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Spectral analysis of monsoonal global energy and enstrophy and their nonlinear exchanges in the zonal wavenumber domain

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Abstract. Vertical distributions of various components of the tropospheric global rotational kinetic energy, enstrophy and available potential energy during July 1979, and the contributions to these from different zonal wave categories were studied. Representative levels in the lower and upper troposphere for the stationary and transient energetics were identified on the basis of different components of energy and enstrophy. The eddy energy and enstrophy contained in different zonal scale components in the lower and upper troposphere were studied to find out the preferred scales for stationary and transient monsoonal motion in the two atmospheric layers. The role of different zonal wave categories in the nonlinear exchanges of energy and enstrophy arising due to stationary-stationary, transient-transient, stationary-transient and observed flow interactions was examined. Stationary and transient global spectra of the aforesaid dynamical variables in terms of the zonal wavenumber(m) with triangular truncation at $m = 42$ were utilized for this purpose.

It was found from the global average kinetic energy in lower and upper troposphere that the global stationary and transient motions were comparable in the lower troposphere while in the upper troposphere stationary motion dominated over the transient motion. The computed zonal and eddy energy confirmed that the stationary motion was predominantly zonal while the transient motion was dominated by eddies. From the time mean nonlinear interaction of kinetic energy (enstrophy) of observed flow it was seen that the long and short waves as well as the zonal flow gained kinetic energy (enstrophy) from medium waves due to nonlinear interactions. The transfer of available potential energy due to nonlinear interaction was down the scale except for short waves in the upper troposphere. The stationary-transient interaction was found to be an important element of the spatial-temporal varying atmospheric flow.

Keywords. Global energy and enstrophy spectra; nonlinear interactions; wave categories; representative levels.

1. Introduction

The global monsoonal circulation has natural variability on a wide spectrum of temporal and spatial scales and involves complex interaction of these scales. Diagnostic studies on variability of monsoonal energetics have a large impact on understanding and simulating the global monsoonal circulation. Studies related to the real atmosphere energetics in the spectral domain may prove helpful in monitoring the model performance in a better way and in identifying some aspects of physical processes to be included in a spectral model.

There have been several spectral energetics studies where Fourier expansions of data at various latitudes were employed. Saltzman (1957) derived equations of energy cycle in the zonal wavenumber domain. Subsequent studies on energetics in terms of the zonal wavenumber were carried out by Saltzman (1970), Kanamitsu *et al* (1972), Krishnamurti *et al* (1973), Murakami (1981), Awade *et al* (1982), and Chakraborty and Mishra (1993). All these studies pertained to selected latitudinal belts over the tropics.

Kubota (1959) used spherical harmonics to represent the large range of spatial scales which are contained in the global atmosphere and derived the available potential energy – kinetic energy budget equations. Baer (1972) suggested a two-dimensional spectral index given by the order of the Legendre polynomials to represent the two-dimensional atmospheric flow. Statistics of time averaged kinetic energy in terms of this index was presented by Baer (1972) and for available potential energy by Baer (1974). These studies and the studies by Burrows (1976), Chen and Wiin-Nielsen (1978) were carried out with northern hemispheric data to compute the energy and enstrophy spectra in terms of the two-dimensional wavenumber. Global calculation in terms of the two-dimensional wavenumber have appeared in Lambert (1981, 1984). Boer and Shepherd (1983), Shepherd (1987) and, Desai and Mishra (1993). Most of these studies were aimed either to study the turbulent behaviour of the tropospheric atmosphere or to study the energy budget in terms of the two-dimensional wavenumber.

The monsoonal circulation which involves a large range of zonal scales of motion is a global phenomenon. The global energetics, therefore, need to be computed in terms of the zonal wavenumber and analysed in details to understand the monsoonal circulation. We intend to study the vertical distributions of:

- the global averaged rotational kinetic energy, available potential energy and enstrophy of the stationary and transient atmospheric flow in the troposphere during the mid-monsoon month of July 1979 and;
- the contributions to these from different zonal wave categories. The nonlinear exchanges of energy and enstrophy among the stationary as well as transient waves, which are important terms of the energetics, are separated out. The stationary-transient nonlinear transfers of these quantities are estimated by residue method. The role of different zonal wave categories in the global energy and enstrophy and their nonlinear interactions are studied.

2. Spectral expressions for energy and enstrophy

The expressions for global averaged rotational kinetic energy, enstrophy and available potential energy in physical space and their transformation to the two-dimensional spectral domain have been discussed in our preceding paper. (Desai and Mishra 1993; hereafter referred as DM). The spectral expressions for these are repeated here for convenience.

$$K = \sum_{n=0}^{M} \sum_{m=0}^{n} K_n^m = \sum_{n=0}^{M} \sum_{m=0}^{n} n(n+1)(2-\delta_{m0})(\psi C_n^{m2} + \psi S_n^{m2})/8a^2, \quad \text{(1a)}$$

$$EN = \sum_{n=0}^{M} \sum_{m=0}^{n} EN_n^m = \sum_{n=0}^{M} \sum_{m=0}^{n} n^2(n+1)^2(2-\delta_{m0})(\psi C_n^{m2} + \psi S_n^{m2})/8a^4 \quad \text{(1b)}$$

$$A = \sum_{n=0}^{M} \sum_{m=0}^{n} A_n^m = \sum_{n=0}^{M} \sum_{m=0}^{n} g(2-\delta_{m0})(TC_n^{m2} + TS_n^{m2})/8\sigma. \quad \text{(1c)}$$

The notations of these equations and of the equations appearing elsewhere in the paper are listed in the appendix. Here, $TC_0^0 = 0$. On summing up right hand sides of 1(a) to 1(c) over the two-dimensional wavenumber (n), we get

$$K = \sum_{m=0}^{M} K_m = \sum_{m=0}^{M} \left(\sum_{n=m}^{M} n(n+1)(2-\delta_{m0})(\psi C_n^{m^2} + \psi S_n^{m^2})/8a^2 \right), \tag{2a}$$

$$EN = \sum_{m=0}^{M} EN_m = \sum_{m=0}^{M} \left(\sum_{n=m}^{M} n^2(n+1)^2(2-\delta_{m0})(\psi C_n^{m^2} + \psi S_n^{m^2})/8a^4 \right), \tag{2b}$$

$$A = \sum_{m=0}^{M} A_m = \sum_{m=0}^{M} \left(\sum_{n=m}^{M} g(2-\delta_{m0})(TC_n^{m^2} + TS_n^{m^2})/8\sigma \right). \tag{2c}$$

