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An analysis of Indian tide-gauge records

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Abstract. The paper presents an analysis of four Indian tide-gauge records. The stations were: Bombay, Madras, Cochin and Vishakhapatnam (Vizag). They were selected because of their reliability.

There was no evidence of a monotonic rising trend at all four stations. The test by Mann and Kendall (loc. cit.) showed a rising trend at Bombay from 1940 to 1986 and at Madras from 1910 to 1933. The other records did not reveal a significant trend.

The records reveal evidence of long-period cycles (50–60 year period), with shorter cycles (4·5 to 5·7-year period) riding on them. Spectral peaks corresponding to shorter cycles passed a false alarm probability test at 95% level of significance. The peaks were identified by computing periodograms and by maximizing the entropy of the time series.

ARIMA models suggest a third order autoregressive model for Bombay and Madras (1953–1986). The remaining records only had a moving average component.

Monthly tide-gauge data of Bombay reveal a 13·4-month cycle which was statistically significant. This was close to the 14·7-month Chandler wobble. But, an interaction between a 13·4-month and an annual cycle could not fully explain the observed short period cycles.

Finally, the paper summarizes evidence to indicate that a pattern exists between fluctuations of monsoon rain and relative sea level at Bombay.

Keywords. Tide gauge records; spectral analysis; ARIMA models; Poletide; monsoon rain.

1. Introduction

Recent concern over global warming has led to a resurgence of interest on fluctuations in relative sealevel. Comprehensive review have been provided on this topic by Charnock (1987) and Stewart *et al* (1990). The evidence for a uniform rise in sealevel is uncertain because coastal movements are not decoupled from changes due to thermal expansion of water or glacier melt. But Peltier and Tushingham (1989) found a more coherent signal of $2{\cdot}40 \pm 0{\cdot}90$ mm/yr if glacial movements were removed. As this was not known with much precision over India, the present study was confined to changes in relative sealevel at four Indian coastal stations.

The purpose of this paper is to examine trends and stochastic features. It was suggested (Munk and Forbes 1989) that warming induced by CO_2 and other greenhouse gases could be around 0·05°C per decade in the coming years. From the time of the industrial revolution (1860) to the present the atmosphere has been estimated to have warmed by 0·5 to 1·0°C. An examination of trends enables us to see if it has left a signal on Indian tide-gauge records.

Emery and Aubrey (1989) used Student's t-test to assess the reliability of records from 12 principal tide-gauges in India. The acceptable records were those with a t-value of 1·0 and a time span of 24 years or more. By this criterion four most accept-

Table 1. Tide-gauge records.

Station	Lat.(°N)	Long.(°E)	Time span
Bombay	18° 55′	72° 50′	1886–1986
Madras (i)	13° 06′	80° 18′	1881–1933
Madras (ii)	—	—	1953–1986
Cochin	09° 58′	76° 16′	1955–1986
Vishakhapatnam	17° 41′	83° 07′	1953–1986

able records were from (i) Bombay, (ii) Madras, (iii) Cochin and (iv) Vishakhapatnam (Vizag). The station at Calcutta (Garden Reach) had a high t-value, but its records were influenced by a high frequency of tropical cyclones and storm surges. Consequently, it was decided not to use it for the present.

2. Data

The location of the stations and the time span of records are shown in table 1.

The Madras records had a gap of 19 years from 1934 to 1952. Hence, the data were divided into two sets, namely, from (i) 1881 to 1933 and (ii) from 1953 to 1986.

The data were kindly provided by the Survey of India. According to information provided by them there were two changes in the reference level at Bombay, but a correction was applied to compensate for these changes. A similar correction was applied at Madras, where one change in the reference level was made in the past. No changes in the reference points for Cochin and Vishakhapatnam were made during the periods shown in table 1.

The accuracy of the readings was estimated by the Survey of India to be $\pm 1{\cdot}0$ cm for Bombay, and $\pm 2{\cdot}0$ cm for the other stations. The typical readings varied from 50 to 300 cm, so the instrumental errors were around 1–4% of the recorded amplitude. In the absence of more precise information, it was assumed that the errors, both positive and negative, were evenly distributed in time, so the trends were not influenced by errors.

By fitting regression lines Emery and Aubrey (1989) found linear trends of 1·3 to $-2{\cdot}1$ mm/yr, with an average of $-0{\cdot}5$ mm/yr for these four stations. This suggests a monotonic trend for the entire time span of each record, which is probably not the case.

