Proc. Indian Acad. Sci. (Earth Planet. Sci.), Vol. 102, No. 1, March 1993, pp. 249–263.

Multiple time scales in rainfall variability

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Abstract. Monthly rainfall data averaged over a selected number of stations in peninsular Malaysia with a long record was subjected to singular spectrum analysis to determine the different modes of fluctuations in the rainfall. The analysis highlights the presence of fluctuations in the QBO time scale to a very long term time scale of 18·5 years which is possibly linked to lunar tidal forcing. There is also evidence of the Malaysian rainfall responding to El-Nino Southern Oscillation. An oscillation with a 7 to 10 year cycle is also evident. The annual cycle as a regular periodic oscillation is well established by the SSA.

Keywords. Singular spectral analysis; time scale; time series; eigenmode; eigenvector; principal component.

1. Introduction

In the tropics, rainfall is the most important climate parameter that has a high degree of variability both temporally and spatially, compared to other atmospheric indicators. Apart from the short term fluctuations, that is, day to day variations or weeks to months, there are longer term fluctuations that range from a few years to even decades. Most of the countries in the tropical region depend largely on agro-based industries for their economic survival and any major change in the rainfall pattern will have a tremendous impact. In view of this, rainfall has always been a subject of detailed study, especially so in the monsoon regions of Asia. For example, the Indian rainfall data which consists of a record of more than 100 years, has been a subject of numerous studies pertaining to short term fluctuations of 2 to 6 years cycle corresponding to the El-Nino Southern Oscillation (ENSO) and up to 22 years cyclic patterns associated with the double sunspot cycle (Bhalme and Jadhav 1984). Vines (1986) using filter analysis also found fluctuations with quasi-periodic cycles of 6 to 7 years, 10 to 11 years and also evidence of an 18 to 19 years fluctuation in the Indian rainfall data. Numerous references to long-term fluctuations in rainfall around the world have been cited in Vines (1986).

In Malaysia no detail synthesis to rainfall variability has been carried out mainly because of the limited number of stations with a very long period of record. Another problem is of missing data. To provide some insight into the long-term fluctuations in the rainfall over peninsular Malaysia, singular spectrum analysis (SSA) was carried out on the time series of monthly rainfall data obtained by averaging the monthly precipitation values for selected stations in peninsular Malaysia. The annual cycle of the rainfall pattern is shown in figure 1, where we notice that when averaging the rainfall for the whole of peninsular Malaysia a rainfall maximum is observed at the end of the year coinciding with the northern hemisphere winter monsoon. A secondary peak is found during the intermonsoon months of April and May. It is also of interest

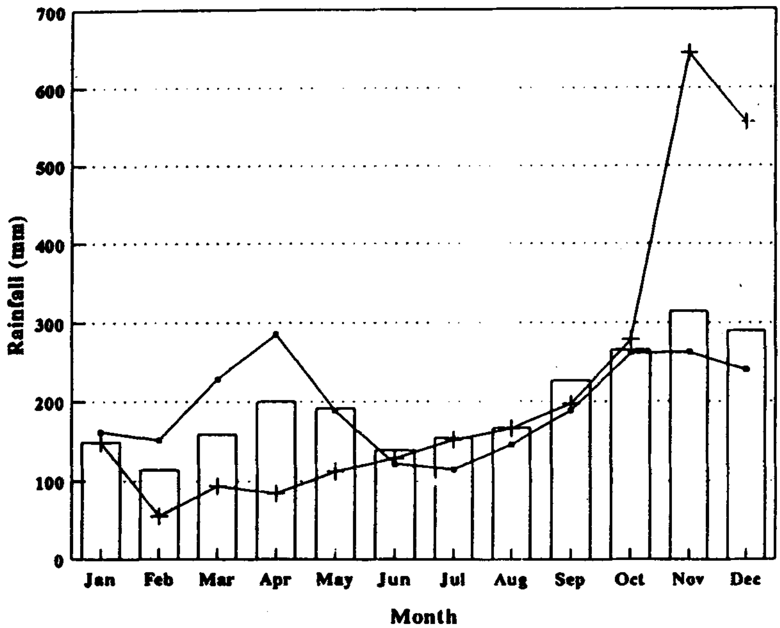


Figure 1. Monthly mean rainfall for Subang station on the west coast (——■——), Kota Baru on the east coast (——|——) and average of selected stations in peninsular Malaysia (rectangular bars).

to note that unlike some other parts of the monsoon region where more than 70% of the rainfall falls in a single season, here the rainfall is distributed throughout the year. For purpose of comparison, the annual rainfall cycle for a station on the west coast (Subang) and a station on the east coast (Kota Baru) of peninsular Malaysia are included in figure 1. The east coast has a single peak at the end of the year whereas the west coast has a semi-annual mode with two maxima coinciding with the two intermonsoon periods. The monthly distribution of rainfall averaged to represent peninsular Malaysia is biased towards the west coast rainfall as most of the stations used are located in the west coast states.

The conventional methods of analysis for determining periodic or quasi periodic oscillation in any climate system is by the direct Fourier analysis technique and Spectral Analysis method. The former provides the amplitude and phase at different harmonic components which are multiples of the fundamental frequency. In the spectral analysis technique the amount of variance contained in different frequency bands is first computed. Then appropriate band-pass filters corresponding to the peaks in the power spectrum are used to filter the original series to determine the amplitude and phase of the fluctuations. To study the combined spatial and temporal variations, principal component or empirical orthogonal function (EOF) analysis is being widely used. In this study, we have chosen the singular spectral analysis technique to determine the different modes of oscillation in the rainfall series for peninsular Malaysia. SSA has certain properties that makes it particularly useful for examining the different modes of fluctuations in climatic records. Vautard and Ghil (1989) applied the SSA technique to paleoclimatic records to examine the principal climatic oscillations and the regime changes in their amplitude. Using SSA, Rasmussen *et al* (1989) found a biennial and a low frequency component in the monthly wind and SST data over the tropical belt.