We split the zonal spectra to separate out the zonal and eddy parts of the global energy and enstrophy. Thus,

$$K = K_0 + \sum_{m=1}^{M} K_m = KZ + KE, \tag{3a}$$

$$EN = EN_0 + \sum_{m=1}^{M} EN_m = ENZ + ENE, \tag{3b}$$

$$A = A_0 + \sum_{m=1}^{M} A_m = AZ + AE, \tag{3c}$$

The divergent kinetic energy components were estimated using the following expression

$$K = \sum_{m=0}^{M} K_m = \sum_{m=0}^{M} \left(\sum_{n=m}^{M} n(n+1)(2-\delta_{m0})(\chi C_n^{m^2} + \chi S_n^{m^2})/8a^2 \right).$$

Spectral expressions for nonlinear exchanges of rotational kinetic energy, enstrophy and available potential energy in the two dimensional wavenumber domain as derived in DM are

$$(K_n^m)_t = (2-\delta_{m0})(\psi C_n^m IC_n^m + \psi S_n^m IS_n^m)/4, \tag{4a}$$

$$(EN_n^m)_t = n(n+1)(2-\delta_{m0})(\psi C_n^m IC_n^m + \psi S_n^m IS_n^m)/4a^2, \tag{4b}$$

$$(A_n^m)_t = -(2-\delta_{m0})g(TC_n^m JC_n^m + TS_n^m JS_n^m)/4\sigma. \tag{4c}$$

The M equations of K, EN and A in the zonal wavenumber domain are obtained by summing up (4a) to (4c) for each m over all n. Thus,

$$(K_m)_t = \sum_{n=m}^{M} (2-\delta_{m0})(\psi C_n^m IC_n^m + \psi S_n^m IS_n^m)/4, \tag{5a}$$

$$(EN_m)_t = \sum_{n=m}^{M} (2-\delta_{m0})n(n+1)(\psi C_n^m IC_n^m + \psi S_n^m IS_n^m)/4a^2, \tag{5b}$$

$$(A_m)_t = -\sum_{n=m}^{M} (2-\delta_{m0})g(TC_n^m JC_n^m + TS_n^m JS_n^m)/4\sigma. \tag{5c}$$

The terms on the right hand sides of these equations represent the gain of K, EN and A by the waves corresponding to wavenumber m due to wave-wave interaction and their interaction with zonal flow. Here, $m=0$ represents the net interaction of the zonal flow with all waves put together. (IC_n^m, IS_n^m) and (JC_n^m, JS_n^m) are the spherical harmonic coefficients of the vorticity and temperature advection terms. The vorticity and thermodynamic equations, definitions of I and J, and the numerical method for the spectral representation of these advection terms are described in DM.

A positive value of nonlinear interaction indicates that energy or enstrophy is being transferred to that wavenumber from other wavenumbers. It may be mentioned here that the nonlinear interaction term may be thought of as being composed of two parts, one of which can be calculated from the data and represents the 'resolved' part of the interaction. The second part is unavailable from the data and may represent the effect of scales of motion which are not contained in the data, on the resolved scales (Boer and Shepherd 1983). In this study the resolved part of the interaction is estimated.

Taking into consideration the continuous interactions of the stationary and transient eddies with eddies of their own type as well as with those of other types, the nonlinear exchanges of energy and enstrophy are shown as sum of the stationary-stationary, transient-transient and stationary-transient components (Desai 1993). Hence,

$$NK_n^m = NKS_n^m + NKT_n^m + NKST_n^m, \tag{6a}$$

$$NEN_n^m = NENS_n^m + NENT_n^m + NENST_n^m, \tag{6b}$$

$$NA_n^m = NAS_n^m + NAT_n^m + NAST_n^m, \tag{6c}$$

where the 1st terms on the right hand sides are the interactions among stationary eddies (stationary-stationary component), 2nd terms are interactions among transient eddies (transient-transient component) and the 3rd terms represent the interactions between the stationary and transient eddies (stationary-transient component). It is worthwhile to mention here that each of the terms on the right hand sides is a true interaction term in the sense that it only redistributes energy or enstrophy among various wavenumbers (Shepherd 1987).

3. Data and the spectral representation

Spherical harmonic coefficients of streamfunction, velocity potential and temperature which represent the stationary and transient components of global horizontal wind field and the temperature field on 10 pressure surfaces (i.e. 1000, 850, 700, 500, 400, 300, 250, 200, 150 and 100 hPa) during July 1979 have been utilized for this study. These were computed from the FGGE IIIb level global grid point analyses of wind and geopotential height provided by ECMWF, U.K. The set of equations to transform the variables between physical and spectral domains, the procedure for spherical harmonic analysis and the numerical methods in the computation of spectral coefficients and the static stability parameter were discussed in detail in DM. The stationary and transient spectra of energy and enstrophy and their nonlinear interactions in terms of the zonal wavenumber (m) were computed using triangular truncation at $M = 42$.

4. Results

The energy and enstrophy spectra were averaged over the lower (1000–500) hPa and upper (500–150) hPa troposphere using trapezoidal rule. The nonlinear exchanges of energy and enstrophy were integrated over a column of unit horizontal area in these layers. The zonal waves were grouped together into the following wave categories depending on their spatial scales. (i) long waves ($1 \leqslant m \leqslant 3$) (ii) medium waves ($4 \leqslant m \leqslant 15$) (iii) short waves ($16 \leqslant m \leqslant 30$) and (iv) extra-short waves ($31 \leqslant m \leqslant 42$). Monsoon belt in this case was considered as the global latitudinal belt between 10S and 30N.

4.1 *Zonal and eddy energy and enstrophy in the troposphere*

4.1a *Lower and upper tropospheric energy and enstrophy*

Global averaged divergent kinetic energy: The global averaged kinetic energy of divergent motion was compared with that of rotational motion. For the comparison, the motion was partitioned into stationary and transient components and each component was further split into zonal and eddy parts separately (values are not presented). It was found that the ratio of divergent to rotational kinetic energy was maximum for the lower tropospheric stationary eddy motion with a value of 9·5% when expressed as percentage. It may be noted that the divergent part accounted for a maximum of 4% of transient eddy kinetic energy in the lower troposphere. The comparison between divergent and rotational kinetic energy suggested that the main features of atmospheric motion can be well depicted by the rotational component. It is well known that the orography (mechanical forcing) and heating (thermal forcing) are mainly responsible for the generation and maintenance of the mean atmospheric circulation. The atmosphere responds to the mechanical and thermal forcings mainly through the vertical motions, which is the manifestation of horizontal divergent motion. A small value of divergent kinetic energy indicates that its resident time is small and its transfer to the rotational kinetic energy is rather fast. The transient motion in the tropics and mid-latitudes is maintained by the transfer of divergent to rotational kinetic energy. It may be pointed out that the boundary layer contributed significantly towards divergent kinetic energy of the lower troposphere. In the monsoon belt, the ratios for eddy motion in the lower and upper troposphere were comparable. The ratios in percentage for stationary and transient eddy in the lower troposphere were 12·5% and 11% respectively. In our further discussion, we have neglected divergent parts and considered only the rotational part of kinetic energy.