3. Methods of analysis

A preliminary study with a limited data set for Bombay was presented by Das (1990). The present paper provides a more complete analysis.

3.1 *Trends*

Correlograms were prepared for the records of each station in table 1. This is a plot of serial correlation coefficients (r_k) against the lag (k). We have

$$r_k = \mathrm{cov}(u_j, u_{j+k}) \div [(\mathrm{var}\, u_j, \mathrm{var}\, u_{j+k})]^{1/2}, \tag{1}$$

where $U_1, U_2, \ldots U_n$ represent individual observations. Correlograms indicate the type of oscillations that might be present in the series (Kendall 1943). There could be (i) oscillations vanishing after a certain point, (ii) indefinite oscillations and (iii) oscillations with a tendency for damping with large values of k. The first is seen in a time series generated by moving averages, while the second is present in a time series made up of harmonic terms. The last one, that is, a correlogram with damping, is found in an autoregressive series. The correlograms of Indian tide-gauge records were indicative of a series made up of either harmonic terms or of an autoregressive nature.

Mann-Kendall's test (Kendall and Stuart 1961) was next applied to find trends. The test seeks to find out if the n-independent observations of a record were ordered in some way. The hypothesis of a trend is tested against a no trend hypothesis based on a measure of the disarray amongst individual observations.

If $U_1, U_2, \ldots U_n$ be the individual observations, then for each observation (U_i) we computed the number of inversions, that is, the number of observations (U_j) which precede $U_i (j < i)$ and for which $U_j < U_i$. If the number be n_i then the test statistic

$$s_k = \sum_{1}^{i=k} n_i \tag{2}$$

is used to measure trend. Under the null hypothesis of all observations being independent, s_k will be normally distributed. Its mean and variance will be expressed by

$$\mu = k(k-1)/4, \tag{3}$$

$$\sigma^2 = k(k-1)(2k+5)/72. \tag{4}$$

To test the departure from a normal distribution we put

$$Z = (s_k - \mu)/\sigma. \tag{5}$$

If Z was a random variable with a normal distribution, then the confidence limits of Z in terms of probability (P) are

$$P(-1{\cdot}96 < Z < 1{\cdot}96) = 0{\cdot}95, \tag{6}$$

S_k and Z were computed for each record to find if Z satisfied 95% confidence limits expressed by (6). If values of Z lay beyond these limits then a trend was indicated.

3.2 *Trend removal*

First differences were used to remove the trend. This provided a new series for each record.

A test was applied to find out if the order of differencing was adequate. This seeks to ascertain if the outcome of differencing led to a random process, that is, it satisfied a test for white noise.

Box and Pierce (1970) and later Ljung and Box (1978) showed that for a random process the statistic

$$\phi(k) = n(n+2) \sum_{k=1}^{K} (1/n-k) r_k^2 \tag{7}$$

will have a χ^2 distribution with K degrees of freedom. By applying this test it was found that first differences were sufficient to remove the trend in each record.

3.3 *Spectral analysis*

3.3a *Periodograms*: As only 4 tide-gauge records were considered, it was decided to make a spectral analysis instead of empirical orthogonal functions (EOF). The procedure for spectral analysis is well documented and only a brief outline is provided for continuity with § 5 on results.

The time series $X(t)$ was assumed to have the Fourier representation

$$X(t) = a_0/2 + \sum_1^\infty a_n \cos n\pi t/T + b_n \sin n\pi t/T, \tag{8}$$

where T was the period and the coefficients a_n, b_n were

$$a_n = 2/T \int_0^T x(t) \cos nt \, \mathrm{d}t,$$

$$b_n = 2/T \int_0^T x(t) \sin nt \, \mathrm{d}t. \tag{9a, b}$$

The total power (or variance) in the interval $(-T, T)$ was

$$\sum_0^\infty c_n^2 = 1/2 \sum_0^\infty (a_n^2 + b_n^2). \tag{10}$$

The aim was to find the contribution of each cycle with a frequency of $(n/2\pi)$ cycles per year (cpy) to the total variance. The variance was expressed by the Fourier transform of $X(t)$. We have

$$X(t) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} G(\omega) \exp i\omega t \, \mathrm{d}\omega \tag{11}$$

whence the variance was

$$\int_{-\infty}^{\infty} x^2(t) \, \mathrm{d}\omega = \int_{-\infty}^{\infty} |G(\omega)|^2 \mathrm{d}\omega. \tag{12}$$

The frequency was represented by ω.

