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Static deformation of two welded half-spaces due to very long strike-slip dislocations

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Abstract. Closed form analytic expressions for displacement and stresses at any point of either of the two homogeneous, isotropic, perfectly elastic half-spaces in welded contact due to very long strike-slip dislocations are obtained. Both cases of vertical and horizontal strike-slip dislocations are discussed in detail. Variation of the displacement with horizontal distance from the fault and with vertical distance from the interface for a vertical strike-slip fault is studied numerically.

Keywords. Vertical strike-slip fault; horizontal strike-slip fault; displacement; shear stresses; static deformation.

1. Introduction

Maruyama (1966) used the method of images to obtain the static deformation of an elastic half-space due to a long strike-slip dislocation. Extending the results of Maruyama (1966), Rybicki (1971) obtained results for a very long strike-slip fault in an elastic half-space with a vertical or horizontal discontinuity. Using the method of images, Chinnery and Jovanovich (1972) calculated the displacement field due to a very long strike-slip fault in earth model consisting of two layers over a half-space. The static surface deformation of a multilayered half-space caused by a very long strike-slip fault was studied by Rybicki (1973), Singh (1985) and Singh and Garg (1985).

Singh (1985) gave an integral representation of seismic sources representing a long strike-slip dislocation in an unbounded medium. In the present paper, we have obtained the closed form analytic expressions for the displacement and stresses at any point of either of two homogeneous isotropic elastic half-spaces in welded contact due to very long strike-slip dislocations. Our method consists of first finding the integral expressions for two half-spaces in welded contact from the corresponding expressions for an unbounded medium by applying the suitable boundary conditions at the interface and then evaluating the integrals analytically. It has been verified that the results for the deformation of a uniform half-space given by Maruyama (1966) can be recovered from our results as a particular case. The results due to a finite strike-slip fault have also been obtained.

2. Basic equations

Let (x, y, z) denote the Cartesian coordinates with z-axis vertically downwards and (u, v, w) be the displacement components. We shall be considering the antiplane strain

Table 1. Source coefficients for various sources. The upper sign is for $z > \beta$ and the lower sign for $z < \beta$.

Source	A_0	B_0
Single couple [12]	$F_{12}/2\pi\mu$	0
Single couple [13]	0	$\pm F_{13}/2\pi\mu$

problem in which $v = w \equiv 0$ and $u = u(y, z)$ satisfies the equilibrium equation

$$(\partial^2 u/\partial y^2) + (\partial^2 u/\partial z^2) = 0 \tag{1}$$

for zero body force. The non-zero stresses are

$$\tau_{12} = \mu(\partial u/\partial y), \quad \tau_{13} = \mu(\partial u/\partial z), \tag{2}$$

μ being the rigidity of the medium.

Suppose that a line source parallel to x-axis passes through a point $P(\alpha, \beta)$ of the yz-plane. The displacement u_0 for the line source parallel to the x-axis in an infinite medium can be written in the form (Singh 1985)

$$u_0 = \int_0^\infty [A_0 \sin k(y-\alpha) + B_0 \cos k(y-\alpha)] \exp(-k|z-\beta|) \mathrm{d}k, \tag{3}$$

where the source coefficients A_0 and B_0 are independent of k. These coefficients for various seismic sources are listed in table 1 for ready reference.

The single couple [12] is a couple in the xy-plane with forces in the x-direction and with its arm in the y-direction. F_{12} is the moment of the couple [12]. Similarly, [13] is the single couple of moment F_{13} in the xz-plane with forces in the x-direction and arm in the z-direction (Ben-Menahem and Singh 1981).