Global averaged rotational kinetic and available potential energy: The zonal and eddy rotational kinetic energy for stationary and transient motion in the lower and upper troposphere were examined (Table 1). The value in the upper troposphere was about 4 times to that in the lower troposphere. Further, in the lower troposphere the stationary and transient kinetic energy was comparable but in the upper troposphere

Table 1. Global averaged rotational kinetic and available potential energy (Unit: Joules/kg).

Layer	Rotational kinetic energy				Available potential energy			
	KZ_{st}	KE_{st}	KZ_{Tr}	KE_{Tr}	AZ_{st}	AE_{st}	AZ_{Tr}	AE_{Tr}
(1000–500)hPa	23·59	9·45	1·52	27·54	503·77	21·86	1·81	21·98
(500–150)hPa	121·60	19·16	3·65	76·30	606·30	21·91	2·94	35·42

the stationary component dominated over the transient component. The stationary motion was predominantly zonal. This was more true for the upper than the lower troposphere. The zonal component (KZ_{St}) accounted for 71·4% (86·4%) of stationary rotational kinetic energy in the lower (upper) troposphere. This global picture of stationary motion does not hold good for the monsoon belt, wherein domination of zonal component is weakened considerably in the lower troposphere. In the monsoon belt, KZ_{St} was 40% (61·8%) of stationary kinetic energy in the lower (upper) troposphere (Chakraborty and Mishra 1993). For global transient motion, the kinetic energy was 29(80) Jkg^{-1} in the lower (upper) troposphere and more than 94% of it was contained in the eddy component. Thus, the transient motion was dominated by eddies. Since KE_{Tr} was three times of KE_{St}, it can also be concluded that the global eddy motion was dominated by transient part. Transient kinetic energy for the monsoon belt in the lower as well as in the upper troposphere was only one third of the global average value but dominated by the eddy component. In the monsoon belt, KE_{St} and KE_{Tr} in the lower (upper) troposphere were 7·2(5·2) Jkg^{-1} and 6·6(8·1) Jkg^{-1} respectively. These values clearly indicate that the eddy motion in the monsoon belt, received nearly comparable contribution from transient and stationary parts.

Stationary available potential energy (APE) in lower and upper tropospheres was 525·5 Jkg^{-1} and 628 JKg^{-1} respectively (Table 1). Stationary zonal available potential energy (AZ_{St}) accounted for more than 95% of APE in both lower and upper tropospheres. Transient APE was 23·8 (38·4) Jkg^{-1} in the lower (upper) troposphere and eddy component accounted for about 90% in both the layers.

A comparison of different components of kinetic and available potential energy in the lower troposphere with the corresponding values at 700 hPa level revealed that the two sets of values were sufficiently close to each other. A similar comparison of upper troposphere and 200 hPa level values indicated that the kinetic (potential) energy values were close to the largest (smallest) values attained in the upper troposphere. The representative level for stationary motion in the upper troposphere was found to be 300 hPa and the level for transient motion was close to 400 hPa. Further, it was seen that around 75% (70%) of KE_{St} was contained in the long waves in the lower (upper) troposphere. Thus, zonal flow and long waves together accounted for more than 90% of stationary rotational kinetic energy (KE) in the troposphere.

4.1b *Vertical distribution of energy and enstrophy*

Vertical distribution of various components of kinetic energy (figure 1) can be grossly understood in terms of a physical system where main dissipation of motion takes place in the lower troposphere and the main source of transient kinetic energy, in

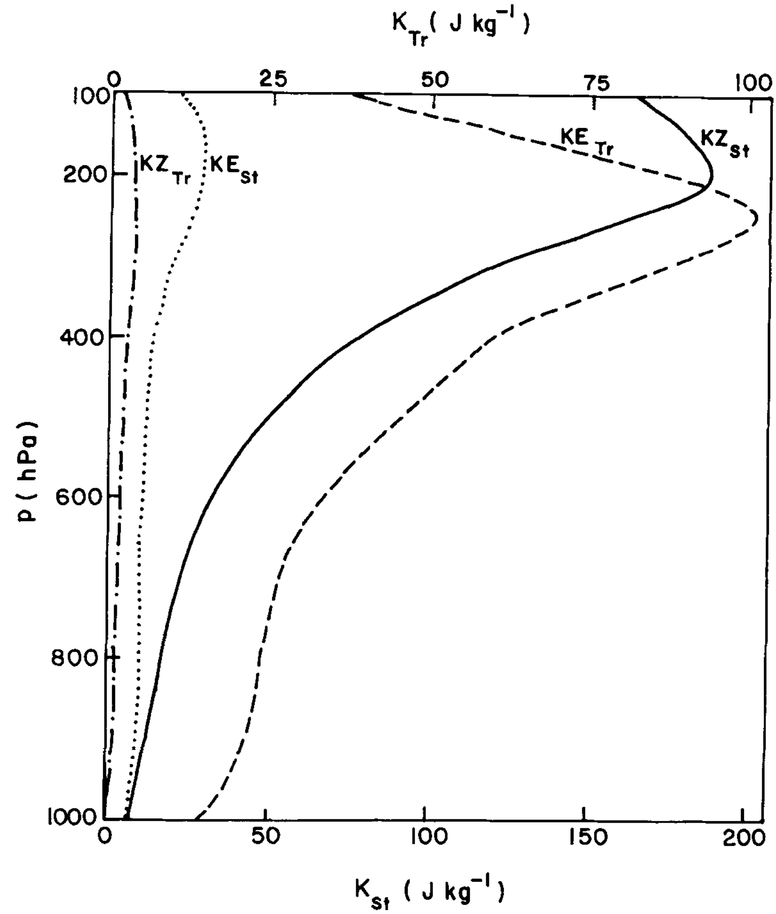

Figure 1. Vertical profiles of global averaged stationary (*St*) and transient (*Tr*) zonal (*KZ*) and eddy (*KE*) rotational kinetic energy during July 1979.

the form of dynamic instability of stationary zonal flow is located in the upper troposphere.