The spectral density $h(\omega)$ was the contribution of each frequency band to the variance. It is related to the autocovariance $R(k)$. Thus,

$$R(k) = 1/n \sum_{t=1}^{n-k} X(t) X(t+k) \tag{13}$$

and

$$h(\omega) = 1/2\pi \left[R(0) + 2 \sum_1^{n-1} R(k) \lambda(k) \cos k\omega \right]. \tag{14}$$

$R(k)$ and the spectral density are Fourier transforms of each other, assuming that the transforms exist. $\lambda(k)$ is a spectral window. It represents a system of weights to cut off sample autocovariances beyond a certain value of n, say M, so that they conform more correctly to the time distribution of $R(k)$. Different spectral windows have been designed (Priestley 1981; Jenkins and Watts 1968). We used a Parzen window (Parzen 1961) because of its simplicity. It does not generate negative values of the spectral density and resembles a Gaussian curve with its peak at $k = 0$, and no tail beyond $k > M$.

The periodogram of a discrete time series of n observations is

$$I_n(\omega)/\sigma^2 = 2/n|X(t)\exp(-i\omega t)|^2, \tag{15}$$

where σ^2 is the variance of the data sampled. It is the equivalent of the Fourier transform of $X(t)$ in (12). The expression on the left of (15) is the normalized periodogram.

3.3b *Maximum entropy*: An alternative approach is an algorithm that seeks to maximize the entropy or information content of a time series. It was proposed by Burg (1967, 1968). Subsequently, different aspects have been described by Ulrych and Bishop (1975); Lacoss (1971) and Priestley (1981). The entropy (E) of a time series is related to the spectral density by

$$E = \int_{-\infty}^{\infty} \ln h(\omega)\, d\omega. \tag{16}$$

This is maximized by a variational approach subject to the constraints

$$\int_{-\infty}^{\infty} h(\omega)\exp(i\omega k)\, d\omega = r_k \tag{17}$$

for $k = 0, \pm 1, \ldots \pm K$. The sample covariances are represented by r_k.

It leads to an expression for the spectral density

$$h_m(\omega) = [h_m \Delta t] \div \left[1 - \sum_{j=1}^{m} a_j^* \exp(-2\pi i j\, \omega t)\right], \tag{18}$$

where Δt is the sampling interval, m is an index of time (t) and the coefficients a_j^* are called the prediction error filters. They are computed by expressions for forward and backward prediction. We have

$$X(t) = \sum_{j=1}^{\infty} a_j X(t-j) \tag{19}$$

$t = m+1, m+2, \ldots$, and

$$\bar{X}(t) = \sum_{j=1}^{\infty} \bar{a}_j \bar{X}(t+j) \tag{20}$$

for $t = 1, 2, \ldots n - m$.

Differences between predictions by (19) and (20) and actual observations of $X(t)$ are residuals which are minimized by least squares. Denoting the sum of residuals by $S(m)$, the expression for $h(m)$ in (18) is

$$h(m) = S(m)/2(n-m), \tag{21}$$

where m is now the number of normal equations needed to determine the coefficients a_j^*. The details of the procedure have been provided by Barrodale and Erickson (1980).

3.3c *False alarm probability*: This is a test to find the statistical significance of peaks revealed by spectral analysis. It ensures that the intensity of a spectral peak was larger than what would appear from noise (MacDonald 1989; Priestley 1981).

The spectral ordinates of the periodogram are located at frequencies $2\pi p/N$, where $p = 0, 1, \ldots N/2$. If they were independently distributed, they follow a χ^2 distribution with 2 degrees of freedom. The probability density here is one of exponential decay (Priestley 1981). Let γ be the maximum value of a peak. Under the null hypothesis of an independent distribution of ordinates we have for any positive number z

$$P(\gamma > z) = 1 - (1 - \exp(-z/2))^{n/2}. \tag{22}$$

By specifying a significance level α, we find a critical region ($\gamma > z_0$) for which the right side of (22) was equal to α. The test was, in brief, to find z_0 and to see if the maximum spectral ordinate exceeded z_0.