3. Line source in two welded half-spaces

Consider two homogeneous, isotropic, perfectly elastic half-spaces that are in welded contact along the plane $z = 0$. The upper half-space ($z < 0$) is called medium I and the lower half-space ($z > 0$), medium II, with rigidities, μ_1 and μ_2, respectively (figure 1). For a line source parallel to the x-axis acting at the point $P(\alpha, \beta)$ of the lower half-space, suitable expressions for the displacement (satisfying the harmonic equation) in the two half-spaces are of the form

$$u^{(1)} = \int_0^\infty [A_1 \sin k(y-\alpha) + B_1 \cos k(y-\alpha)] \exp(kz) \mathrm{d}k, \tag{4}$$

$$u^{(2)} = u_0 + \int_0^\infty [A_2 \sin k(y-\alpha) + B_2 \cos k(y-\alpha)] \exp(-kz) \mathrm{d}k. \tag{5}$$

The superscript (1) denotes quantities related to the medium I and the superscript (2) those related to medium II. The unknowns A_1, B_1, A_2 and B_2 are to be found from the boundary conditions at the interface $z = 0$. Since the two half-spaces are

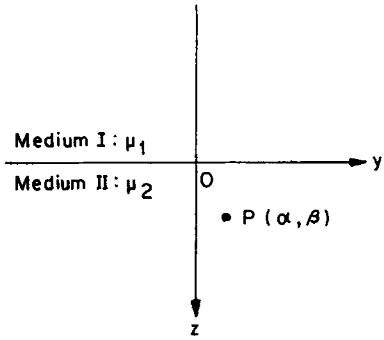

Figure 1. Section $x = 0$ of the model consisting of two half-spaces in welded contact with a line source in the lower half-space.

assumed to be in welded contact, the displacement and stress across the plane $z = 0$ are continuous. Hence

$$u^{(1)} = u^{(2)}, \quad \tau_{13}^{(1)} = \tau_{13}^{(2)} \text{ at } z = 0. \tag{6}$$

The stresses $\tau_{13}^{(1)}$ and $\tau_{13}^{(2)}$ are

$$\tau_{13}^{(1)} = \mu_1 \int_0^\infty [A_1 \sin k(y-\alpha) + B_1 \cos k(y-\alpha)] \exp(kz) k \, dk, \tag{7}$$

$$\tau_{13}^{(2)} = \mu_2 \int_0^\infty [\mp \{A_0 \sin k(y-\alpha) + B_0 \cos k(y-\alpha)\} \exp(-k|z-\beta|) - \{A_2 \sin k(y-\alpha) + B_2 \cos k(y-\alpha)\} \exp(-kz)] k \, dk. \tag{8}$$

From table 1, we note that the source coefficient B_0 has different values for $z < \beta$ and $z > \beta$. We write B_0^1 for B_0 when $0 \leqslant z < \beta$. Equations (4)–(8) yield

$$A_1 = \left(\frac{2\mu_2}{\mu_2 + \mu_1}\right) A_0 \exp(-k\beta), \quad B_1 = \left(\frac{2\mu_2}{\mu_2 + \mu_1}\right) B_0^1 \exp(-k\beta), \tag{9}$$

$$A_2 = \left(\frac{\mu_2 - \mu_1}{\mu_2 + \mu_1}\right) A_0 \exp(-k\beta), \quad B_2 = \left(\frac{\mu_2 - \mu_1}{\mu_2 + \mu_1}\right) B_0^1 \exp(-k\beta). \tag{10}$$

Putting these values of the constants A_1, B_1, etc in equations (4), (5), (7) and (8), we obtain the integral expressions for displacement and stress in the two welded half-spaces. These integrals have been evaluated analytically by using the standard transform integrals given in Appendix.

