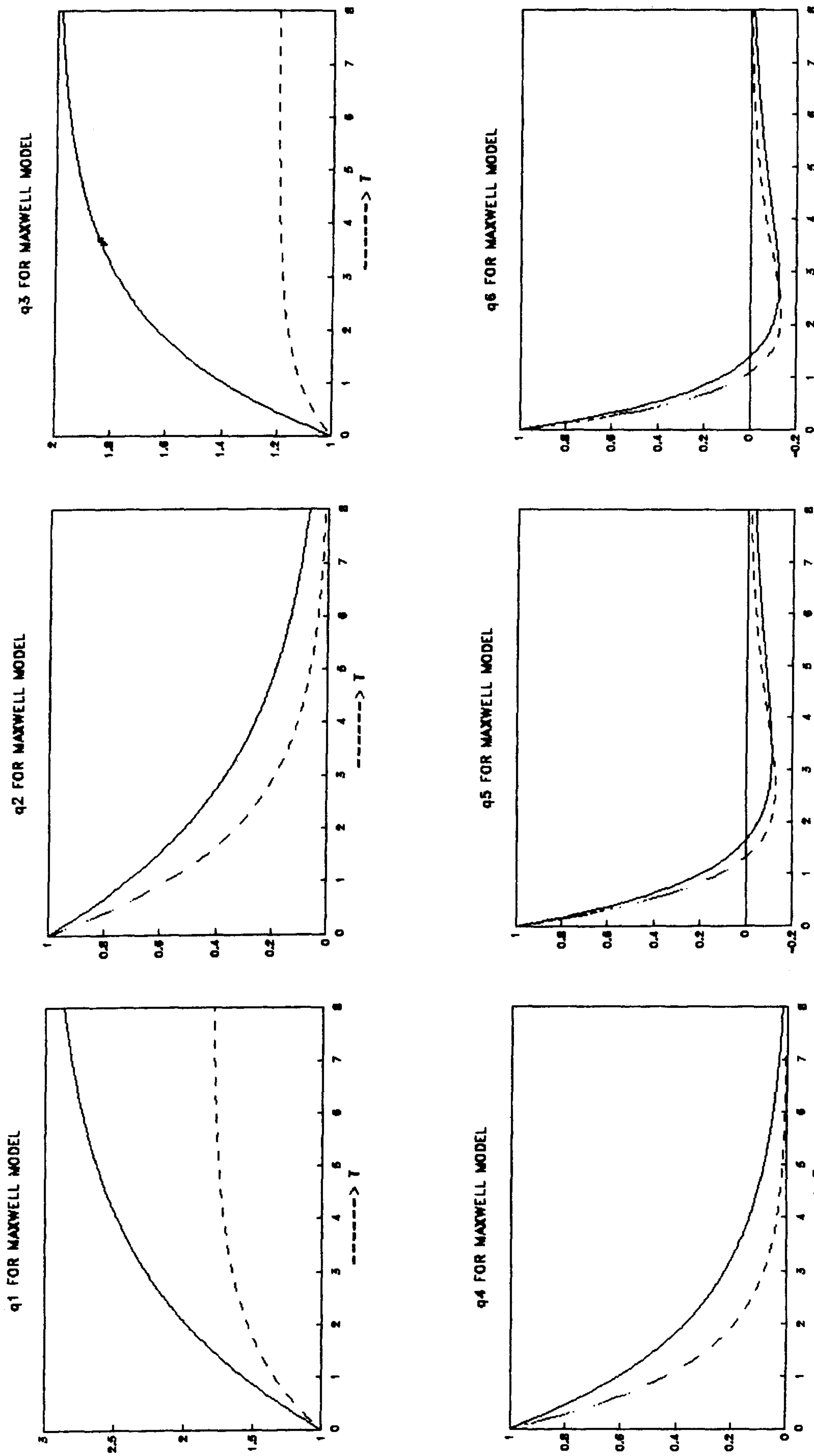

Figure 1. Variation of auxiliary functions with dimensionless time $T = t/t_2$ for the Maxwell model, under the assumption $\lambda = \mu$. The solid line is for λ = const. and the broken line is for k = const.