2. Data

A single time series of monthly rainfall data was obtained by averaging the monthly rainfall data from 21 selected climatological stations with records dating back to 1901. The data covers the period from January 1901 to December 1988. There was no record or the data were not reliable for the period between 1941 and 1945 for all the stations. For each of the individual stations, missing records and defective records were filled with data simulated by the Monte Carlo technique. Since we are interested in the interannual fluctuations the data were smoothed using a 13-point parzen filter to remove inter month and intraseasonal variability.

3. Formulation of SSA for a discrete time series

SSA is applied to a one dimensional time series of data at uniformly spaced points in time. The principles of SSA are similar to that of the classical EOF analysis of a two dimensional time series. The eigenvectors define dominant modes of variability while the associated principal components provide information relating to the intermittency and variations in amplitude that are not obtainable from conventional spectrum analysis. A comprehensive explanation of the theory of singular spectral analysis and its explanation are given in Vautard and Ghil (1989).

Consider a finite time series $y(t)$, of length N. i.e.

$$y(t) = y(Kt_s)K = 1, \ldots N \tag{1}$$

where t_s is the sampling interval.

Construct the normalized series x_K from y_K

$$x_k = \frac{y_k - \bar{y}}{\sigma_y} \tag{2}$$

$$k = 1, \ldots, M$$

where y and σ_y are the mean and standard deviation respectively.

Given the series $x_k, k = 1, \ldots N$, an M-dimensional subspace (or viewing window; $M < N$) is constructed by taking as state vectors the consecutive sequences

$$\mathbf{Z}_{n,m} = (x_n, x_{n+1}, \ldots, x_{n+m+1}) \tag{3}$$

for $n = 1$ to $N + M - 1$ and $m = 1$ to M

The state vectors form the rows in the matrix $\mathbf{Z}$. Within this embedding phase space, the dynamics of the system $\mathbf{Z}_{n,m}$ which is equivalent to a single time series X_n can be described statistically in a linear way by its principal axes.

The principal axes are obtained by solving the eigenvalue problem

$$\beta \mathbf{C}\mathbf{e} = \alpha \mathbf{e} \tag{4}$$

where β is a scaling constant for numerical convergence, typically $\beta = 1/M$; **C** is the symmetric autocovariance matrix; α and **e** are the eigenvalues and associated eigenvectors respectively.

Equation (4) in its scalar form is given by;

$$\alpha_k e_{k,j} = \frac{1}{M} \sum_{l=1}^{M} C_{j-1} e_{k,l} \tag{5}$$

$$i = 1, \ldots, M; \; j = 1, \ldots, M$$

where α_k is the eigenvalue of the kth eigenmode; $e_{k,l}$ is the lth element of the kth eigenmode.

Now the original time series can be expressed as

$$x_{i+j} = \sum_{k=1}^{M} A_{k,i} e_{k,j} \tag{6}$$

$$i = 1, \ldots N - M; j = 1, \ldots M$$

where $A_{k,i}$ is the ith component of the kth principal component (PC) or the amplitude function corresponding to the kth eigenvector.

Using the orthonormal property of the eigenvectors in equation (7) an exact expression for the kth PC is

$$A_k(t) \equiv A_{k,i} = \frac{1}{M} \sum_{j=1}^{M} x_{i+j} e_{k,j} \tag{7}$$

This represents a filtered version of the original series $x(t)$ with the amount of variance explained by the kth PC being α_k.

4. Results

The SSA was applied to the monthly anomaly rainfall with a window size (M) equal to 61. The eigenvectors for the 10 leading modes of fluctuations in the rainfall time series is shown from top to bottom in figure 2. These eigenvectors represent moving average time filters that have been determined directly from the data. We note that the eigenvectors are generally not sinusoidal. However, a pure oscillation rising above the red noise background with time scales shorter than the window length will appear as an even and odd eigenvector pair in the shape of the oscillation but in quadrature with each other. For those oscillations whose time scales are much longer than the window size even and odd eigenvectors will represent a running mean and a trend respectively.

The first eigenvector, EV1, is even and it is applied as low-pass running mean filter on the original series. This very low frequency mode accounts for nearly 16·9% of the variance (see table 1). The spectrum for this mode shown in figure 3, points to a quasi-periodic oscillation centered around a period of 18·5 years. The time series of the principal components on this time scale is shown in figure 4. Fluctuations on this time scale has also been found in the Indian rainfall data (Vines 1986 and Curie 1984). From an analysis of June rainfall data for northern India, Campbell *et al* (1983) had earlier postulated a lunar tidal forcing of 18·6 year period as one of the