3.1 *Single couple* [12]

The closed form expressions for the displacement and stresses at any point of the medium I due to a single couple [12] in the medium II are

$$u^{(1)} = \frac{F_{12}}{\pi(\mu_1 + \mu_2)} \left[\frac{y-\alpha}{R^2}\right], \tag{11}$$

$$\tau_{12}^{(1)}=\frac{\mu_1 F_{12}}{\pi(\mu_1+\mu_2)}\left[\frac{(z-\beta)^2-(y-\alpha)^2}{R^4}\right], \tag{12}$$

$$\tau_{13}^{(1)}=\frac{2\mu_1 F_{12}}{\pi(\mu_1+\mu_2)}\left[\frac{(y-\alpha)(\beta-z)}{R^2}\right], \tag{13}$$

where

$$R^2=(y-\alpha)^2+(z-\beta)^2. \tag{14}$$

For medium II, the displacement and stresses are ($z \neq \beta$)

$$u^{(2)}=\frac{F_{12}}{2\pi\mu_2}\left[\frac{y-\alpha}{R^2}+\left(\frac{\mu_2-\mu_1}{\mu_2+\mu_1}\right)\frac{y-\alpha}{S^2}\right], \tag{15}$$

$$\tau_{12}^{(2)}=\frac{F_{12}}{2\pi}\left[\frac{(z-\beta)^2-(y-\alpha)^2}{R^4}+\left(\frac{\mu_2-\mu_1}{\mu_2+\mu_1}\right)\left\{\frac{(z+\beta)^2-(y-\alpha)^2}{S^4}\right\}\right], \tag{16}$$

$$\tau_{13}^{(2)}=\frac{F_{12}}{\pi}\left[\frac{\beta-z}{R^4}-\left(\frac{\mu_2-\mu_1}{\mu_2+\mu_1}\right)\frac{z+\beta}{S^4}\right](y-\alpha), \tag{17}$$

where

$$S^2=(y-\alpha)^2+(z+\beta)^2. \tag{18}$$

3.2 *Single couple* [13]

The deformation field caused by a single couple [13] is given by

$$u^{(1)}=\frac{F_{13}}{\pi(\mu_1+\mu_2)}\left[\frac{z-\beta}{R^2}\right], \tag{19}$$

$$\tau_{12}^{(1)}=\frac{2\mu_1 F_{13}}{\pi(\mu_1+\mu_2)}\left[\frac{(y-\alpha)(\beta-z)}{R^4}\right], \tag{20}$$

$$\tau_{13}^{(1)}=\frac{\mu_1 F_{13}}{\pi(\mu_1+\mu_2)}\left[\frac{(y-\alpha)^2-(z-\beta)^2}{R^4}\right], \tag{21}$$

$$u^{(2)}=\frac{F_{13}}{2\pi\mu_2}\left[\frac{z-\beta}{R^2}-\left(\frac{\mu_2-\mu_1}{\mu_2+\mu_1}\right)\frac{z+\beta}{S^2}\right], \tag{22}$$

$$\tau_{12}^{(2)}=\frac{F_{13}}{\pi}\left[\frac{\beta-z}{R^4}+\left(\frac{\mu_2-\mu_1}{\mu_2+\mu_1}\right)\frac{z+\beta}{S^4}\right](y-\alpha), \tag{23}$$

$$\tau_{13}^{(2)}=\frac{F_{13}}{2\pi}\left[\frac{(y-\alpha)^2-(z-\beta)^2}{R^4}+\left(\frac{\mu_2-\mu_1}{\mu_2+\mu_1}\right)\left\{\frac{(z+\beta)^2-(y-\alpha)^2}{S^4}\right\}\right]. \tag{24}$$

4. Strike-slip dislocations in medium II

The displacement field due to a long strike-slip line source on an inclined plane is expressible in terms of displacements due to vertical strike-slip and horizontal strike-slip line sources (Singh and Garg 1985).

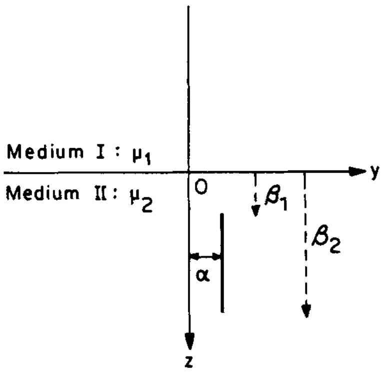

Figure 2. Section $x = 0$ of a finite vertical strike-slip fault ($0 \leqslant \beta_1 \leqslant z \leqslant \beta_2 < \infty$) in the lower half-space.

