

Lightning-generated waves escaping out through plasma holes in the nightside Venus ionosphere

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Abstract. The plasma waves in the Venus ionosphere measured by OEFD aboard PVO are analysed. It is shown that these waves are generated by lightning like cloud-to-cloud discharges anywhere in the Venus ionosphere-surface waveguide. The theoretical minimum attenuation for waveguide mode propagation at 5.4 kHz is consistent with the maximum occurrence rate at this frequency. The lightning-generated and globally-propagating signals when encountered with plasma holes or ion-trough structures escape out partially and are detected by the OEFD aboard PVO. The 100 Hz signals can propagate upwards in whistler mode. Even the localized electrostatic mode waves would be converted into electromagnetic waves in the plasma holes and ion-trough regions.

Keywords. Planetary lightning; waves in planetary atmosphere; nightside Venus ionosphere; plasma holes; escaping waves.

1. Introduction

The electric field detector (OEFD) aboard Pioneer Venus Orbiter measures field strengths at four spot frequencies; 100, 730 Hz, 5.4 and 30 kHz (Scarf *et al* 1980b). The OEFD-recorded signals are known to be contaminated by sunlight and are not recorded by OEFD in dayside sector of the Venus atmosphere. The exact nature of these signals, whether electrostatic or electromagnetic, is not known and is to be inferred by some additional and indirect features of these signals. The OEFD measures strong impulsive electric field bursts at all the four frequencies whenever lower PVO nightside periapsis is encountered. Various features of 100 Hz signals led Taylor *et al* (1979) and Scarf *et al* (1980a) to find closer semblance with the earlier in situ measurements of electromagnetic noise using Venera-11 lander. The Venera measurements were made at four frequencies below the Venus cloud-top and were attributed to originate from Venus lightning (Ksanfomality 1979, 1980). Out of four frequencies aboard PVO, the 100 Hz signals were interpreted to be escaping out from the ionospheric region in whistler mode. The occurrence statistics of 100 Hz signals showed a positive correlation with the highland topography of Venus (Scarf and Russel 1983). This correlation implied the lightning origin of these signals and their stimulation by active volcanoes in the Venus highlands. This interpretation of Scarf and Russel (1983) was questioned by Taylor *et al* (1985) and an alternative suggestion that these waves are correlated with the localized ion density 'troughs' and are non-propagating electrostatic waves, was made. The exact process of generation of PVO recorded waves is unknown and the origin of these waves are attributed to the cloud-to-cloud lightning process.

These electromagnetic waves propagate through the Venus-ionosphere waveguide and propagate upwards whenever and wherever plasma holes are encountered on the nightside of Venus. Singh and Russell (1986) analysed the OEFD data and noted that apart from the 100 Hz signals, the higher frequency signals 730 Hz, 5.4 kHz and 30 kHz and at times also escape from the ionosphere and exhibit simultaneity and semblance with the impulsive nature of 100 Hz signals. The escape of signals is interpreted in terms of partial transmission through plasma holes or ion troughs whenever the ideal conditions of geometrical optics resulting in specular reflections are violated in the wake region of the nightside Venus ionosphere.