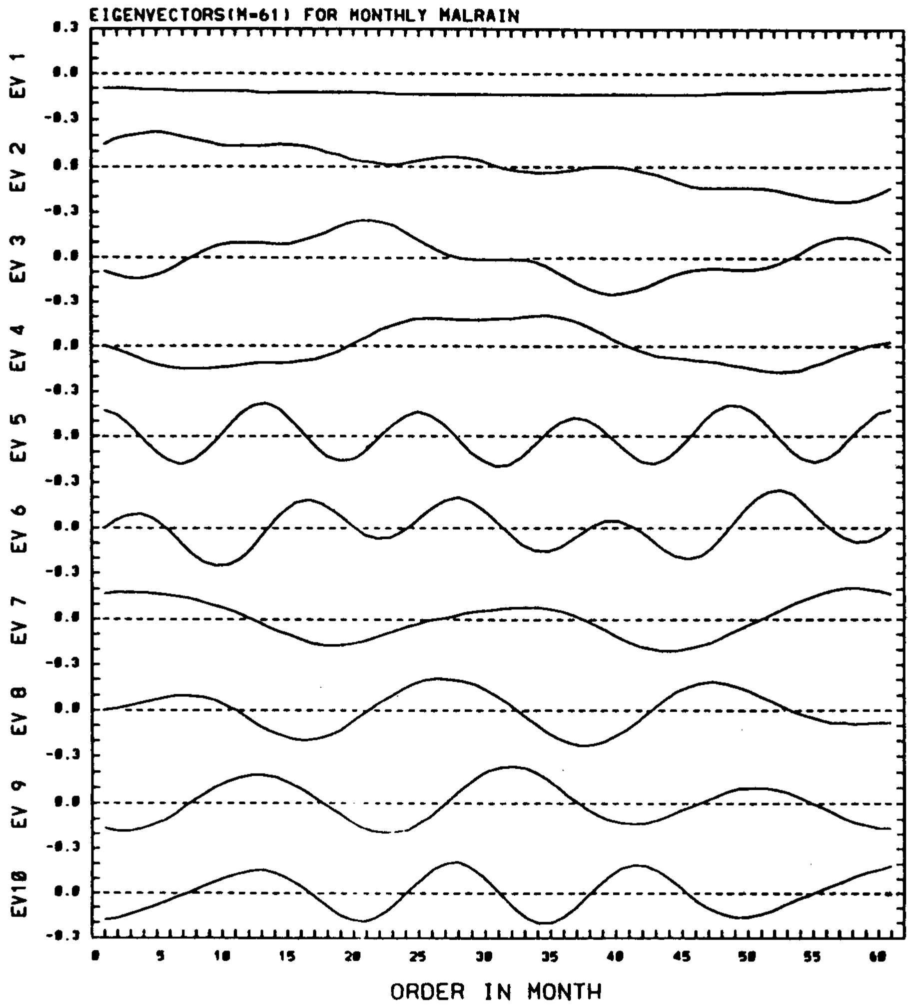


Figure 2. Eigenvectors for the first ten leading modes for monthly rainfall data.

Table 1. Percentage variance for the first 10 eigenmodes.

Mode	% Variance
1	16·9
2	11·1
3	9·6
4	9·5
5	8·8
6	8·5
7	7·9
8	4·4
9	4·2
10	2·3

contributors to the Indian floods and droughts. The 18·6 year cycle as a response to tidal forcing has also been detected in other parts of the world, namely the west coast of USA (Curie 1981) and east coast of Australia (Vines and Tomlinson 1985). It is then possible to speculate that the 18·5 year fluctuation in the Malaysian rainfall may be associated with the long-term lunar (M_N) tidal potential generated by gravitational interactions in the sun-earth-moon system.

The second mode which carries a variance of 11·1%, isolates an oscillation in the period range of about 7 to 10 years. No strong physical reasoning can be attributed with this cycle. It cannot be linked to the single sunspot cycle which has a 11 year cycle. Two possibilities arise; first, it may be a result of aliasing of the shorter period ENSO mode and second, it may have been produced by the interactions of different longer term cycles.