4.1 *Vertical strike-slip dislocation in the lower half-space*

Maruyama (1966) showed that the single couple [12] is equivalent to a long vertical strike-slip line source such that

$$F_{12} = \mu_2 U_0 \mathrm{d}\beta, \tag{25}$$

where $\Delta u = U_0$ denotes the slip. Equations (11)–(18) and (25) determine the deformation field at any point of the medium consisting of two welded half-spaces due to a long vertical strike-slip line source situated in the lower half-space.

The deformation field at any point of the welded medium due to a long vertical strike-slip fault with finite vertical extent $\beta_1 \leqslant \beta \leqslant \beta_2$ is obtained by integrating with respect to β from β_1 to β_2 (figure 2). We find

$$u^{(1)} = \frac{\mu_2 U_0}{\pi(\mu_1+\mu_2)}\left[\tan^{-1}\left(\frac{\beta_2 - z}{y-\alpha}\right) - \tan^{-1}\left(\frac{\beta_1 - z}{y-\alpha}\right)\right], \tag{26}$$

$$\tau_{12}^{(1)} = \frac{\mu_1\mu_2 U_0}{\pi(\mu_1+\mu_2)}\left[\frac{z-\beta_2}{(y-\alpha)^2+(z-\beta_2)^2} - \frac{z-\beta_1}{(y-\alpha)^2+(z-\beta_1)^2}\right], \tag{27}$$

$$\tau_{13}^{(1)} = \frac{\mu_1\mu_2 U_0}{\pi(\mu_1+\mu_2)}\left[\frac{y-\alpha}{(y-\alpha)^2+(z-\beta_2)^2} - \frac{y-\alpha}{(y-\alpha)^2+(z-\beta_1)^2}\right], \tag{28}$$

$$u^{(2)} = \frac{U_0}{2\pi}\left[\tan^{-1}\left(\frac{\beta_2 - z}{y-\alpha}\right) - \tan^{-1}\left(\frac{\beta_1 - z}{y-\alpha}\right) + \left(\frac{\mu_2-\mu_1}{\mu_2+\mu_1}\right) \times \left\{\tan^{-1}\left(\frac{\beta_2 + z}{y-\alpha}\right) - \tan^{-1}\left(\frac{\beta_1 + z}{y-\alpha}\right)\right\}\right], \tag{29}$$

$$\tau_{12}^{(2)} = \frac{\mu_2 U_0}{2\pi}\left[\frac{z-\beta_2}{(y-\alpha)^2+(z-\beta_2)^2} - \frac{z-\beta_1}{(y-\alpha)^2+(z-\beta_1)^2} - \left(\frac{\mu_2-\mu_1}{\mu_2+\mu_1}\right)\left\{\frac{z+\beta_2}{(y-\alpha)^2+(z+\beta_2)^2} - \frac{z+\beta_1}{(y-\alpha)^2+(z+\beta_1)^2}\right\}\right], \tag{30}$$

$$\tau_{13}^{(2)}=\frac{-\mu_2 U_0}{2\pi}\left[\frac{y-\alpha}{(y-\alpha)^2+(z-\beta_2)^2}-\frac{y-\alpha}{(y-\alpha)^2+(z-\beta_1)^2}-\left(\frac{\mu_2-\mu_1}{\mu_2+\mu_1}\right)\right.$$
$$\left.\times\left\{\frac{y-\alpha}{(y-\alpha)^2+(z+\beta_2)^2}-\frac{y-\alpha}{(y-\alpha)^2+(z+\beta_1)^2}\right\}\right]. \tag{31}$$

4.2 *Horizontal strike-slip dislocation*

A long horizontal strike-slip line source is represented by the single couple [13] with

$$F_{13}=\mu_2 U_0 \mathrm{d}\alpha. \tag{32}$$

The deformation field at any point of the welded medium due to a long horizontal strike-slip line source can be obtained from (19)–(24) and (32).