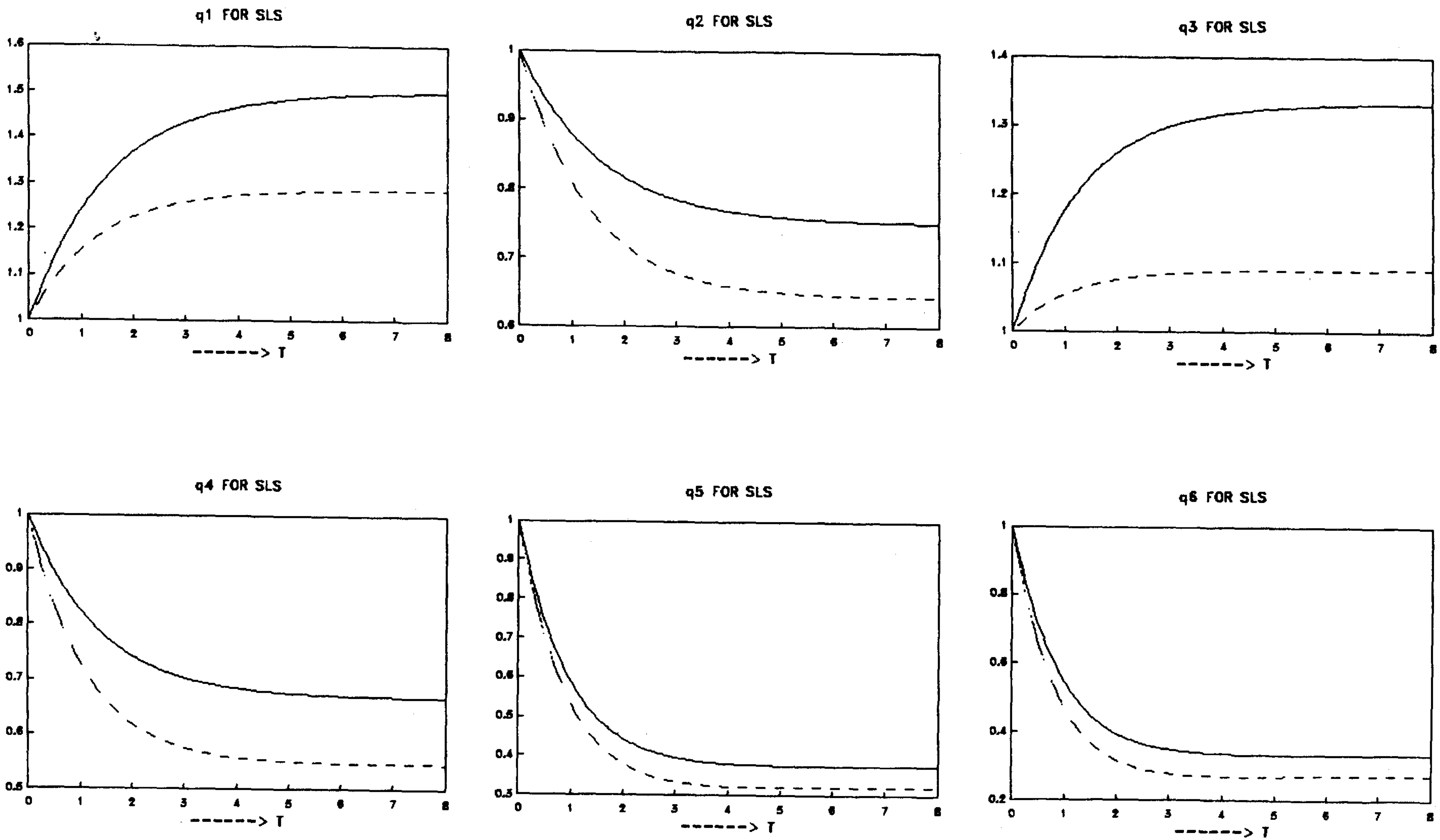

Figure 2. Variation of the auxiliary functions with dimensionless time $T = t/t_2$ for the Standard Linear Solid (SLS), under the assumption that the elastic constants of the two springs of the SLS model are equal and that $\lambda = \mu$. The solid line is for λ = const. and the broken line is for k = const.

so that while relaxing the shear modulus, the bulk modulus also gets relaxed. In this case, we find the following expressions for the auxiliary functions for a Poissonian material ($\lambda = \mu$).

3.1 Maxwell model

$$
\begin{aligned}
q_1 &= 3 - 2\exp(-\tfrac{1}{3}T),\\
q_2 &= \exp(-\tfrac{1}{3}T),\\
q_3 &= 2 - \exp(-\tfrac{1}{2}T),\\
q_4 &= \exp(-\tfrac{1}{2}T),\\
q_5 &= \tfrac{3}{2}\exp(-T) - \tfrac{1}{2}\exp(-\tfrac{1}{3}T),\\
q_6 &= 2\exp(-T) - \exp(-\tfrac{1}{2}T).
\end{aligned} \tag{9}
$$

3.2 Standard linear solid

$$
\begin{aligned}
q_1 &= \tfrac{3}{2} - \tfrac{1}{2}\exp(-\tfrac{2}{3}T),\\
q_2 &= \tfrac{3}{4} + \tfrac{1}{4}\exp(-\tfrac{2}{3}T),\\
q_3 &= \tfrac{4}{3} - \tfrac{1}{3}\exp(-\tfrac{3}{4}T),\\
q_4 &= \tfrac{2}{3} + \tfrac{1}{3}\exp(-\tfrac{3}{4}T),\\
q_5 &= \tfrac{3}{8} - \tfrac{1}{8}\exp(-\tfrac{2}{3}T) + \tfrac{3}{4}\exp(-T),\\
q_6 &= \tfrac{1}{3} - \tfrac{1}{3}\exp(-\tfrac{3}{4}T) + \exp(-T).
\end{aligned} \tag{10}
$$

Table 1 gives the limiting values of $q_i(t)$ as $t \to \infty$. We note that the limiting values of q_i when λ is kept fixed are much larger than the corresponding values when k is kept fixed. However, in all cases, $q_i(t) \to 1$ as $t \to +0$.

4. Numerical results and discussion

Figure 1 shows the variation of the auxiliary functions q_i with dimensionless time $T = t/t_2$ for the Maxwell model, assuming $\lambda = \mu$. The solid line is for $\lambda =$ const. and the broken line is for $k =$ const. We note that, for all times, the values of q_1 and q_3 when λ is fixed are much larger than the corresponding values when k is fixed. As $t \to \infty$, q_2 and q_4 tend to zero much faster under the assumption $k =$ const. The difference between the values of q_5 and q_6 under the two assumptions is not significant.

Figure 2 shows the variation of q_i for SLS. For this model, for all times, the values of q_i when λ is held fixed are significantly larger than the corresponding values when k is held fixed.

Since the values of the auxiliary functions q_i under the two assumptions are significantly different, the choice between the two assumptions $k =$ const. and $\lambda =$ const. becomes important. The assumption $k =$ const. seems more relevant since it is based on the physical consideration that the medium behaves elastically for dilatational changes.

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