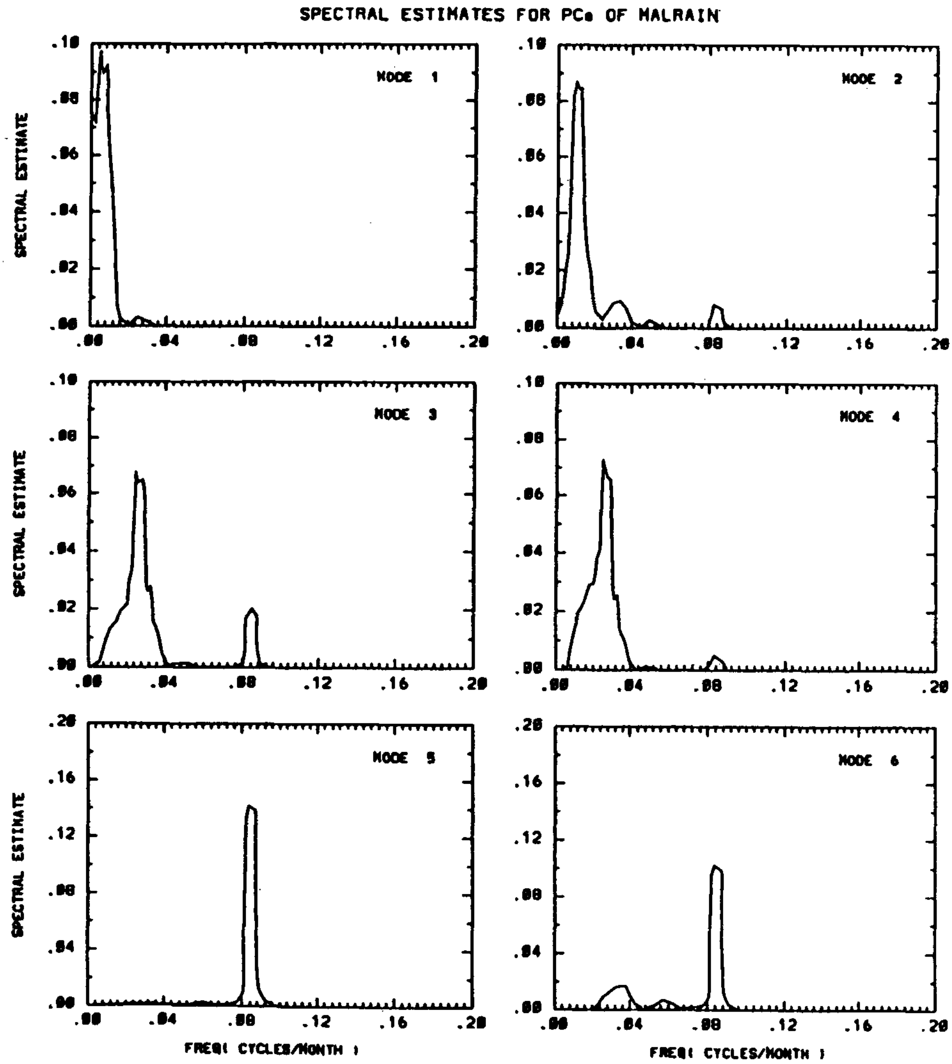


Figure 3(a).

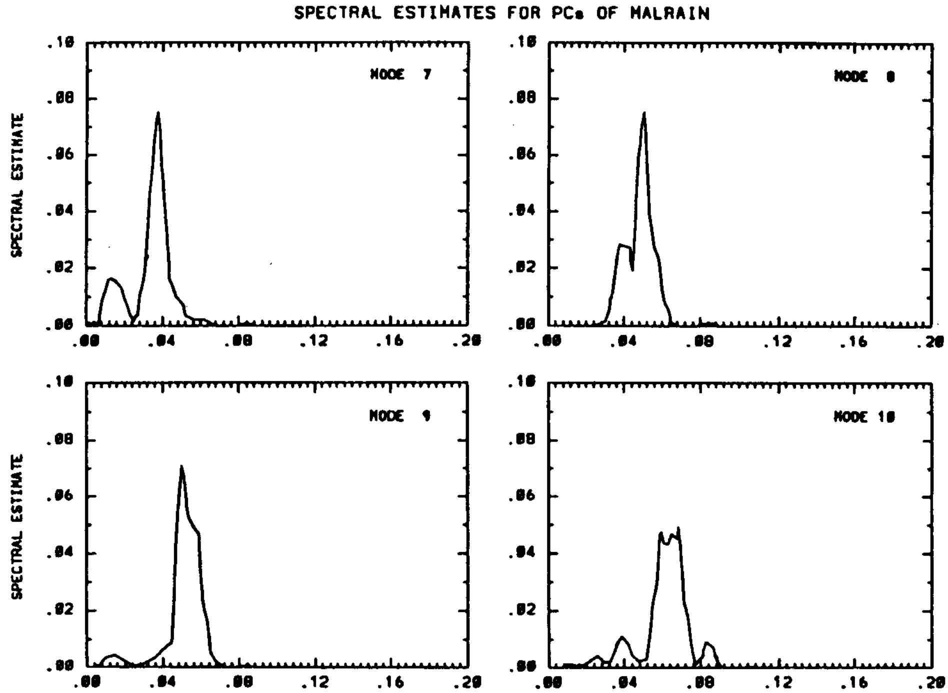


Figure 3(b).

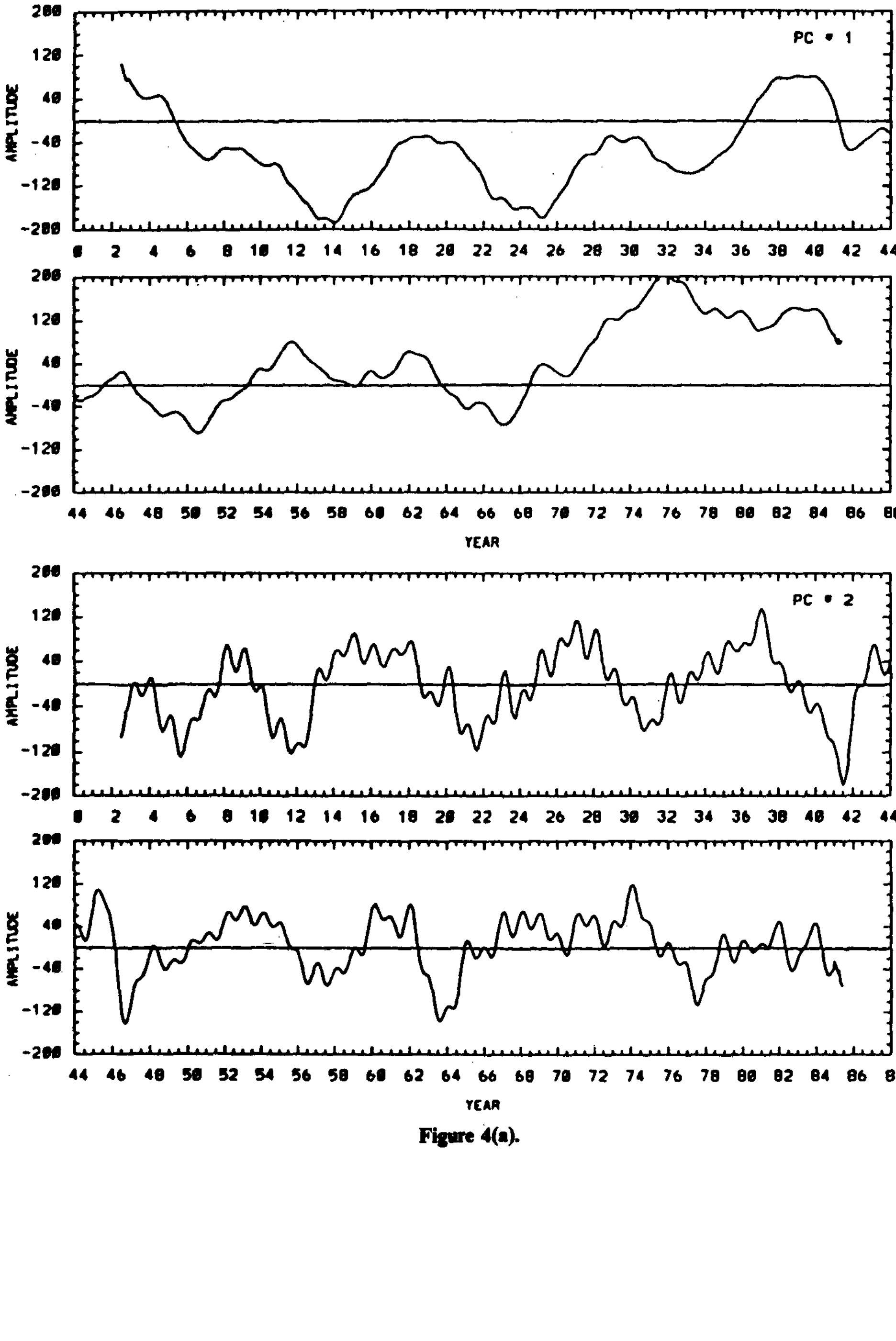
Figure 3(a–b). Power spectral estimates for the principal components corresponding to the ten leading eigenmodes.