The transient eddy kinetic energy (KE_{Tr}) attained a maximum value at 250 hPa below the tropopause and also below the level of maximum stationary zonal kinetic energy (KZ_{St}) i.e. 200 hPa which is the level of mid-latitude westerly jet in summer (figure 1). KE_{St} attained maximum value at 150 hPa level. The vertical variation of AZ_{St} and AE_{Tr} was similar in the layer (1000–250) hPa (figure 2). AZ_{St} was about 20 times as that of AE_{Tr}. AE_{Tr} distribution exhibited primary maximum at 400 hPa and secondary maxima at 700 and 150 hPa levels. The location of maximum KE_{Tr} above the level of primary maximum of AE_{Tr} can be qualitatively understood from the results for computed energetics of numerically simulated northern hemispheric atmosphere using a general circulation model by Smagorinsky *et al* (1965). AZ_{St} acts as a source of energy for AE_{Tr} which is achieved by the meridional transport of sensible heat down the zonal averaged temperature gradient. AE_{Tr} is converted to KE_{Tr} following a vertical distribution close to itself. KE_{Tr} is transferred upward to the level of the jet stream and downward to the level of surface boundary layer by

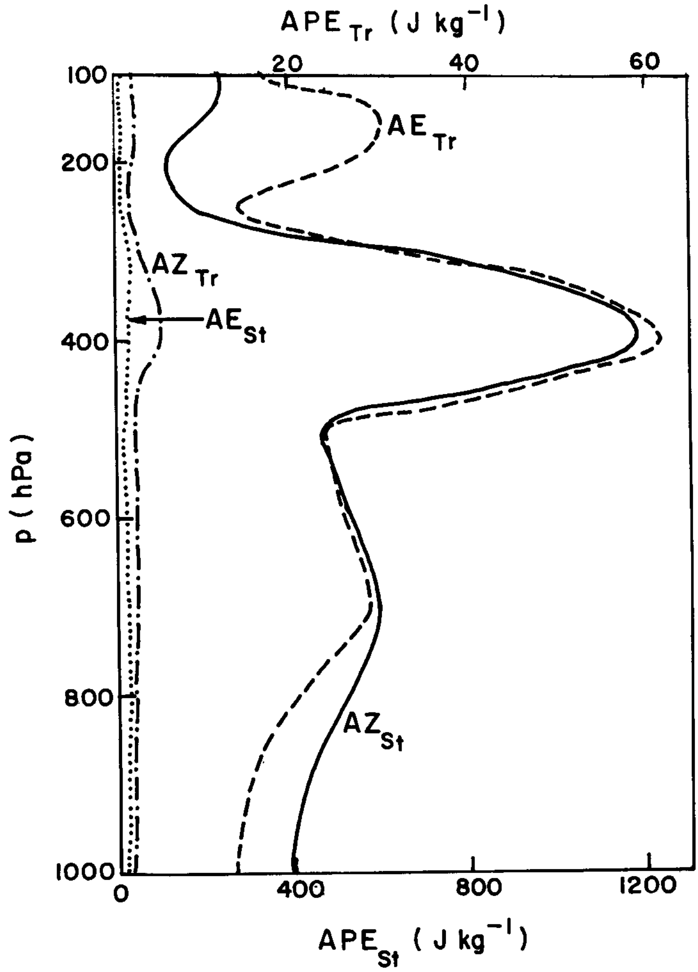

Figure 2. Same as figure 1 but for available potential energy.

the eddy pressure interaction mechanism. The double maxima in the vertical distribution of KE_{Tr} is expected on the basis of the above consideration. The lower maximum of KE_{Tr} is not seen because the eddy kinetic energy dissipation in the boundary layer is nearly one order more than that of the upper layer.

It was seen from the tropospheric distribution of the zonal and eddy parts of stationary and transient enstrophy that the transient eddy enstrophy dominated over all the other components and its value exceeded the other components by about one order of magnitude. All the four components attained their maximum at 250 hPa which is also the level of maximum KE_{Tr}. Thus, the enstrophy can be described as a transient eddy like parameter.

4.1c *Eddy energy and enstrophy in different zonal scale components*

To describe the energy in different zonal scale components as a function of zonal wavenumber, the energy values are plotted against the zonal wavenumber on log–log

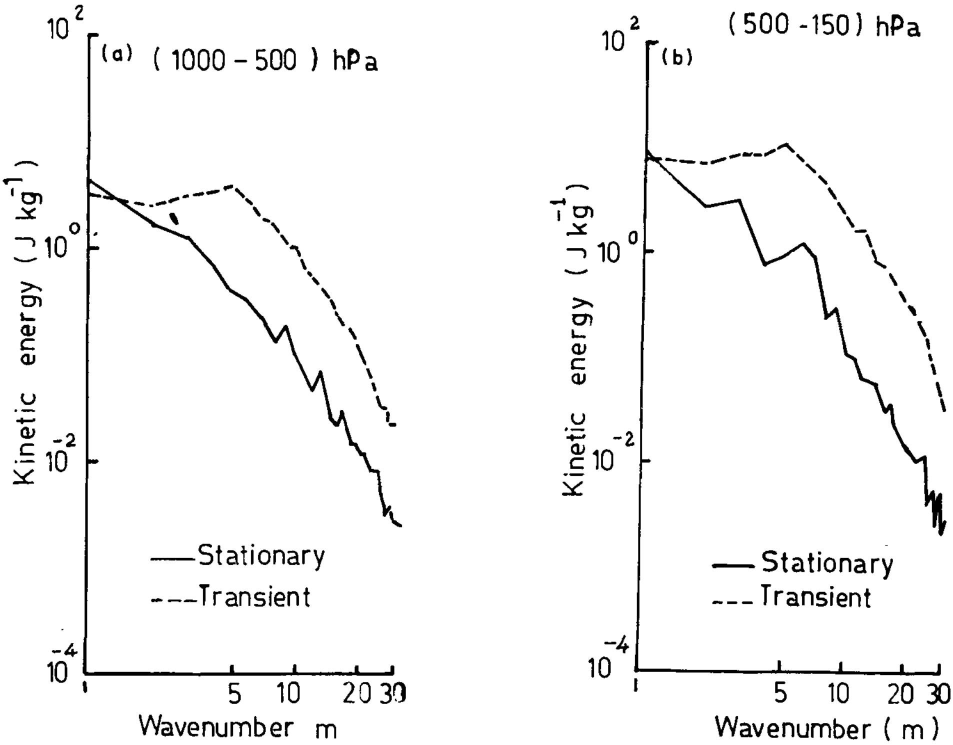

Figure 3. Zonal spectra of global averaged stationary and transient kinetic energy during July 1979 in the a) lower troposphere b) upper troposphere.

scale. The stationary and transient components of kinetic energy in the lower and upper troposphere are presented in figures 3a and 3b. For $m = 1$, the transient KE was slightly less than the stationary KE in the lower and upper troposphere. For waves with $m > 2$, the transient KE was more than the stationary KE in the lower and upper troposphere. The transient KE had broad peak in the wavenumber domain $1 \leqslant m \leqslant 5$ while the stationary KE has its peak at $m = 1$. The magnitudes of the two components were comparable for $1 \leqslant m \leqslant 3$ in both the atmospheric layers.