The displacement and stresses of the welded medium due to a finite horizontal strike-slip fault with horizontal range, $\alpha_1 \leqslant \alpha \leqslant \alpha_2$, are

$$u^{(1)}=\frac{\mu_2 U_0}{\pi(\mu_1+\mu_2)}\left[\tan^{-1}\left(\frac{\alpha_2-y}{z-\beta}\right)-\tan^{-1}\left(\frac{\alpha_1-y}{z-\beta}\right)\right], \tag{33}$$

$$\tau_{12}^{(1)}=\frac{\mu_1\mu_2 U_0}{\pi(\mu_1+\mu_2)}\left[\frac{z-\beta}{(y-\alpha_2)^2+(z-\beta)^2}-\frac{z-\beta}{(y-\alpha_1)^2+(z-\beta)^2}\right], \tag{34}$$

$$\tau_{13}^{(1)}=\frac{\mu_1\mu_2 U_0}{\pi(\mu_1+\mu_2)}\left[\frac{y-\alpha_2}{(y-\alpha_2)^2+(z-\beta)^2}-\frac{y-\alpha_1}{(y-\alpha_1)^2+(z-\beta)^2}\right], \tag{35}$$

$$u^{(2)}=\frac{U_0}{2\pi}\left[\tan^{-1}\left(\frac{\alpha_2-y}{z-\beta}\right)-\tan^{-1}\left(\frac{\alpha_1-y}{z-\beta}\right)-\left(\frac{\mu_2-\mu_1}{\mu_2+\mu_1}\right)\right.$$
$$\left.\times\left\{\tan^{-1}\left(\frac{\alpha_2-y}{z+\beta}\right)-\tan^{-1}\left(\frac{\alpha_1-y}{z+\beta}\right)\right\}\right], \tag{36}$$

$$\tau_{12}^{(2)}=\frac{-\mu_2 U_0}{2\pi}\left[\frac{z-\beta}{(y-\alpha_2)^2+(z-\beta)^2}-\frac{z-\beta}{(y-\alpha_1)^2+(z-\beta)^2}-\left(\frac{\mu_2-\mu_1}{\mu_2+\mu_1}\right)\right.$$
$$\left.\times\left\{\frac{z+\beta}{(y-\alpha_2)^2+(z+\beta)^2}-\frac{z+\beta}{(y-\alpha_1)^2+(z+\beta)^2}\right\}\right], \tag{37}$$

$$\tau_{13}^{(2)}=\frac{\mu_2 U_0}{2\pi}\left[\frac{y-\alpha_2}{(y-\alpha_2)^2+(z-\beta)^2}-\frac{y-\alpha_1}{(y-\alpha_1)^2+(z-\beta)^2}-\left(\frac{\mu_2-\mu_1}{\mu_2+\mu_1}\right)\right.$$
$$\left.\times\left\{\frac{y-\alpha_2}{(y-\alpha_2)^2+(z+\beta)^2}-\frac{y-\alpha_1}{(y-\alpha_1)^2+(z+\beta)^2}\right\}\right]. \tag{38}$$

5. Numerical results and concluding remarks

We study numerically the variation of the displacement component u due to a finite vertical strike-slip fault situated in the lower half-space. For simplicity, we take $\beta_1=0$, $\beta_2=H$ and $\alpha=0$ so that the strike-slip fault of finite extent H now lies on the z-axis.