Taylor and Cloutier (1987) and Taylor *et al* (1987) have continued questioning the lightning interpretation of OEFD measured signals and have connected this interpretation with the question whether Venus is dead or alive. Despite raising these questions, no serious effort was made by Taylor and Cloutier (1987) to unfold the earlier suggestion that these signals are generated by some of the plasma instabilities. A detailed statistical study of occurrence rates of OEFD signals has shown that the impulsive wave bursts appear on all the four frequencies and these impulsive signals are different from the monotonic and conspicuous interference signals which appear at times (Russell *et al* 1988a, b, c). The existing provision of two antennae on OEFD has been used to show that these impulsive signals are polarized (Scarf and Russell 1988). Since the OEFD measurement does not measure the magnetic field vector of the wave, the question of wave mode is to be settled only by indirect evidences and circumstantial support. If these signals are generated by the Venus lightning, it must conform to global distribution of lightning, as in the case of lightning in the earth's atmosphere. The lightning in the Venus cloud should take place in the dayside, as well as in the nightside. The OEFD wave bursts are mostly recorded in the nightside of Venus. We proceed to analyse some features of the OEFD signals to show that these signals conform to the generation by Venus cloud-to-cloud lightning. These signals essentially propagate in the waveguide mode and fill the entire annular space between Venus surface and its ionosphere. Using the waveguide mode propagation theory, we show that the 6 kHz signals suffer minimum attenuation through the Venus-ionosphere waveguide and the attenuation increases at lower, as well as, at higher frequencies. The wake region of the Venus ionosphere is highly inhomogeneous showing plasma density holes and ion troughs. The magnetic field configuration in the hole region is such that the 100 Hz propagating waveguide waves escapes out in the whistler mode (Scarf and Russell 1983). In addition, we show that the OEFD recording of the higher frequency signals may not provide the exact location of lightning in the Venus clouds and the higher frequency signals may propagate long distances before their detection. The occurrence rate analysis of the OEFD signals at four frequencies is a combined effect of dayside and nightside lightning-generated signals propagating and escaping out of the shielding ionosphere. The escape of the higher frequency signals is primarily governed by the scale length of the wake region plasma inhomogeneities and configuration of the magnetic field in the region. The maximum occurrence rate of the 5.4 kHz signals is seen to change slightly from one PVO season to another.

2. Correlation of OEFD signals with nightside plasma holes

The electron density and ion density measurements have depicted the existence of significant structures with hung-up magnetic field in the wake region of the Venus

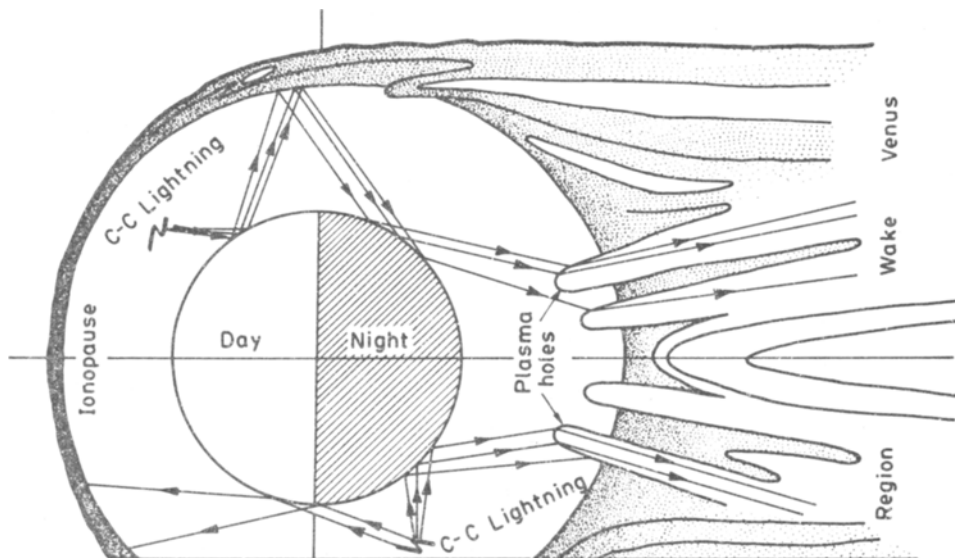


Figure 1. Schematic showing the waveguide mode propagation of Venus lightning generated signals and their escape upwards through the plasma 'holes' having radial magnetic field configuration.