The log-log scale plot of stationary and transient available potential energy in the lower and upper troposphere as a function of zonal wavenumber is presented in figures 4a and 4b. The stationary AE in long waves with $m = 1$ and 2 was found to be more than the transient AE in the lower and upper troposphere. In the wavenumber region $4 \leqslant m \leqslant 23$, the transient component was more in both the layers. Beyond $m > 23$ both components were comparable in the lower troposphere while in the upper troposphere the transient component continued to be stronger even beyond $m > 23$.

The transient enstrophy had broad peak in the wavenumber domain $2 \leqslant m \leqslant 5$ and was maximum at $m = 5$ in the lower and upper troposphere. The stationary enstrophy was maximum for $m = 1$ in both the atmospheric layers. Further, the transient enstrophy was more than stationary enstrophy for all the zonal scales (figure not presented).

Thus, it can be concluded from the energy and enstrophy distribution among

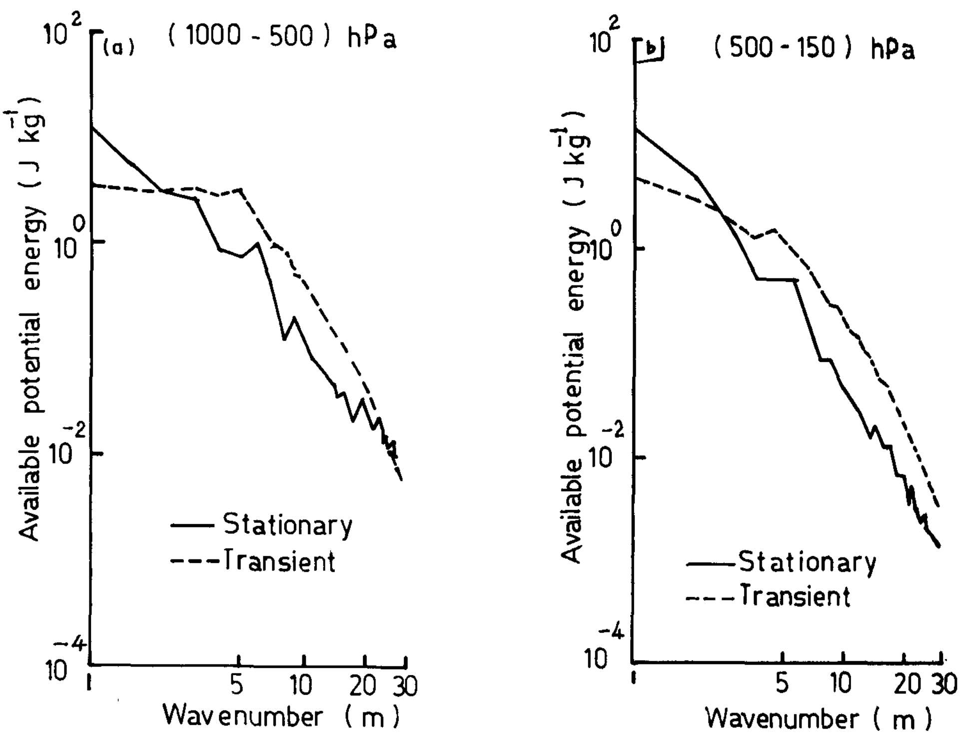

Figure 4. (a) and (b) are same as figures 3(a) and (b) but for available potential energy.

various zonal components that the planetary scale wave of wavenumber 1 represents the preferred zonal scale for stationary monsoonal motion in the lower and upper troposphere over the entire globe. For global transient motion the preferred zonal scale was found to vary between wavenumbers 2 to 5 from upper to lower troposphere. The study needs to be repeated for different monsoon seasons to determine the most preferred scale for transient motion.

4.1d *Energy and enstrophy in zonal wave groups – its tropospheric distribution*

Vertical distributions of eddy kinetic and available potential energy for long, medium and short waves are presented in figures 5 and 6. The short waves contained less than 1% and 4% of KE_{st} and KE_{Tr} respectively, in the troposphere. In case of AE the contribution of the short waves did not exceed 2%. The long waves contained more than ˜0% of KE_{st}, while the medium waves accounted for about 64% of KE_{Tr}. In case of available potential energy, the domination of long waves was increased to 86% for stationary eddy motion, while that of medium waves decreased to 52% in transient eddy motion. The vertical level of maximum stationary eddy kinetic energy was seen to shift downward from 150 hPa for the long waves to 250 hPa for short waves. This is in agreement with the theoretical results that the response of the atmosphere is dependent on the horizontal wavelength of forcings. Asnani and Mishra (1975) have shown that wavelengths of the order 15,000 km produce maximum geopotential variations around 150 hPa level and for wavelengths of order 5000 km

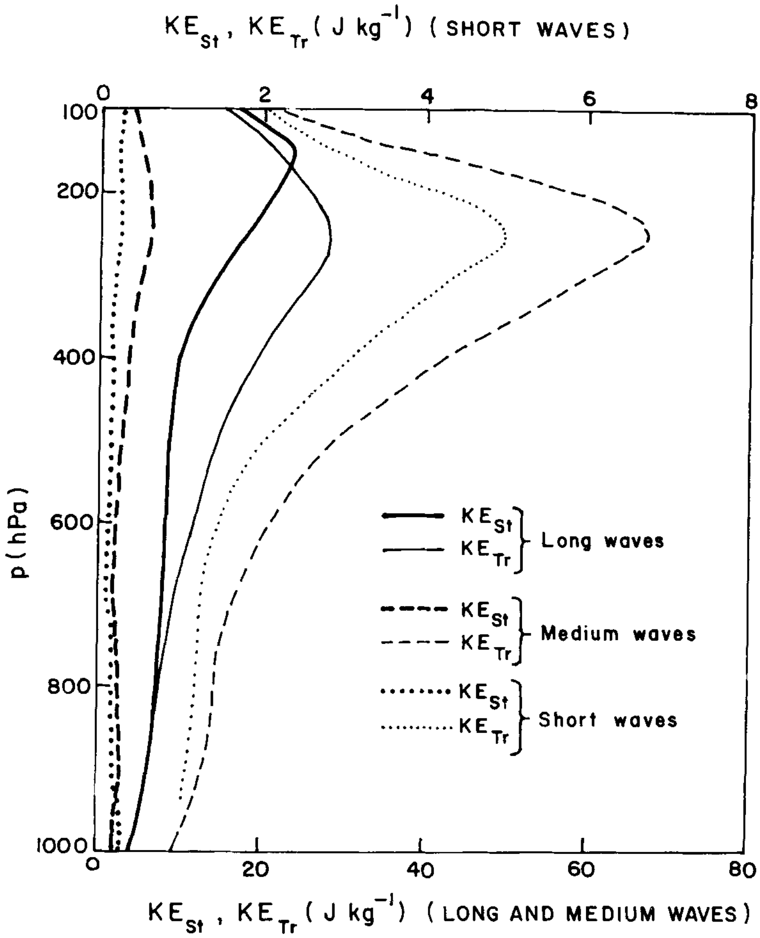

Figure 5. Stationary and transient global averaged rotational kinetic energy of long, medium and short waves during July 1979 shown as function of pressure.