4. Stochastic models

Stochastic models have an autoregressive (AR) and a moving average (MA) part. The time series was first made stationary by finite differencing. As this is a process of integration (I) these models are referred to as ARIMA models. If after taking d finite differences the series is stationary, its structure may be represented by

$$X_t + \phi_1 X_{t-1} + \cdots + \phi_p X_{t-p} = Z_t + \theta_1 Z_{t-1} \cdots + \theta_q Z_{t-q}. \tag{23}$$

The order of the model is (p, d, q) where p and q are the orders of the autoregressive and moving average parts. Standard procedures for determining the coefficients $\phi_1 \ldots \phi_p$ and $\theta_1 \ldots \theta_q$ have been described by (Pankratz 1983) and will not be repeated here. These techniques are described by Box and Jenkins (1976) who pioneered this approach. The procedure involves finding solutions to a set of simultaneous equations known as the Yule-Walker equations.

The adequacy of a prescribed model was tested by a cumulative periodogram of residuals. As before, the residuals are deviations of model-generated values from actual observations. If the specified model was adequate then the residuals satisfy a test for white noise. The cumulative spectrum is

$$C(\omega_j) = \sum_{i=1}^{j} R^2(\omega_i)/ns^2. \tag{24}$$

where

$$R^2(\omega_i) = 2/n\left[\left(\sum_1^n a_t \cos 2\pi\omega_i t\right)^2 + \left(\sum_1^n a_t \sin 2\pi\omega_i t\right)^2\right]. \tag{25}$$

The residuals are represented by a_t, while s^2 is an estimate of error variance. The frequencies are expressed by ω_i.

The Kolmogorov-Smirnov bounds were used to test confidence limits of the cumulative spectrum. If $C(\omega_j)$ be plotted against the frequency the two should be related by a straight line inclined at an angle of $\pi/2$ to the axes of reference (Vandaele 1983). But, due to model deficiencies there are deviations from the theoretically predicted straight line. Kolmogorov and Smirnov (Box and Jenkins 1976) have provided confidence limits for the test. For a 95% confidence interval, the critical region is bounded by $\pm 1{\cdot}36/(m)^{1/2}$ on both sides of the theoretical line, where $m = (n-2)/2$ for even values of n, and $m = (n-1)/2$ for odd n. If the cumulative periodogram lies within these bounds then the white noise test is satisfied.

5. Results

5.1 *Statistical features*

To save space we show the time series for Bombay in figure 1. The time series for the remaining three stations, including Madras (i) and (ii) were also considered. Their main features are summarized in table 2.

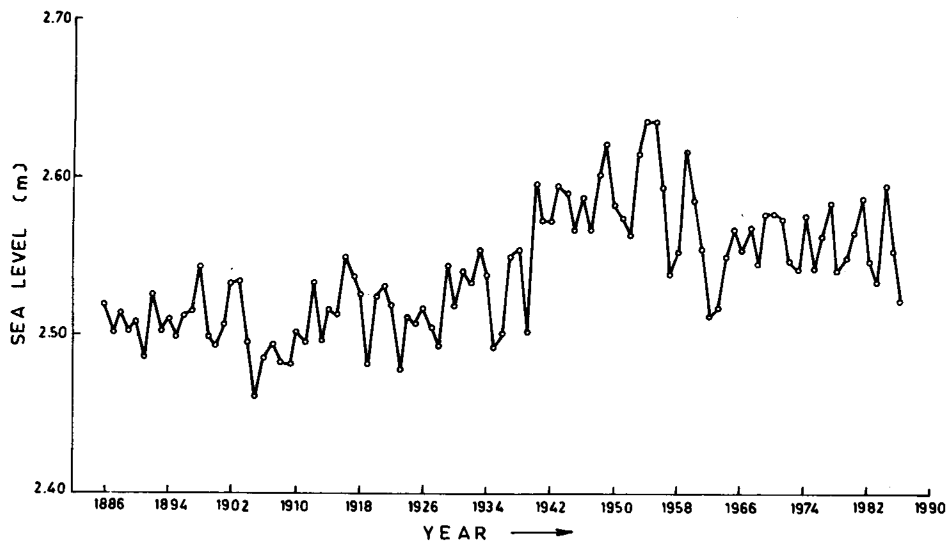


Figure 1. Time series of tide gauge records: Bombay (levels).

Table 2. Statistical features.