ionosphere as schematically shown in figure 1. The depletions of electron density known as plasma 'holes' have been measured by the pioneer Venus orbiter Electron temperature probe (OETP) (Brace *et al* 1982, 1983). These plasma 'holes' are distributed on either side of the midnight meridian having varying north-south and east-west extents. The magnetic field configuration in the plasma 'holes' tend to be radial (Luhmann *et al* 1982; Marubashi *et al* 1985). The 100 Hz OEFD signal attributed to Venus lightning by Taylor *et al* (1979) and Scarf *et al* (1980a) propagates upward in the whistler mode along the radial magnetic field in the plasma 'holes' region. Taylor *et al* (1985) suggested that these waves are essentially electrostatic and are generated by solar wind plasma interaction with the nightside Venus ionospheric plasma but no concrete plasma wave generation mechanism has been worked out as yet. Taylor *et al* (1985, 1987) have also shown that 100 Hz signals tend to cluster in the region of ion density 'troughs'. The plasma 'holes' and ion 'troughs' are the same plasma structures measured by two different probes (Taylor 1989). A basic controversy exists regarding the origin and mode of the higher frequency waves detected by OEFD. It seems that with the available OEFD measurements and their limitations this controversy cannot be fully resolved. Analysis of the OEFD data, at best, can only provide indirect observational and interpretational support in resolving this controversy and this is presented in this paper. The 100 Hz signals escape out in whistler mode. The existence of plasma 'holes' plays an important role in allowing the upward propagation of higher frequency OEFD signals i.e. 730 Hz, 5.4 and 30 kHz, by means of wave diffraction. Therefore, these two signals show quite a different type of wave burst occurrence statistics. Since attenuation of 5.4 kHz is minimum, this shows maximum occurrence rate among the three higher frequencies.

3. Propagation and attenuation of lightning generated waves

The lightning phenomena is a well-known source of wideband electromagnetic wave radiation (Uman 1969). As on today, the existence of lightning in Venus cloud can neither be totally accepted nor ruled out. The high surface temperature of the planet and its decrease with altitude are important factors against the terrestrial-like lightning. However, dense and thick aqueous sulphuric acid cloud layer at higher altitude which correspond to the maximum cosmic ray ionization (Borucki *et al* 1982) are some of the important features which support the charging of cloud deck and the possibility of existence of cloud-to-cloud discharges. The signals generated due to these discharges propagate through the Venus-ionosphere waveguide and leak or reradiate upwards when plasma 'holes' in the nightside Venus ionosphere are encountered. The impulsive nature and the recorded intensity of these signals depend on the total path traversed by the signals within the waveguide before getting leaked or reradiated upward through the nightside plasma 'holes'. We use the standard mode theory formulation for estimating the attenuation of lightning generated signals while propagating through the Venus-ionosphere waveguide. In order to apply this theory, we assume the Venus surface to be sharply bounded and infinitely conducting. The ionospheric boundary is characterized by finite conductivity. The height of the waveguide is governed by the changing altitude between the Venus surface and its ionosphere. The wave propagation depends on other parameters such as the plasma frequency and the collision frequency of the ionospheric boundary. The global concentric space between the Venus surface and the ionosphere behaves like a waveguide structure with a certain cut-off frequency. The orientation and positional occurrence of plasma holes or ion troughs are likely to govern the escape or reradiation of signals from the ionosphere. These features also govern the statistics of the signals recorded by the OEFD aboard PVO.

The electromagnetic wave propagation between the two spherical parallel conducting plates is generally theoretically discussed using the mode theory (Bremmer 1949; Budden 1952 and Wait 1957a, b). The VLF waves reflecting between the parallel spherical shell formed by the Venus lower ionospheric and the Venus surface would likewise propagate in various modes satisfying the transcendental wave mode equation (Wait 1957a, b, 1962)

$$R_i R_v = \exp(+i4\pi h C_n / \lambda), \quad (1)$$

where h is the height between Venus spherical surface and the Venus spherical reflecting ionospheric layer, λ is the wavelength of the propagating waves, the cosine of the complex angle of incidence is denoted by C_n and $n = 0, 1, 2, \dots, n$ defines various modes of wave propagation of lightning generated electromagnetic waves. The derivation of this equation is based on many simplifying physical and mathematical assumptions that are not detailed here and readers may refer to the original derivation. The left hand side of (1) is the product of reflection coefficients R_v and R_i which refer to the Venus surface and the Venus ionospheric layer respectively. The approximate solution of the wave mode equation (1), for the parallel guiding spherical shell formed by the Earth's surface and the Earth's ionosphere, was obtained by Wait (1957). With appropriate change of parameters, the solution of (1) can also be used in the case of

