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Deformation path concepts in strain analysis: Applications for study of ductile shear zones

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Abstract. The internal fabric of a deformed rock represents the state of finite strain. In some special cases the fabrics also record the strain history of the deformed body. These special cases can profitably be utilized to compare the predictions of dynamic models and strain paths in natural deformations. In this contribution, the concept of deformation path in the study of ductile shear zones has been demonstrated.

Keywords. Incremental strain; steady state; simple shear; dilation.

1. Introduction

The concept of deformation path was first introduced in structural geology by Flinn (1962). The mathematical formulation of this concept has been discussed by Elliott (1972) and Ramberg (1975). In view of its growing applications in structural geology, the concept is first reviewed in this paper and subsequently applied for the study of ductile shear zones.

2. Principles of progressive deformation

Finite strain is the net change in geometry and/or volume between an initial and a final state which is accomplished over some specific length of time. During this time interval the rock passes through a series of changing geometrical states (incremental strain). These changing states of a deformation process define the deformation history or path. In principle, there is an infinity of geometrically possible deformation paths that give rise to a given finite deformation but only one of which actually occurred.

3. Types of deformation paths

Depending upon the relationship between each incremental strain, deformation paths can be divided into two types: coaxial and non-coaxial deformation. The two simple geometric types of deformation viz., pure shear and simple shear, denote coaxial and non-coaxial deformation, respectively. Deformation paths can also be steady state (continuous) path or non-steady state (pulsatory) path. In the former case the magnitude of the strain rate vector remains constant and in the latter the strain rate curves show discontinuities.

4. Mathematical framework for study of deformation paths

4.1 *General conditions*

When a rock undergoes deformation, a spherical marker embedded in it changes to a finite strain ellipse through a sequence of continuously changing ellipses. Each stage of the increment is characterized by a unique ellipse. For understanding the deformation paths, two conditions to be satisfied are: (**a**) the deformation path must be smooth and continuous during one period of deformation; and (**b**) sudden reversals in principal direction or jumps in amount of strain should not occur.

4.2 *Different methods of study*

Different parameters for the study of deformation paths include displacement, strain and rotation. The last two parameters can be calculated from the first one by standard equations. All these parameters can be analysed by three different methods, i.e., incremental, differential and graphical. The incremental method deals with the analysis of incremental displacement, incremental strain and incremental rotation. In the differential method, the rate of change of different parameters with respect to time is studied. The graphical method involves the study of the particle movement path (Ramberg 1975), and presentation in strain space: Flinn plot and Ramsay plot (Ramsay and Huber 1983). The differential and incremental methods are easily handled by matrix algebra. The nature of the transformation required for a particular analysis has to be determined from the boundary conditions of deformation. For any analysis, the order of application of successive strains is important in non-coaxial paths, but not in coaxial deformation.

4.3 *Incremental methods*

A general two-dimensional deformation can be expressed by the linear equations:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{1}$$

where, (x, y) and (x', y') represent co-ordinates of a point before and after deformation, respectively, and a, b, c and d are transformation constants.

Let the strain matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be called $\mathbf{D}$.

When a rock deforms, the total final deformation ($\mathbf{D}_f$) from the original geometry ($\mathbf{D}$) is given by the equation:

$$\mathbf{D}_f = \mathbf{D}_i\mathbf{D} \tag{2}$$

where $\mathbf{D}_i$ is the deformation increment (Elliott 1972). The finite deformation accumulation for a general path is given by the equation:

$$\mathbf{D}_f = \mathbf{D}_{in} \ldots \mathbf{D}_{i3}\mathbf{D}_{i2}\mathbf{D}_{i1} \tag{3}$$

where, $\mathbf{D}_{in}$ represents incremental deformation at each successive stage. Assuming the necessary smoothness constraints, the conventional measures of finite strain quantities can be calculated from the transformation constants.

4.4 *Time in progressive deformation*

To plot the deformation path, some sort of time unit has to be chosen. A record of true time is difficult to extract from geological information. Monotonously increasing scalar parameters like arc-length of growing fibers, radius of a growing porphyroblast, increasing strain axial ratio, or changing dip of a bed or foliation, can be correlated with the progress of deformation and provide dimensionless time-like parameter (Elliott 1972).

5. Interpretation of deformation paths in nature

Deformation paths can be interpreted by two different approaches: (**a**) *Direct observations*, in the field and in thin sections, of boudinage, development of folds, ductile shear zones, extension veins, formation of cleavage, trails of inclusions in minerals, pressure shadows, etc., and (**b**) *Computer simulation* of a structure by assuming all the material properties and boundary conditions. Theoretical methods can be used to model natural geological structures and to test hypothetical paths by imposing restrictions on the total number of deformation paths.