Figures 3–6 show the variation of the displacement with the horizontal distance

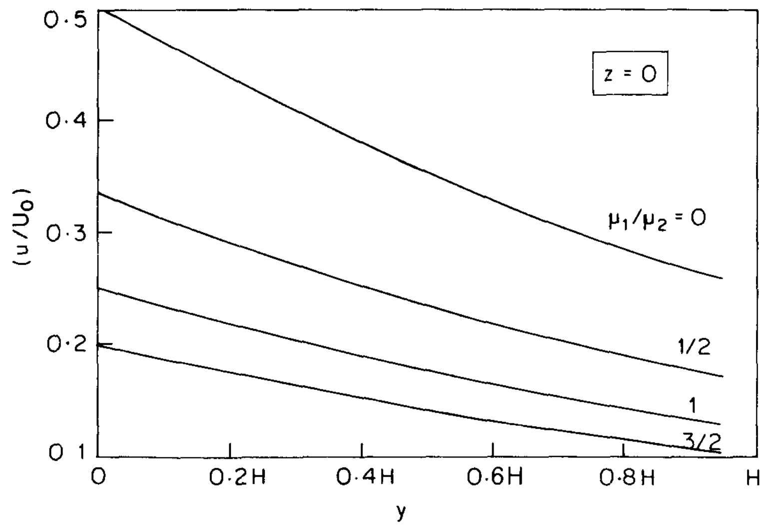

Figure 3. Variation of the displacement u/U_0 with the horizontal distance (y) from a vertical strike-slip fault for $z = 0$.

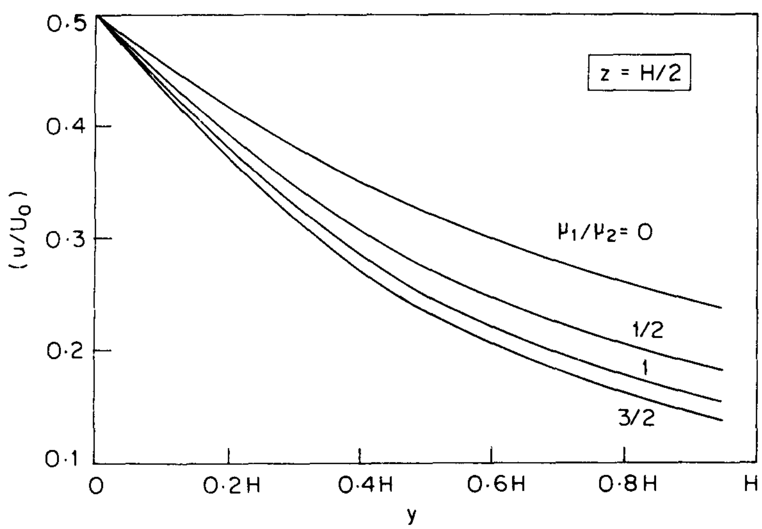

Figure 4. Variation of u/U_0 with y for $z = H/2$.

from the fault for four different positions of the observer, namely, $z = 0$, $H/2$, H and $3H/2$. In each figure, four values of the ratio of rigidities of the two welded half-spaces are considered, namely, $\mu_1/\mu_2 = 0$, 1/2, 1 and 3/2. The case $\mu_1/\mu_2 = 0$ corresponds to a uniform half-space and $\mu_1/\mu_2 = 1$ to an unbounded uniform infinite medium. When $z = 0$, the observer is at the interface and when $z = 3H/2$, the observer is below the strike-slip fault. In figures 3–5, we note that the displacement decreases as the distance from the fault increases for fixed values of μ_1/μ_2 and z. For fixed values of z and y, the displacement decreases as μ_1/μ_2 increases.

In figure 6, the displacement first increases as y increases and after attaining a