guiding shell formed by the Venus surface and its ionosphere. We assume that Venus surface is perfectly reflecting ($\sigma \rightarrow \infty$) and the ionospheric reflection is partial which is governed by the varying collision frequency ν and the plasma frequency ω_p of the Venus lower ionospheric layer. Taking asymptotic expansion for the Legendré function, the maximum electromagnetic wave radiation from a vertical radiating dipole at a radial distance d within the parallel spherical guiding shell is written as the mode sum (Wait 1957a, b)

$$E = E_0 W \text{ mV m}^{-1}, \tag{2}$$

where E_0 is the electric field radiated by a lightning stroke assumed as a vertical dipole here. If the radiated power P is expressed in kilowatts and the radial distance from an assumed vertical lightning discharge d (expressed in km), the electric field is written as

$$E_0 = 300(P)^{1/2}/d \text{ mV m}^{-1}. \tag{3}$$

The dimensionless factor W in (2) represents the changes taking place in the signals while propagating through the waveguide and also includes signal attenuation. It also depends on various waveguide parameters and the complex functions of angles satisfying the wave mode equation (1).

$$W = \left(\frac{d/a}{\sin d/a} \right)^{1/2} \frac{(d/\lambda)^{1/2}}{(h/\lambda)} \left| \sum_{n=0}^{\infty} \delta_n S_n^{3/2} \exp[-i2\pi S_n(d/\lambda)] \right|, \tag{4}$$

where h and λ are expressed in kilometres $d \simeq a\theta$, a is the Venus radius in km

$$\begin{aligned} \delta_n &= [1 + \sin(2khC_n)/2khC_n]^{-1} \simeq 1/2 \quad \text{for } n = 0 \\ &\simeq 1 \quad \text{for } n = 1, \dots, n. \end{aligned}$$

The waveguide mode attenuation of the lightning signal is mainly governed by the exponential term in (4). The complex sine function appearing in the exponential when separated in real and imaginary parts are rewritten as

$$\text{Re } S_n = \bar{S}_n + \frac{1}{(2)^{3/2} \pi (h/\lambda) \bar{S}_n} \left[\frac{(n - \frac{1}{2})^2}{(2h/\lambda)^2} \left((L)^\ddagger - \frac{1}{(L)^\ddagger} \right) \right], \tag{5}$$

$$\text{Im}(S_n) = -\frac{1}{(2)^{3/2} \pi (h/\lambda) \bar{S}_n} \left[\frac{(n - \frac{1}{2})^2}{(2h/\lambda)^2} \left((L)^\ddagger + \frac{1}{(L)^\ddagger} \right) \right], \tag{6}$$

$$\bar{S}_n = (1 - C_n^2)^\ddagger = [1 - (n - \frac{1}{2})^2/(2h/\lambda)^2]^\ddagger, \tag{7}$$

and

$$L = \omega\nu/\omega_p^2.$$

The attenuation of lightning-generated signals satisfying the wave mode equation in decibels per km of propagation path in the Venus-ionosphere waveguide is thus written as

$$\alpha_n = -8.68 \times \frac{2\pi}{\lambda} \text{Im}(S_n) \quad n = 0, 1, 2, \dots, n. \tag{8}$$