Station	Time span (yr)	Mean (m)	Variance $(m^2) \times 10^{-2}$
Bombay	100	2·542	0·2680
Madras (i)	53	0·587	0·1038
Madras (ii)	34	0·624	0·1434
Cochin	32	0·655	0·8025
Vizag	34	0·817	0·1959

5.2 *Correlograms*

The correlogram for Bombay is shown in figure 2. Similar correlograms were also prepared for the other records. As we can see, the correlogram oscillates in an irregular manner. There was a tendency for slight damping of the oscillations up to $k = 40$, but subsequently the oscillations increased in amplitude. The correlograms for other stations showed similar oscillations but the tendency for damping was not pronounced. This suggests that the records were made up of harmonic terms, with tendency for an autoregressive nature.

5.3 *Trends*

The variation of the test statistic Z (equation (5)) with time is shown in figures 3(A, B, C, D, E) for each record.

There was evidence for an increasing trend in Bombay records after 1940, and in Madras between 1910 and 1933. The dotted lines on each figure show the confidence limits of Z. Variations within the confidence limits do not indicate a significant trend.

The trend at Bombay for the period 1940–1986 was estimated to be approximately

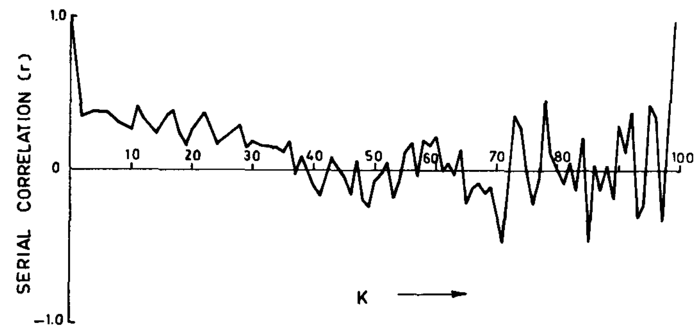
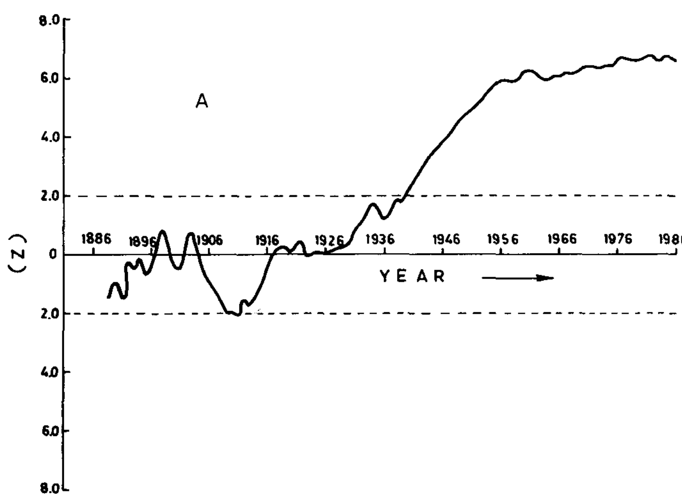


Figure 2. Correlogram: Bombay.