The spectra (figure 3) for the PC's of eigenmodes 3 and 4 isolates an oscillation with a 3 to 5 year cycle which corresponds to the ENSO mode. Eigenvectors EV3 and EV4 which account for a combined variance of 19·1% are in quadrature and have approximately the same variance. EV3 and EV4 which are odd and even respectively are not quite nearly identical which implies that the ENSO signal in the rainfall is not a regular oscillatory feature unlike the annual cycle which is depicted by eigenvectors EV5 and EV6.

In figure 4, together with PC3, the ENSO years are indicated by solid circles, and it is interesting to note that during the majority of the ENSO years, below normal rainfall is observed.

Eigenvectors EV5 and EV6 are nearly identical and in quadrature with one another and have an oscillation pattern with a 12 month cycle. The PC's (figure 4) for these two modes clearly show the annual fluctuation and the spectra (figure 3) for the corresponding PC's indicate that this pair clearly isolates the annual cycle which contributes about 17·3% of the total variance.

The biennial mode is revealed by mode 7. The variance carried by this biennial signal is 7·9%. The principal component of this mode which corresponds to the eigenvector, EV7 is shown in figure 4. The quasi-biennial oscillation (QBO) is well amplified in PC7. To compare the phase relationship of the stratospheric QBO with our rainfall QBO, a four station average of the 50 mb zonal wind anomaly in the tropics from 1951–1987 is shown in figure 5. The time when the westerly anomaly reaches its maximum is shown alongside PC7 as solid circles in figure 4. It is interesting


PC # 1
PC # 2
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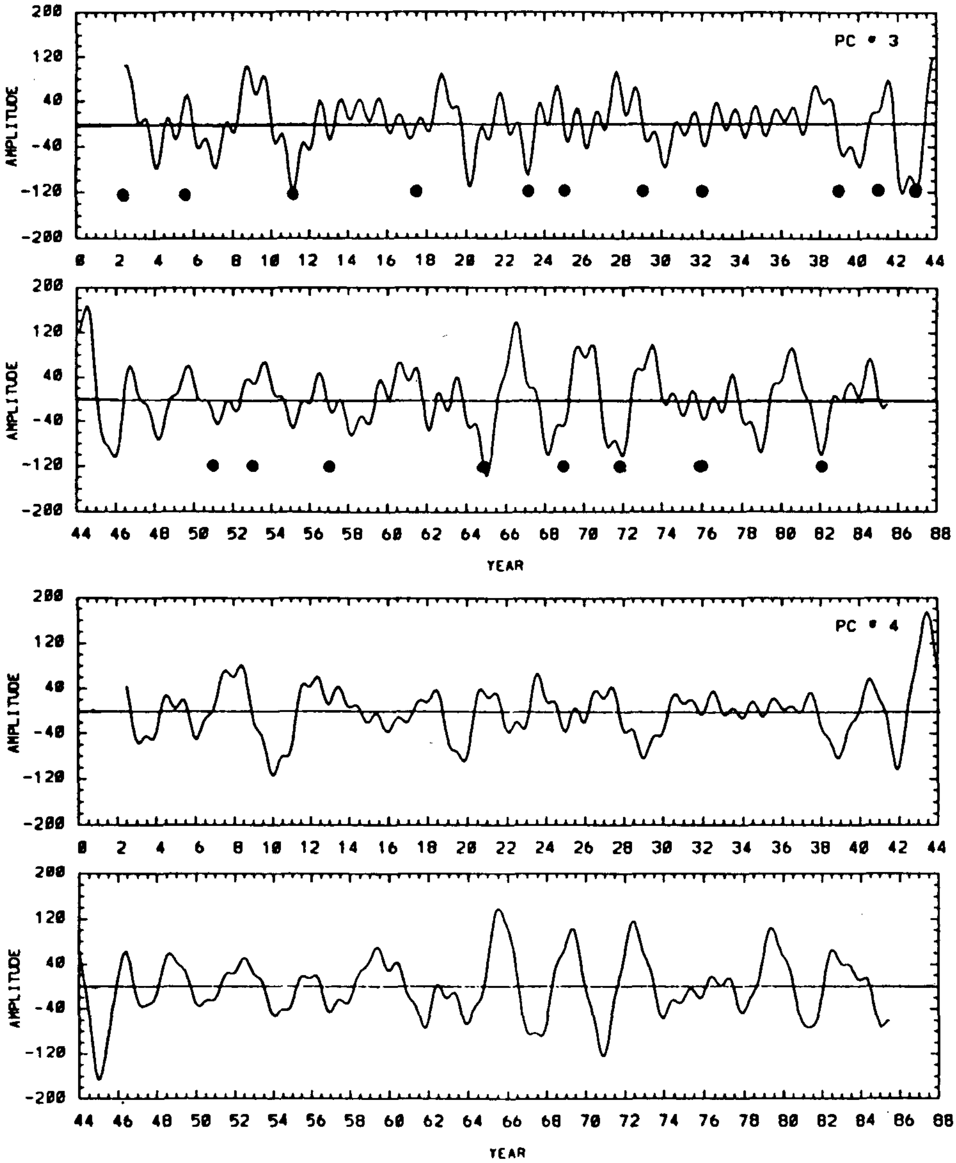
YEAR
200
120
40
-40
-120
-200
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44
44 46 48 50 52 54 56 58 60 62 64 66 68 70 72 74 76 78 80 82 84 86 88