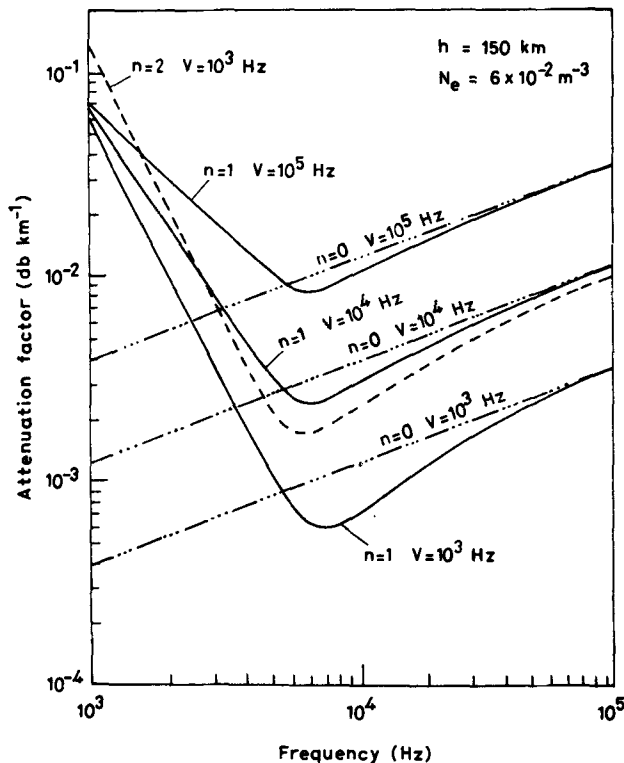


Figure 2. Variation of attenuation factor computed using mode theory of VLF wave propagation through the Venus-ionosphere waveguide. The wave attenuation shows a minimum attenuation for $n > \text{zero}$ modes around $f \approx 6 \text{ kHz}$.

The variation of wave attenuation with frequencies satisfying the wave mode equation (1) for infinite Venus surface conductivity, different values of ionospheric parameters and for different wave modes is computed and the results are shown in figure 2.

4. Results and discussion

The wave measurements carried out by OEFD have provided a rich data base. Since the magnetic field of the waves has not been measured, the interpretation of these waves as electromagnetic or electrostatic is not yet fully agreed and settled. Two schools of thought have emerged over the years. Indirect evidences have been forwarded by one school to establish that OEFD signals are lightning-generated (Taylor *et al* 1979; Scarf *et al* 1980; Scarf and Russell 1983; Singh and Russell 1986) while the other school believes that the OEFD signals are basically localized electrostatic waves generated by some of the plasma instability mechanisms (Taylor *et al* 1985). The polarization measurements of OEFD signals (Scarf and Russell 1988) and detailed occurrence rate analyses of all the OEFD signals: 100, 730 Hz, 5.4 and 30 kHz have established beyond doubt that these are real impulsive wave bursts and, at times, these signals appear at all the four frequencies Russell *et al* 1988a, b, c).

The 100 Hz signals were noticed first and have been studied extensively and are believed to propagate out of the Venus ionosphere in whistler mode. The occurrence rate of 100 Hz signals is comparatively larger than the occurrence rates of other three frequencies namely 730 Hz, 5.4 and 30 kHz. The geometrical optics approach does not allow electromagnetic waves of frequency ω to propagate out of the ionospheric region whose plasma frequency $\omega_p > \omega$. A mechanism of partial transmission of waves illuminating the plasma inhomogeneity or plasma 'hole' has been discussed by Singh *et al* (1987).