the level of maximum variations descend to the 300 hPa level. The feature that the level of maximum eddy energy descends with the decrease of wavelength was also shown by stationary available potential energy, where the level of maximum were 350 hPa and 400 hPa for the long and medium waves, respectively. The transient eddy energy for all categories attained maximum value at the same level of 250 hPa (400 hPa) for kinetic (available potential) energy.

The basic nature of enstrophy parameter is such that the relative contribution of zonal and long waves decreases while the contribution of short waves is enhanced compared to the values for rotational kinetic energy. As expected, the short waves accounted for about 10% (15%) of stationary (transient) eddy enstrophy in the troposphere. In stationary enstrophy it was not the zonal component but the eddy component which dominated unlike the case with kinetic energy. It is important to note that not only the transient but also the stationary eddy enstrophy in all three categories attained their maximum values at the same level at 250 hPa (vertical distribution of enstrophy is not presented).

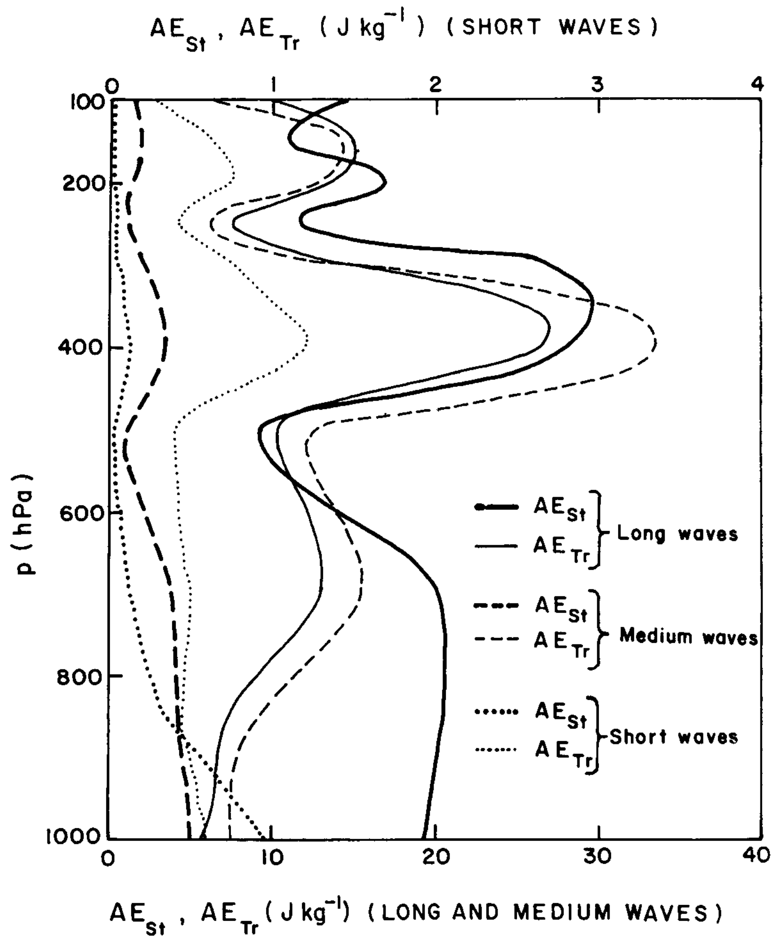

Figure 6. Same as figure 6 but for available potential energy.

It has already been pointed out that the stationary motion was dominated by zonal component and transient motion by eddy component. This alongwith the vertical distributions of *KE* and *APE* for different wave categories indicated that the wave truncation at zonal wavenumber 15 is adequate to depict to a large extent the major features of global atmospheric circulation.

4.2 *Nonlinear interactions of energy and enstrophy*

Time mean nonlinear interaction of kinetic energy (enstrophy) for different zonal wave categories computed from its zonal wavenumber representation is presented in table 2 (table 3). The stationary-stationary and transient-transient nonlinear interactions are also presented in the table. It is seen from the table that the nonlinear interaction for extra-short waves can be neglected in comparison to the other wave categories. Therefore a spectral model with zonal wavenumber truncation at 30 on the average can adequately describe the nonlinear interactions of kinetic energy in the troposphere. The medium waves were found to lose kinetic energy (enstrophy), while the long and

Table 2. Nonlinear interactions of kinetic energy in different zonal wave categories (Unit: $10^{-3} Wm^{-2}$).

Zonal wave no./ Layer	Combined					Stationary					Transient				
	$m=0$	1–3	4–15	16–30	31–42	$m=0$	1–3	4–15	16–30	31–42	$m=0$	1–3	4–15	16–30	31–42
1000–500 hPa	118·2	14·38	−138·8	5·26	0·96	27·74	−23·01	−4·59	−·02	−·12	−3·22	18·23	−24·88	9·18	0·70
500–150 hPa	234·2	47·61	−311·1	26·0	3·24	53·23	−37·53	−15·05	−·63	−·02	17·34	34·92	−88·83	33·82	2·76

Table 3. Nonlinear interactions of enstrophy in different zonal wave categories (Unit: $10^{-14} kg\,m^{-2} s^{-3}$).

Zonal wave no./ Layer	Combined					Stationary					Transient				
	$m=0$	1–3	4–15	16–30	31–42	$m=0$	1–3	4–15	16–30	31–42	$m=0$	1–3	4–15	16–30	31–42
1000–500 hPa	7·48	−1·19	−45·51	35·10	4·12	1·71	−3·68	1·49	0·83	−0·36	·9988	−4·85	−29·20	32·23	2·82
500–150 hPa	4·57	−9·40	−117·0	108·2	13·56	2·86	−2·99	−0·28	0·34	0·07	5·93	−19·92	−88·55	91·75	10·79

short waves as well as zonal flow gained kinetic energy (enstrophy) due to the nonlinear interactions. The gain of kinetic energy (enstrophy) by zonal flow ($m = 0$) and long waves was more (less) than that of short waves. It can be concluded that it was the nonlinear interaction which contributed towards maintenance of long waves. Further, it can also be said that it was eddies through zonal-wave interaction which were mainly responsible for maintenance of zonal flow. It is interesting to note that in the tropics also the zonal-wave interaction contributed towards the maintenance of zonal motion (Chakraborty and Mishra 1993). The picture of nonlinear exchanges of kinetic energy between medium waves to long and short waves as emerged in this study is in agreement with the theory developed by Fjortoft (1953) for non-divergent two-dimensional flow in two-dimensional wavenumber domain.