Figure 4(a).


PC # 3
PC # 4
AMPLITUDE
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200
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-120
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44 46 48 50 52 54 56 58 60 62 64 66 68 70 72 74 76 78 80 82 84 86 88

Figure 4(b).

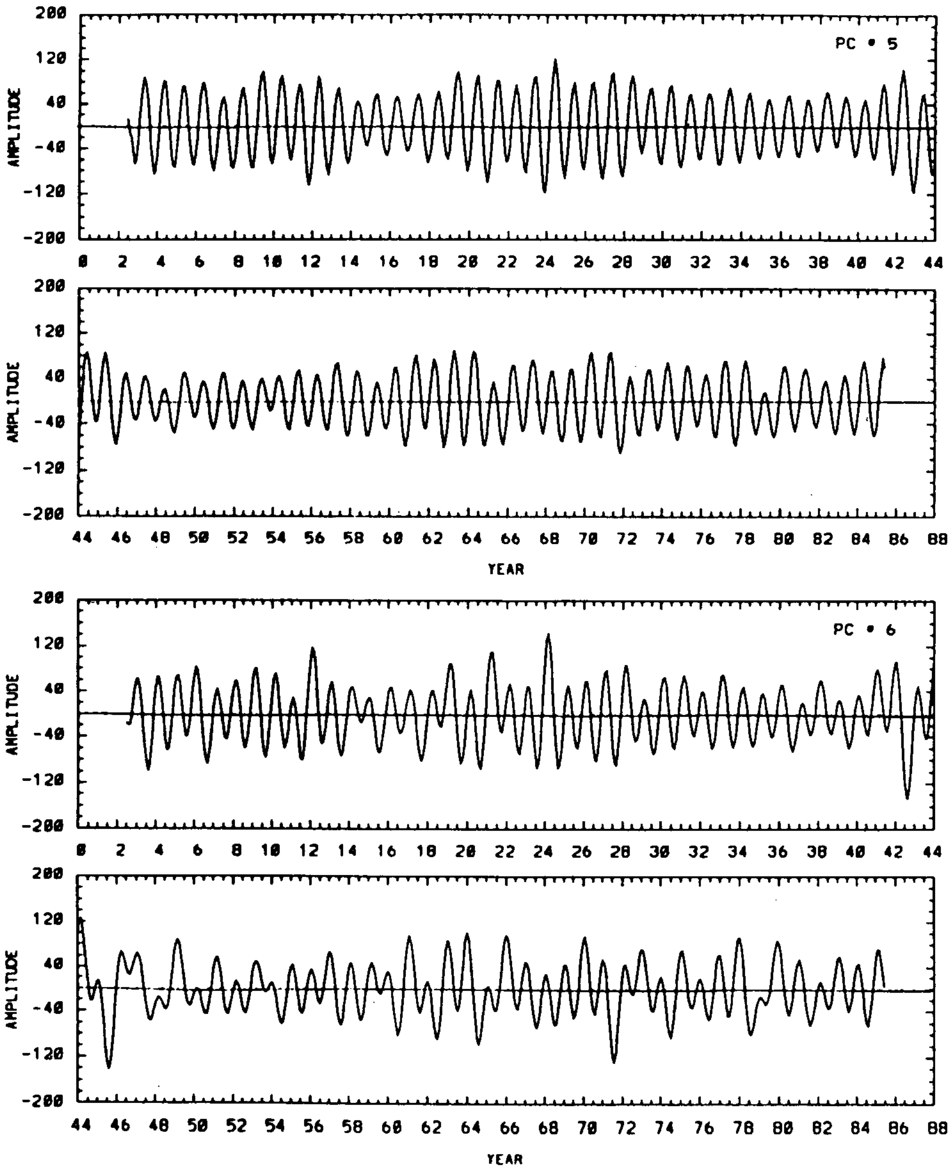
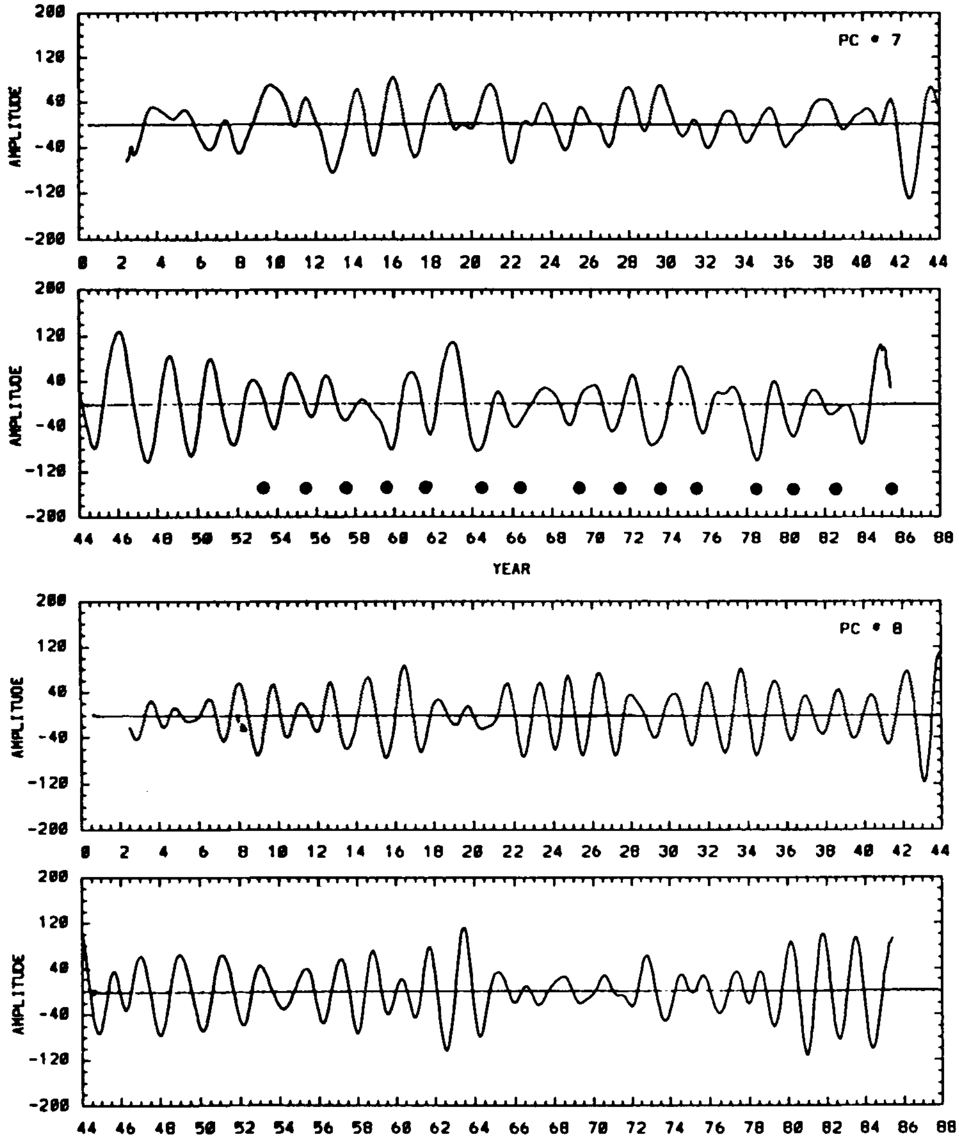