The Venus cloud-to-cloud lightning produces a wideband frequency spectrum with a field intensity peak in VLF range. The radiated intensity of the lightning signals decrease fast at lower frequencies (f^{-2}) as compared to f^{-1} variation at higher frequencies. If Venus lightning takes place, in all probability, it would be a global phenomenon. This implies that the VLF signal generated by the lightning process would also be propagating globally through the Venus-ionosphere waveguide. Thus it is quite obvious that the lightning signals generated in the dayside region of Venus cloud-top can propagate through the waveguide to nightside sector of Venus and vice-versa. The nightside ionosphere contains plasma 'holes' and ion 'troughs' which manifest the escape of the signals upwards (Singh *et al* 1987). Such an escape of signals is not possible on the dayside. This scenario has been schematically shown in figure 1. In view of this scenario some important facts emerge out: (i) We find that invariably the signals generated from the lightning source propagate through the waveguide before these signals encounter plasma 'holes' in the nightside region of the Venus ionosphere, (ii) Each of these events are characterized by varying degrees of wave mode attenuation through the waveguide, (iii) The 100 Hz signals do not satisfy the waveguide mode equation and propagate in cavity mode. In addition, the 100 Hz wave have distinct features and are capable of escaping out of the Venus ionosphere in the whistler mode, (iv) Since lightning sources are necessarily not just below the observation site, only a limited number of carefully chosen 100 Hz events can be used for extrapolating their propagating path and correlating their origin with the highland topography of Venus, (v) Because of limited escape of signals through plasma 'holes' and 'ion-troughs' it appears that the impulsive wave burst events measured by OEFD are only a small fraction of the total global Venus lightning events. The OEFD measured occurrence rate statistics therefore cannot be used for comparing it with the observed terrestrial lightning rate.

The attenuation of the lightning in the frequency range 1–30 kHz for two values of the L -parameters and for $n = 0, 1, 2$ have been computed and its variation is shown in figure 2. The attenuation for $n = 0$ wave mode is minimum at 1 kHz and is seen to increase with increasing frequencies. The change of collision frequency is shown to affect the wave attenuation significantly. For $n = 1, 2$, the wave attenuation shows a 'knee' with a minimum attenuation. The wave attenuation around this 'knee' is seen to decrease with increasing collision frequencies. The maximum in the wave amplitude spectrum of OEFD measured signals is also found to appear in this frequency. The signal strength was found to decrease on either side of this peak (Singh and Russell 1986) as reproduced in figure 3. Exception to general attenuation at 100 Hz is well known. This feature of the 100 Hz OEFD signal arises because of its unattenuated escape out of the Venus ionosphere in whistler mode (Singh and Russell 1986). The appearance of minimum attenuation in the Venus-ionosphere waveguide as well as an amplitude peak obtained from the analysis of OEFD measured signal demonstrate

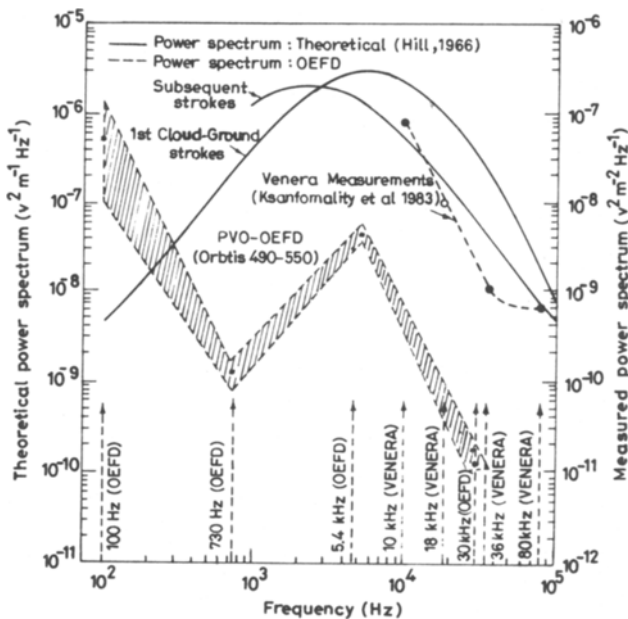


Figure 3. Power spectral distribution of OEFD signals showing a peak around $f \approx 6$ kHz. The signal at low frequencies shows further increase because of its unattenuated escape out of the Venus ionosphere in whistler mode. Comparison with theoretical and experimental results shown by Singh and Russell (1986).

the possibility of waveguide mode propagation of lightning-generated signals. Therefore the outcome of the present theoretical analysis and its comparison with the experimental wave amplitude spectrum are important though it provides only an indirect support in favour of lightning interpretation of OEFD signals.