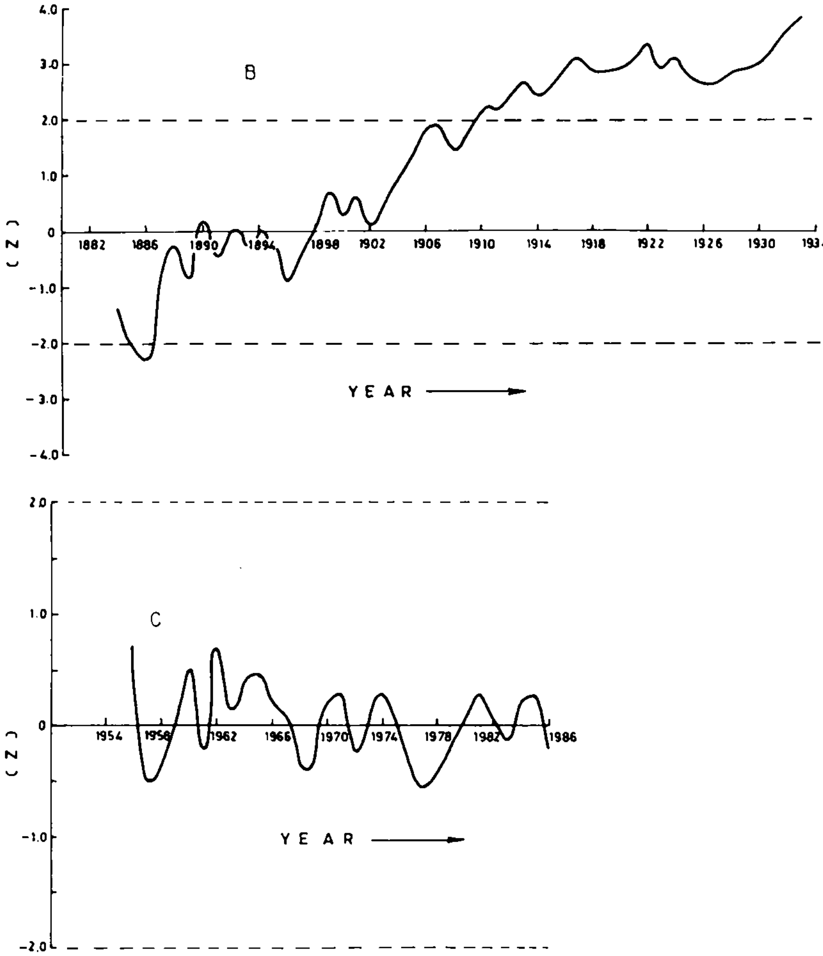
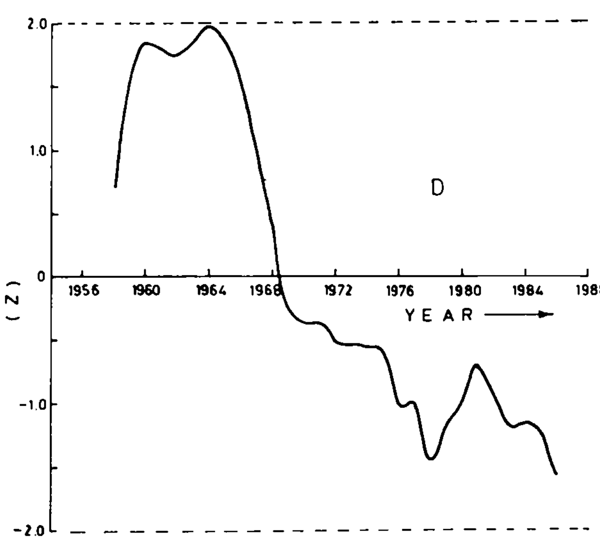


Figure 3A–C. Mann-Kendall tests for trends: (**A**) Bombay; (**B**) Madras (i); (**C**) Madras (ii).



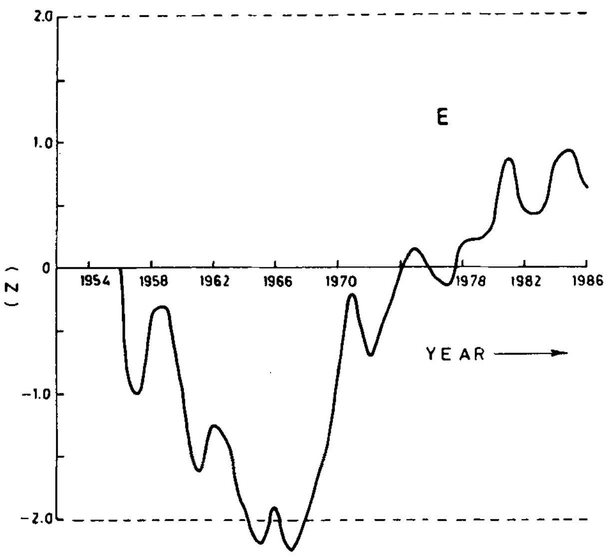


Figure 3D–E. Mann-Kendall tests for trends. **D.** Cochin. **E.** Vizag.

0·8 mm/yr, but no trend was visible in the records prior to 1940. The trend at Madras from 1910 to 1933 was 0·4 mm/yr but, interestingly, no trend was visible during the subsequent period from 1953 to 1986.

From these observations it is difficult to infer that the rising trend at Bombay was the outcome of rapid growth in industry after 1940. Firstly, the values are for relative sealevel and, secondly, a similar trend was not observed at the other stations. Moreover, after 1986 the rising trend at Bombay showed a tendency towards decrease. It is difficult at this stage to associate the observed trends with uniform warming of the atmosphere.

5.4 *Spectral peaks*

An analysis was first made of the data at levels, that is, without differencing. The results showed that most of the variance resided in long-period cycles whose periods were around 50 years. The spectrum for Bombay is shown in figure 4, but similar spectra were computed for the other records.