As expected from the theory, the zonal-wave interaction of kinetic energy (enstrophy) was stronger (weaker) compared to the wave-wave interaction. It was also noted that the nonlinear interaction of kinetic energy and enstrophy was much stronger in the upper troposphere than in the lower troposphere.

The nonlinear interaction among stationary zonal waves transferred the kinetic energy up the scale. All resolved waves had been losing kinetic energy to the zonal flow. It seems that short and extra-short waves did not play any important role in the stationary nonlinear interaction of kinetic energy. It was zonal-wave interaction between zonal flow ($m = 0$) and the stationary long waves which dominated the stationary nonlinear interaction. An inspection of transient nonlinear interactions for different zonal wave categories indicated that the loss of kinetic energy by medium transient waves was nearly balanced by equal gain by long and short transient waves. Thus, the nonlinear interaction among transient zonal waves was quite different from that among stationary zonal waves. The transfer of energy due to transient and stationary nonlinear interaction in zonal space deviated considerably from the Fjortoft's theory in two-dimensional wavenumber space.

The computation of stationary-transient nonlinear interaction for kinetic energy was based on the fact that the nonlinear interaction is the sum of stationary, transient and stationary-transient interactions. The stationary-transient interaction for different zonal-wave categories is presented in table 4. It is seen that zonal-wave interaction and wave-wave interaction for medium waves dominated the stationary-transient interactions. The medium and short waves lost kinetic energy. The former loss was about 25 times to that of the latter. The zonal and long waves gained kinetic energy. The former gain was more than the latter due to stationary-transient nonlinear interaction.

An examination of tables 2 and 4 leads to the following characteristics of the nonlinear interaction of kinetic energy in the troposphere during monsoon. The

Table 4. Stationary-transient nonlinear interaction of kinetic energy for different zonal wave categories (Unit: $10^{-3}\,Wm^{-2}$).

Zonal wave no./ Layer	$m = 0$	1–3	4–15	16–30	31–42
(1000–500) hPa	93·68	19·16	−109·33	−3·9	0·38
(500–150) hPa	163·63	50·22	−207·22	−7·19	0·5

stationary-transient interaction dominated for the medium waves as well as for zonal flow. In addition to stationary-transient interaction, the transient interaction was significant for the medium waves. The transient interaction was the most important source of kinetic energy for short waves. For long waves all the three interactions were equally significant in controlling their dynamics. The transient and stationary-transient interactions acted as a source while stationary interaction as a sink of kinetic energy.

It can be inferred from a comparison between stationary and transient nonlinear interaction of enstrophy (table 3) that the former was much weaker than the latter for all wave categories, even for the zonal flow ($m = 0$) and long waves. This was in contrast to the case of kinetic energy where the stationary nonlinear interaction for the zonal and long waves dominated over the transient interaction. This contrasting behaviour between enstrophy and kinetic energy transfer due to nonlinear interaction may be linked to the fact that the transient percentage of enstrophy was more than 80% whereas the transient kinetic energy percentage was below 50%. Further, the transient long and medium waves lost enstrophy while the short and extra-short waves gained enstrophy due to nonlinear interaction. Once again it was noticed that the transient component of motion behaves differently from the combined (stationary + transient) motion with respect to the nonlinear interactions. The stationary-transient nonlinear interaction of enstrophy (table 5) was weaker than the transient interaction. The medium waves lost enstrophy while the gain of enstrophy by long and short waves was nearly equal.

The nonlinear interactions of available potential energy of combined (stationary + transient), stationary and transient flows for different zonal wave categories are shown in table 6. The zonal-wave interaction ($m = 0$) dominated over nonlinear interactions for all the wave categories. The zonal-wave interaction maintained the eddy available potential energy. The rate of transfer of kinetic energy to zonal component through zonal-wave interaction (table 2) was much less than the rate of transfer of available potential energy away from the zonal component. Thus, there is a need for generation of zonal available potential energy.

It can be concluded from the table that the nonlinear interaction of APE is significant upto medium waves. Retaining the waves in the zonal wavenumber intervals $0 \leqslant m \leqslant 15$ is adequate for the purpose of describing the nonlinear interaction of APE.

The transfer of available potential energy due to nonlinear interaction was down the scale, except for the extra-short waves in the upper troposphere. The zonal mode was losing available potential energy. The maximum gain was noticed for the medium waves. It is interesting to note that the nonlinear interaction was much stronger in

Table 5. Stationary-transient nonlinear interaction of enstrophy in different zonal wave categories (Unit: 10^{-14} kg m^{-2} s^{-3}).

Zonal wave no./ Layer	$m=0$	1–3	4–15	16–30	31–42
(1000–500)hPa	6·76	7·34	−17·80	2·04	1·66
(500–150)hPa	−4·22	13·51	−28·17	16·11	2·7

Table 6. Nonlinear interactions of APE in different zonal wave categories (Unit: $10^{-3}\,Wm^{-2}$).

Zonal wave no./ Layer	Combined					Stationary					Transient				
	$m=0$	1–3	4–15	16–30	31–42	$m=0$	1–3	4–15	16–30	31–42	$m=0$	1–3	4–15	16–30	31–42
1000–500 hPa	−847·6	185·8	643·2	17·7	0·88	−112·2	32·87	78·43	0·62	0·28	−0·02	1·15	−10·7	9·16	0·42
500–150 hPa	−459·5	97·4	360·8	1·37	−0·12	−54·95	25·3	28·61	0·97	0·08	−9·65	−6·24	9·50	6·09	0·3

Table 7. Stationary-transient nonlinear interaction of available potential energy for different zonal wave categories (Unit: 10^{-3} Wm^{-2}).

Zonal wave no./layer	$m=0$	1–3	4–15	16–30	31–42
(1000–500)hPa	−735·38	151·78	575·47	7·92	0·18
(500–150)hPa	−394·9	78·34	322·69	−5·69	−0·5

the lower troposphere compared to upper troposphere. This was in contrast to behaviour noticed in case of kinetic energy. In this connection it may be mentioned that the eddy kinetic (available potential) energy was more (less) in the upper troposphere compared to the lower troposphere.