The impulsive wave bursts occurrence rate at 5.4 kHz has been replotted for six PVO seasons (Russell *et al* 1988a) and is shown in figure 4. The occurrence rates of impulsive wave bursts at 5.4 kHz seem to be at closely spaced altitudes for all the six PVO seasons. The occurrence rates of impulsive bursts do not maximize at the lowest altitude. Since the average occurrence rate does not bring out details of statistical importance, some of the important features of these events can best be studied by an event-to-event analysis. The correlation of higher frequency occurrence rates with plasma 'holes' or ion 'troughs' has not been reported. A careful study of such correlations is important to ascertain their propagation characteristic inferred by indirect means.

5. Conclusion

The OEFD data aboard PVO are available from the nightside orbits of Venus. In the nightside of Venus the wave burst events are not uniformly distributed and show maximum occurrence rate on either side of midnight meridian. These features agree well with the scenario of cloud-to-cloud electrical discharges producing electromagnetic waves. These waves seem to propagate through the Venus surface-

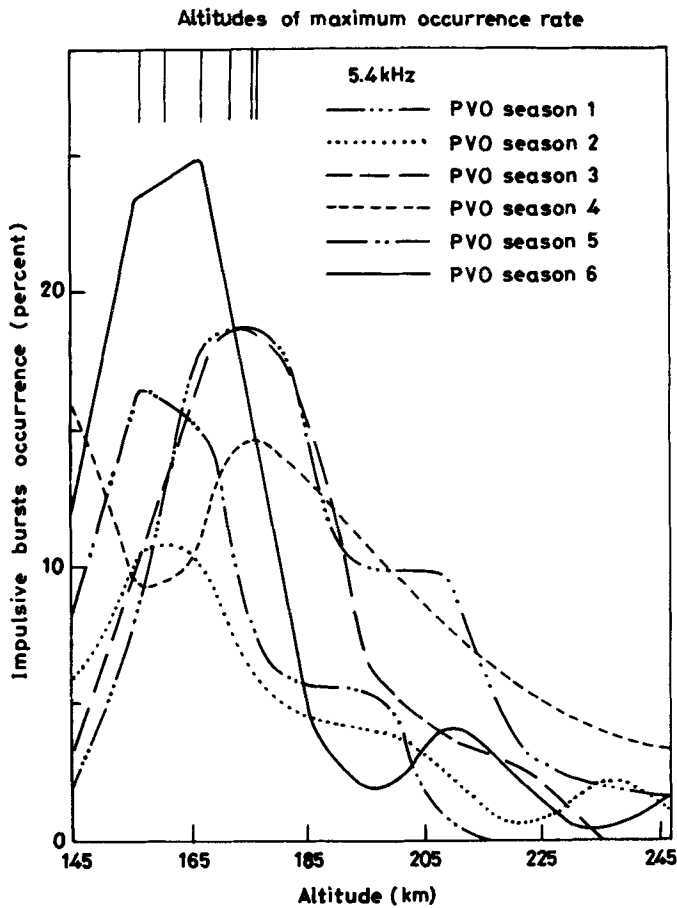


Figure 4. Occurrence rate statistics of 5.4 kHz OEFD signals for six PVO seasons showing some variation in the altitudes of occurrence rate peaks from one season to another season (replotted from Russell *et al* 1988a).

ionospheric waveguide and may escape out whenever plasma ‘holes’ or ‘ion-troughs’ are encountered. Most of the statistical features of these waves conform well with the electromagnetic wave mode. The electromagnetic wave interpretation has been disputed and an alternative suggestion of electrostatic mode waves has been made and pursued. The observed broadband spectral feature with a peak around 6 kHz rules out the electrostatic mode of these waves. Further, the chances of their escaping out enabling detection at higher altitudes would be almost impossible. Thus on the basis of available information, it seems certain that OEFD-measured waves are generated by lightning activity in the Venus clouds. The optical measurements and acoustic wave measurements may provide supporting evidences.

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