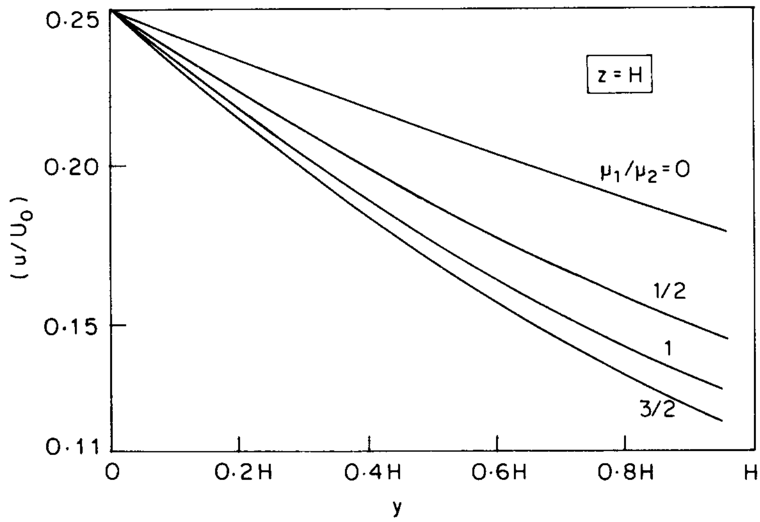

Figure 5. Variation of u/U_0 with y for $z = H$.

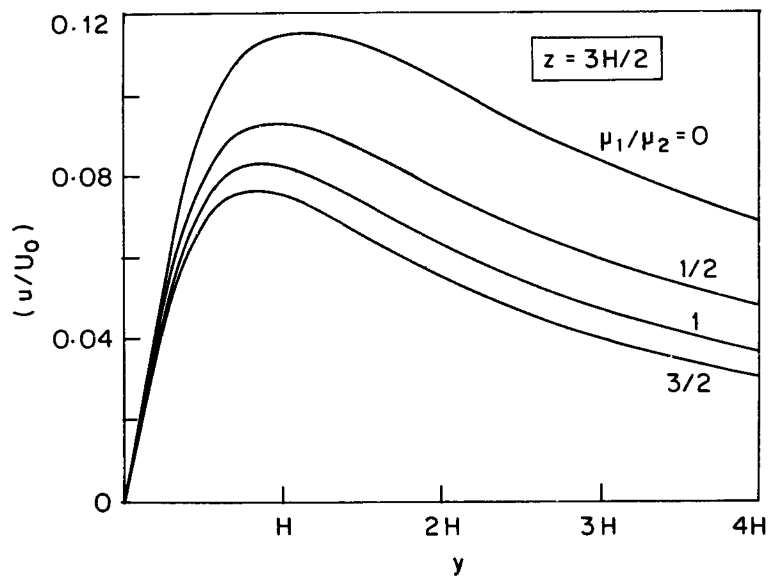

Figure 6. Variation of u/U_0 with y for $z = 3H/2$.

maxima, the displacement starts decreasing. The displacement u/U_0 has maximum at $y = H$ (approximately) when μ_1/μ_2 is 0 and 1/2. For $\mu_1/\mu_2 = 1$ and 3/2, the maximum occur at $y = 3H/4$ (approximately).

Figures 7–8 exhibit the variation of displacement u/U_0 with the distance from the interface for fixed values of μ_1/μ_2 and y. The range of the distance z is taken from $z = -3H/5$ to $z = 2H$. In these figures, the negative values of the distance from the interface indicate that the observer is in medium I and the positive values imply that the point of observation lies in medium II in which the source lies. We note that the

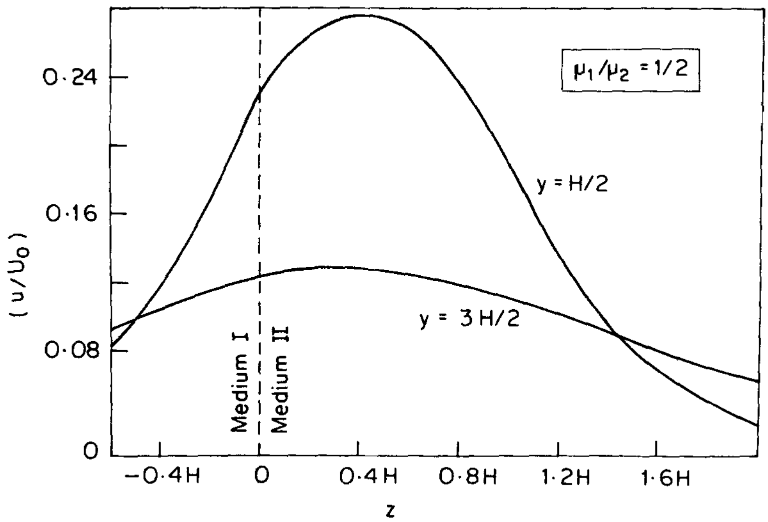

Figure 7. Variation of u/U_0 with the vertical distance (z) from the interface for $\mu_1/\mu_2 = 1/2$.