The nonlinear interaction of available potential energy was stronger in stationary component compared to transient component of the flow. The cascade of available potential energy down the scale was seen more clearly in the stationary component. It may be noted that the waves in the upper and lower troposphere had in general the same direction of energy and enstrophy transfer due to nonlinear interactions except for transient nonlinear interaction of available potential energy.

The stationary-transient interaction in all wave categories dominated over the stationary and transient interactions for available potential energy while transient interaction could be neglected in comparison to the other two nonlinear interactions (table 7). It seems that the available potential energy of medium transient waves was maintained by the stationary-transient zonal-wave interaction.

It may be mentioned that the past studies have indicated that the mid-latitude synoptic scale transient eddies are generated as a result of baroclinic instability of stationary zonal flow. Further, the kinetic energy of transient eddies is provided by the conversion from APE on the same scale. It is well known, that the mid-latitude westerly jet is maintained by synoptic scale transient eddies. On the basis of the above, it can be said that the stationary-transient component of zonal-wave interaction of APE is the source of energy to the medium waves and kinetic energy of stationary zonal motion is maintained by stationary-transient component of zonal-wave interaction of *KE*. The same picture of energy transfer has emerged in the study. Further, it can be concluded that the stationary-transient interaction is an important element of the spatial-temporal varying atmospheric flow.

5. Conclusions

It was found from the global average kinetic energy in lower and upper troposphere that the stationary and transient motions were comparable in lower troposphere while in the upper troposphere stationary motion dominated over the transient motion. The computed zonal and eddy kinetic and available potential energy confirmed that the stationary motion was predominantly zonal while the transient motion was dominated by the eddies. From different components of kinetic and available potential energy 700 hPa was found to be the representative level in the lower troposphere for stationary and transient motions. In the upper troposphere the representative levels for stationary and transient motions were identified as 300 hPa and 400 hPa, respectively. From the tropospheric distribution of various components of enstrophy, its transient eddy component was found to dominate over rest of the components.

From the lower and upper tropospheric eddy kinetic and available potential energy of the different zonal scale components, the planetary wave with zonal wavenumber 1 was found to represent the preferred zonal scale for stationary global monsoonal motion in both lower and upper troposphere. The preferred zonal scale for the corresponding transient motion was found to vary between wavenumbers 2 to 5 from upper troposphere to lower troposphere. The contribution of short waves in the case of enstrophy was quite large as compared to their contributions in kinetic energy and available potential energy.

On the basis of the tropospheric distributions of stationary and transient global average kinetic and available potential energy of different zonal wave categories the long waves were found to contribute maximum towards stationary eddy energy as compared to the other wave categories. The medium waves had their maximum contribution in the transient component of eddy energy. Thus it can be concluded that the long waves are stationary in nature while the medium waves are transient in nature. The vertical distributions of *KE* and APE for different zonal wave categories indicated that the wave truncation at zonal wavenumber 15 is adequate to describe the major features of global atmospheric circulation.

From the time mean nonlinear interaction of kinetic energy of the tropospheric observed (stationary + transient) flow for different zonal wave categories, the medium waves were found to lose *KE* while the long and short waves as well as the zonal flow gained *KE* due to nonlinear interactions. The nonlinear interaction was found to contribute towards the maintenance of long waves. Further, the eddies through zonal-wave interaction were mainly responsible for maintenance of zonal flow. A spectral model with zonal wavenumber truncation at 30 is adequate to describe the nonlinear interaction of *KE* in the troposphere.

The transfer of available potential energy due to nonlinear interaction was down the scale except for extra-short waves in upper troposphere. The zonal mode was found to lose available potential energy. Further, the zonal-wave interaction which dominated over all wave categories was responsible for maintaining the eddy available potential energy. The nonlinear interaction of APE in the troposphere can be adequately described with a zonal wavenumber truncation at 15.

The stationary-transient nonlinear interaction of kinetic energy dominated for the medium waves as well as for zonal flow. The transient interaction was significant for medium waves and was the most important source of *KE* for short waves. For long waves all the three interactions of *KE* were important in controlling their dynamics. The transient and stationary-transient interactions acted as a source while stationary interaction as a sink of kinetic energy.

The stationary-transient nonlinear interaction in all wave categories dominated over the stationary and transient interaction for APE while transient interaction could be neglected in comparison to other two nonlinear interactions. The stationary-transient zonal-wave interaction was responsible for maintaining the APE of medium transient waves. The stationary-transient nonlinear interaction is thus found to be an important element of the tropospheric atmospheric flow.

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APPENDIX

List of symbols and variables

K	Kinetic energy per unit mass.
EN	Enstrophy per unit mass.
A	Available potential energy per unit mass.
$(\psi C_n^m, \psi S_n^m)$, $(\chi C_n^m, \chi S_n^m)$, (TC_n^m, TS_n^m)	Cosine and sine component pairs of spherical harmonic coefficients of stream function, velocity potential and temperature.
δ_{m0}	Arbitrary constant defined as $\delta_{m0} = 0$ for $m = 0$ and $\delta_{m0} = 1$ for $m \neq 0$.
K_n^m, EN_n^m A_n^m	Kinetic energy, enstrophy and available potential energy of the (m, n) mode of motion.
m	Zonal wavenumber.
n	Two-dimensional wavenumber.
M	Truncation value for m and n.
a	Radius of earth.
g	Acceleration due to gravity.
σ	Static stability $= g\{[T]_p C_p^{-1} - pR^{-1}(\partial[T]_p/\partial p)\}$.
C_p	Specific heat of air at constant pressure.
R	Gas constant for air.
$[T]_p$	Global mean temperature over the pressure surface p.
K_m, EN_m, A_m	K, EN and A in the mth zonal mode.
KZ, ENZ, AZ	K, EN and A of the mean zonal flow.
K_n, EN_n, A_n	K, EN and A of the nth two-dimensional mode.
KE, ENE, AE	K, EN and A of the zonal eddies.
$(\ )_t$	Partial derivative of the bracketed quantity with respect to time (t).
NK_n^m, NEN_n^m, NA_n^m	Nonlinear exchange of K, EN and A by the (m, n) mode of motion.
$NKS_n^m, NENS_n^m, NAS_n^m$	Nonlinear exchange of K, EN and A by the stationary (m, n) mode.
$NKT_n^m, NENT_n^m, NAT_n^m$	Nonlinear exchange of K, EN and A by the transient (m, n) mode.
$NKST_n^m, NENST_n^m, NAST_n^m$	Nonlinear exchange of K, EN and A by the mixed stationary-transient (m, n) mode.

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