The long-period cycles were subsequently removed by first differences. A diagnostic check was made with $\phi(k)$, the Ljung-Box statistic (equation (7)). For the Bombay records this test statistic did not exceed the critical χ^2 value for 30 degrees of freedom at 95% or 99% level of significance for first differences. But, the critical value was exceeded when second differences were considered. Similar results were found for all the other records. It was thus inferred that first differences were adequate to remove long-period trends.

Periodograms were computed for the residuals, that is, after removal of long-period cycles by first differences. Figure 5 shows the periodogram for Bombay. The angular frequency is shown in units of radians/year. This is the product of 2π and the circular frequency, and the period (T) is the reciprocal of the circular frequency. The Nyquist

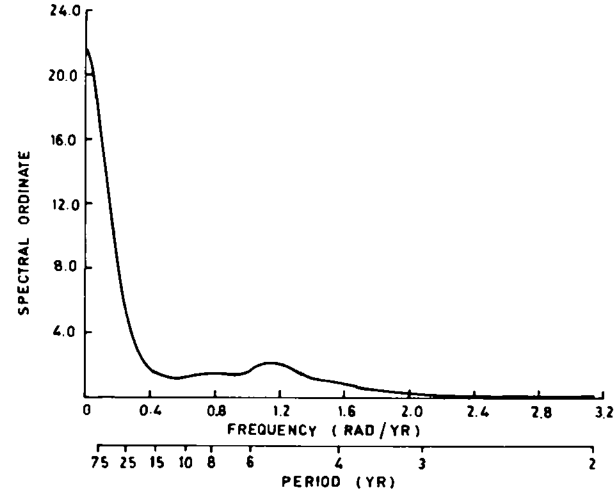


Figure 4. Power spectrum: Bombay (levels).

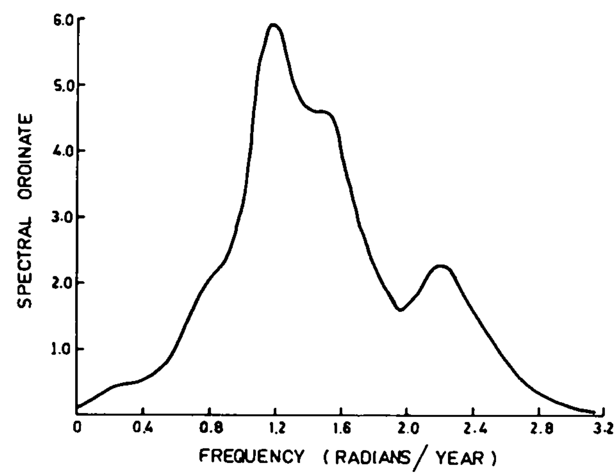


Figure 5. Power spectrum: Bombay (first differences). (M = 30, L = 64).

frequency was 3·14 rads/yr for each record. The principal spectral peaks are summarized in table 3.

Figure 5 contains information on the number of autocorrelation lags (M) and the number of ordinates (L). To save space the periodograms of other stations have not been reproduced, but they were considered and the results are summarized in table 3. A two-point moving average was used to filter out high frequency noise in each record.

Table 3 indicates 4·5–5·7 year cycles at all stations except Vishakhapatnam (Vizag). These cycles appear to ride on the longer-period (50-year) cycles. The spectral peaks revealed by the maximum entropy algorithm are shown in table 4 for comparison.

The results of tables 3 and 4 are comparable but for small differences. This lends support to the authenticity of 4·5–5·7-year cycles.

Table 3. Principal spectral peaks.

Station	Angular frequency (rads/yr)	Period (yrs)
Bombay	1·2	5·2
Madras (i)	1·1	5·7
Madras (ii)	1·4	4·5
Cochin	1·2	5·2
Vizag	1·7	3·7

Table 4. Spectral peaks (maximum entropy).

Station	Angular frequency (rads/yr)	Period (yrs)
Bombay	1·4	4·5
Madras (i)	1·4	4·5
Madras (ii)	1·6	3·9
Cochin	1·1	5·7
Vizag	1·7	3·7

Table 5. False alarm probability (95% level).

Station	Maximum normalized periodogram peak	Z_0	Number of frequencies
Bombay	5·9	4·8	64
Madras (i)	4·7	3·9	40
Madras (ii)	4·6	3·9	40
Cochin	3·8	3·9	40
Vizag	6·0	3·9	40

The spectral peaks of the table 3 were tested for false alarm probability at 95% level of significance. The results are shown in table 5.