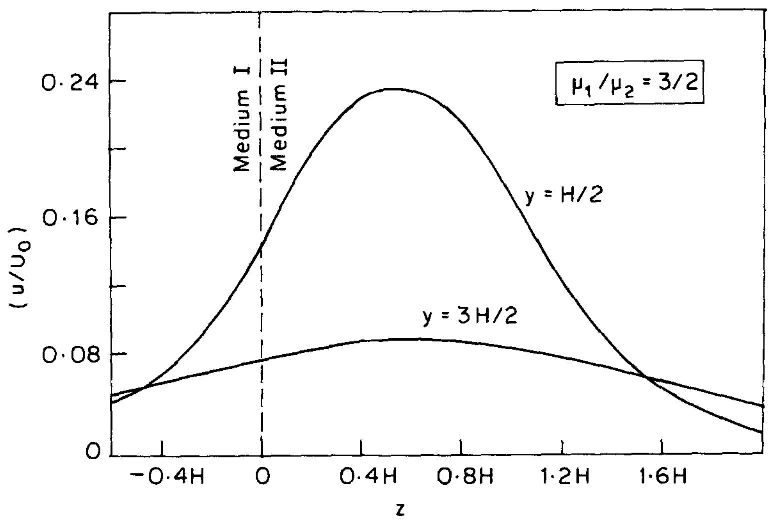

Figure 8. Variation of u/U_0 with z for $\mu_1/\mu_2 = 3/2$.

curve for the displacement corresponding to $y = H/2$ is smooth and peaked while the curve for $y = 3H/2$ is smooth and almost flat.

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Appendix ($\xi > 0$)

$$\int_0^\infty \exp(-k\xi)\sin k\eta \, dk = \eta/(\xi^2 + \eta^2) \tag{A1}$$

$$\int_0^\infty \exp(-k\xi)\cos k\eta \, dk = \xi/(\xi^2 + \eta^2) \tag{A2}$$

$$\int_0^\infty k\exp(-k\xi)\sin k\eta \, dk = 2\xi\eta/(\xi^2 + \eta^2)^2 \tag{A3}$$

$$\int_0^\infty k\exp(-k\xi)\cos k\eta \, dk = (\xi^2 - \eta^2)/(\xi^2 + \eta^2)^2 \tag{A4}$$

References

Ben-Menahem A and Singh S J 1981 *Seismic waves and sources* (New York: Springer Verlag)

Chinnery M A and Jovanovich D B 1972 Effect of earth layering on earthquake displacement fields; *Bull. Seismol. Soc. Am.* **62** 1629–1639

Maruyama T 1966 On two-dimensional elastic dislocations in an infinite and semi-infinite medium; *Bull. Earthquake Res. Inst., Tokyo Univ.* **44** 811–871

Rybicki K 1971 The elastic residual field of a very long strike-slip fault in the presence of discontinuity; *Bull. Seismol. Soc. Am.* **61** 79–92

Rybicki K 1973 Static deformation of a multilayered half-space by a very long strike-slip fault; *Pure Appl. Geophys.* **110** 1955–1966

Singh S J 1985 Static deformation of a multilayered half-space by two dimensional sources; *Acta Geophys. Pol.* **33** 123–134

Singh S J and Garg N R 1985 On two-dimensional elastic dislocations in a multilayered half-space; *Phys. Earth. Planet. Inter.* **40** 135–145