Figure 4(c).


PC • 7
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AMPLITUDE
200
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0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44
44 46 48 50 52 54 56 58 60 62 64 66 68 70 72 74 76 78 80 82 84 86 88
YEAR

Figure 4(d).

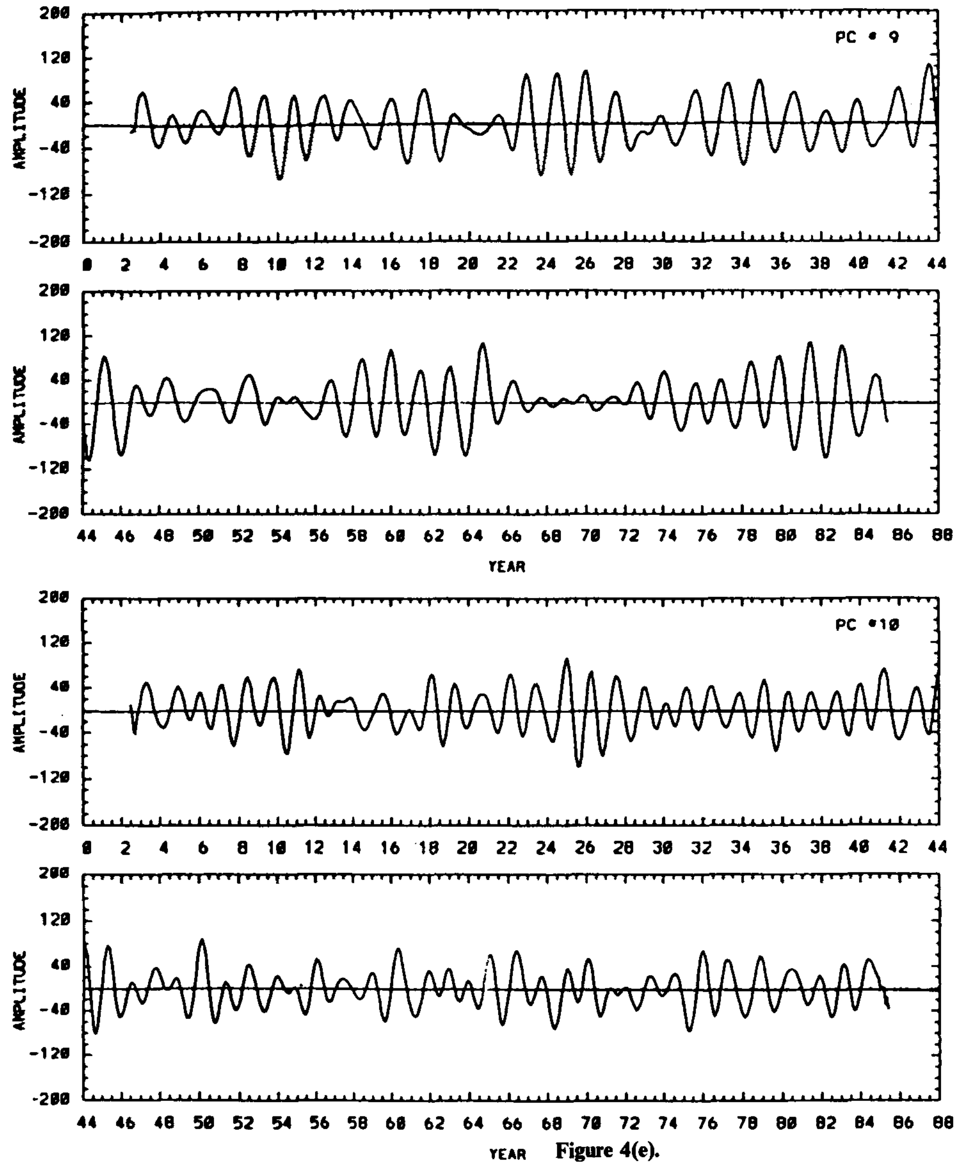

Figure 4(e).

Figure 4(a–e). Time series of filtered components (principal components) corresponding to the ten leading eigenmodes. The solid circles underneath PC3 indicate the ENSO years. The solid circles underneath PC7 indicate the westerly maximum in the stratospheric zonal wind QBO.

to note that the minimum in the rainfall QBO is almost in phase with westerly maximum in the stratospheric zonal wind QBO.

The last three of the leading 10 modes explain very little variance to the overall interannual variability. Eigenvectors EV8 and EV9 are identical and in quadrature with each other. The spectrum for the corresponding principal components isolates an oscillation with a $1\frac{1}{2}$ years cycle. There is no real physical basis for this mode

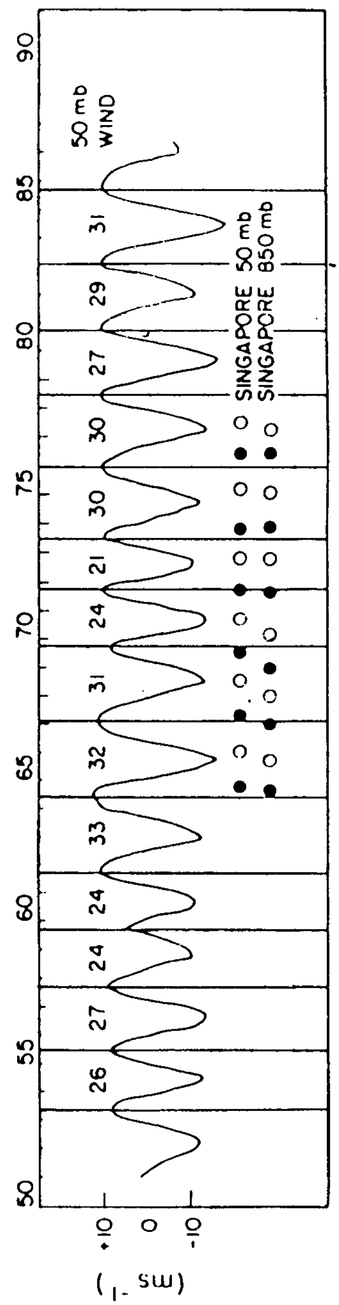

Figure 5. Four station averaged zonal wind anomaly at 50 mb. The numbers indicate spacing in months for successive westerly maxima. The open and full circles immediately below represent easterly and westerly wind maxima for 50 mb and 850 mb at Singapore (from Kane 1992).

except that a double mode in the annual cycle could have produced an oscillation in these time scales.

5. Conclusion

Singular spectrum analysis provides an alternate method of determining the naturally occurring multiple time scales in the interannual variability of the rainfall series. The eigenvalues of the singular spectrum provides information relating to the contribution made by each mode towards the overall variability. Whereas the corresponding eigenvectors act as data-adaptive filters which when applied to the original time series provides the principal components for each mode.

This study is really a first attempt, carried out to detect the various quasi-periodic fluctuations in the Malaysian rainfall data. The results though encouraging are preliminary because the number of stations used is very small. The homogeneity of the data for the years before 1950 is also uncertain. Another unavoidable problem is missing data. However a more detailed study using 43 years of data from 1950 to 1992 using a larger set of stations is currently being undertaken to verify some of the findings here.

Among the various fluctuations detected those that are of interest and are associated with other physical phenomena in the same time scale are:

- Fluctuations with a 18·5 year cycle, possibly linked to soli-lunar tidal forcing.
- A 3 to 5 year fluctuation associated with the ENSO. Below normal rainfall has been observed during the ENSO years on this time scale.
- A QBO signal which is out of phase with the stratospheric zonal wind QBO.

Similar oscillations in the various time scales described here have also been observed in other regions of the world. Although no single mode of fluctuation stands above the rest as the variance explained by each mode are comparatively small, it is interesting to note that the dominant modes of variability are forced naturally by inter-planetary forcings at the very long time scale, like the 18·5 year cycle and planetary scale forcings like the ENSO and QBO cycles.

Acknowledgements

I would like to thank the Director General of the Malaysian Meteorological Service for his encouragement and support and to Dr. Lim Joo Tick for his advice and helpful suggestions.

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