6. Deformation paths from ductile shear zones

Ductile shear zones with parallel boundaries develop under any combination of three types of displacement: heterogeneous simple shear, heterogeneous volume change and uniform homogeneous strain (Ramsay 1980). The finite strain matrix for these boundary conditions is given by

$$\mathbf{D} = \begin{bmatrix} 1 & -\gamma \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1+\Delta \end{bmatrix} \begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} p - r\,\gamma(1+\Delta) & q - s\,\gamma(1+\Delta) \\ r(1+\Delta) & s(1+\Delta) \end{bmatrix}. \tag{4}$$

For shear zones with undeformed walls, the above equation changes to

$$\mathbf{D} = \begin{bmatrix} 1 & -\gamma \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1+\Delta \end{bmatrix} = \begin{bmatrix} 1 & -\gamma(1+\Delta) \\ 0 & (1+\Delta) \end{bmatrix}. \tag{5}$$

Deformation path for an ideal ductile shear zone with undeformed walls can be calculated by substituting equation 5 in equation 3 (see Ramsay and Huber 1983, p. 217–234, for detail calculations). For steady state process, a constant proportion of incremental dilation (Δ_i) to incremental shear strain (γ_i) needs to be chosen. At the end of each increment, dilation and shear strain values can be computed from the finite strain matrix by using equation 5, and the relationship between both the parameters can be graphically analysed.

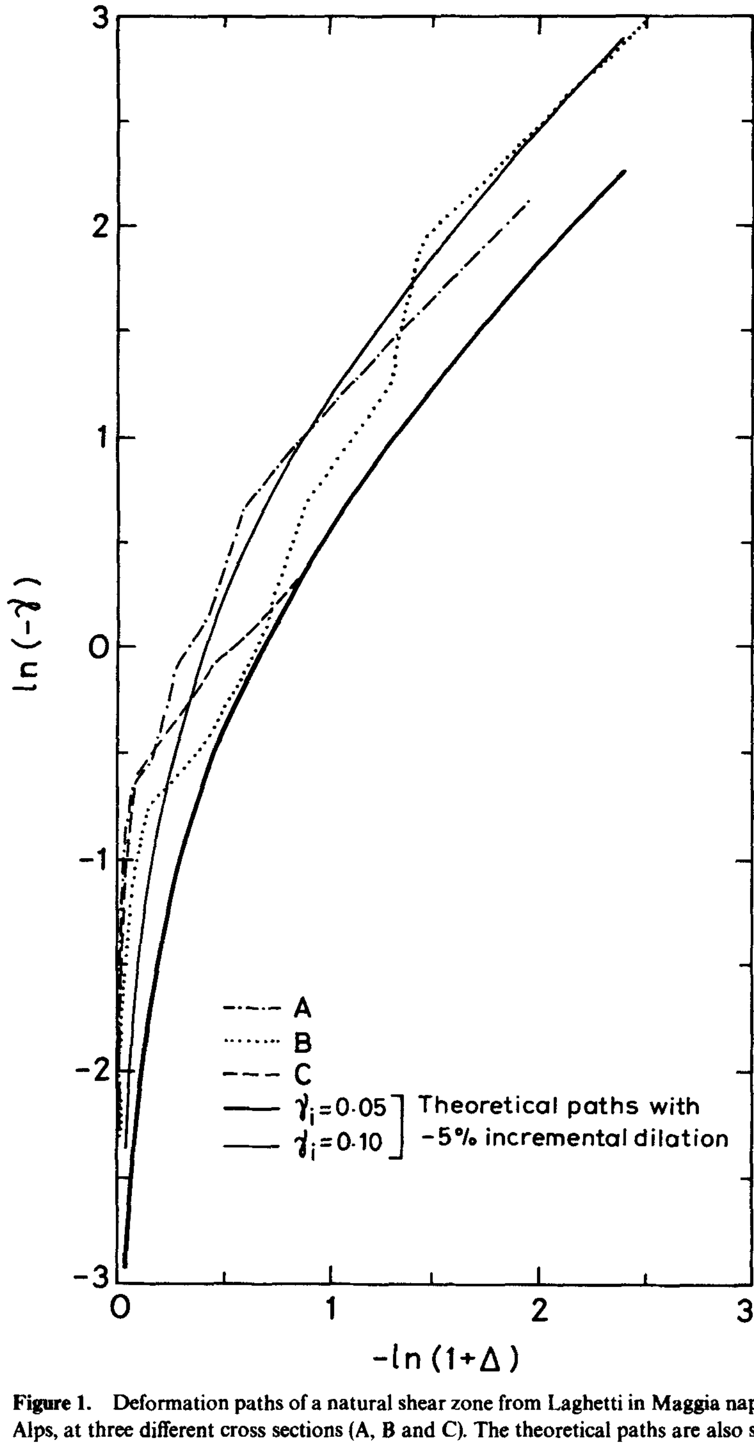

Figure 1. Deformation paths of a natural shear zone from Laghetti in Maggia nappe, Swiss Alps, at three different cross sections (A, B and C). The theoretical paths are also shown for comparison (taken from Mohanty and Ramsay 1994).

7. Case study

The principle outlined above was applied to the ductile shear zones in Hercynian granites in the Maggia nappe, a Lower Pennine nappe of the Swiss Alps (Mohanty and Ramsay 1994). Both heterogeneous simple shear and heterogeneous volume change played an important role in the formation of these shear zones. A comparative analysis of the theoretical paths with the strain data computed from field observations was made (figure 1). The natural path is parallel to the theoretical path at high strain region, indicating steady state process, but at low strain zone the natural path lies above the theoretical path and cuts across it, indicating non-steady state deformation path.

8. Conclusions

The concepts of deformation path provide a powerful mathematical tool for strain analysis in ductile shear zones. These concepts were applied in an area in the Pennine nappe of the Swiss Alps. A comparative study of theoretical paths and the natural results indicates constant shear strain rate but changing dilation rate during deformation.

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