Table 5 shows that the maximum peaks at all stations were significant except Cochin, which was marginally not significant. This test was based on the assumption that the time variance of the time series was represented by the variance of the sampled data. This is a limitation, but as there was a fairly long record for Bombay and the other stations had records exceeding thirty years, we felt the test could be relied upon.

6. Autoregressive models

An ARIMA model was fitted to each time series by computing autocorrelation functions (acf) and partial autocorrelation functions (pacf) after taking first differences. The procedure has been described by Pankratz (1983).

The models that best fitted the data are summarized in table 6.

The models for Cochin and Vizag are marked with an asterisk because the confidence in models for these stations was low. The small length of record led to low confidence.

Table 6. ARIMA models ($d = 1$).

Station	Order (p, d, q)	AR ϕ_1	AR ϕ_2	AR ϕ_3	MA θ_1	MA θ_2	MA θ_3
Bombay	(3, 1, 0)	−0·42	−0·44	−0·39			
Madras (i)	(0, 1, 1)				−0·85		
Madras (ii)	(3, 1, 0)	−0·85	−0·77	−0·59			
Cochin*	(0, 1, 3)				−0·32	−0·19	−0·54
Vizag*	(3, 1, 0)	−0·73	−0·73	−0·29			

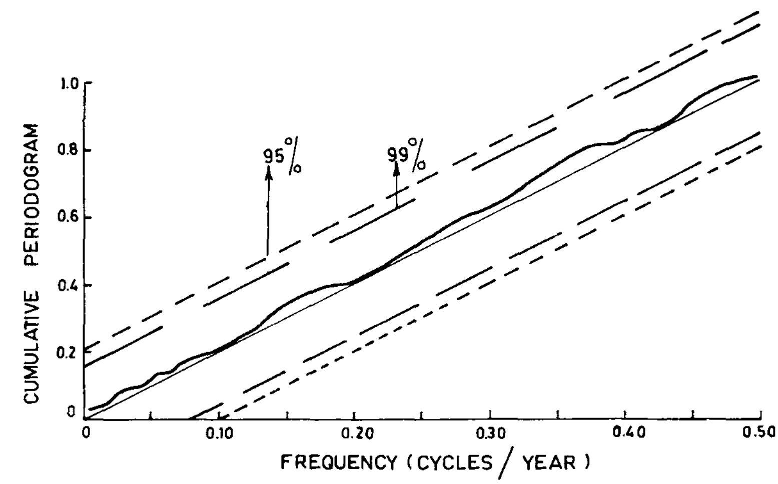


Figure 6. Kolmogorov – Smirnov test: Bombay.

To test the adequacy of each model a cumulative spectrum was prepared for each record. The cumulative spectrum for Bombay is shown in figure 6. The Kolmogorov–Smirnov bounds are also shown. It will be observed that the spectrum does not exceed the upper and lower bounds. The spectra for the other records are not shown, but in each case it was ensured that they lay within the Kolmogorov–Smirnov bounds (loc. cit) and satisfied the test for white noise.

The results shown in table 6 suggest that an AR model (3, 1, 0) is adequate for Bombay, Madras (ii) and possibly Vizag. On the other hand, Madras (i) and Cochin were MA models. In view of some evidence of an element of determinism in AR models we examined two forcing mechanisms that might generate small period cycles.

7. The pole tide

Many references have been made to a 5–6 year cycle in tide-gauge records (Holland and Murty 1970; Currie 1975, 1976). This is because of interactions between a 14·7-month pole tide and a 12·0-month annual cycle. The former is the result of periodic motion of the earth's axis which causes the pole to execute wavy motion. It was discovered by S C Chandler in 1891 and is referred to as a Chandler wobble. The annual cycle is generated by an annual shift of air mass from one hemisphere to another. Many investigators have also found evidence of the Chandler wobble in the atmosphere (Maksimov 1966; Bryson and Starr 1977). This is the atmospheric pole tide.

The frequencies of the annual cycle and the Chandler wobble are 0·0833 cpm and 0·0680 cpm. The difference between these two frequencies is 0·0153 cpm, which corresponds to a 5·5-year cycle. It is close to what has been observed in many parts of the world. Our observations also support a 4·5- to 5·7-year cycle. If a nonlinear system is perturbed by two oscillations with frequencies of f_1 and f_2, an interaction between them could generate oscillation with a frequency of $(f_1 - f_2)$.

In support of this mechanism Hameed and Currie (1989) quote the outputs from a coupled ocean-atmosphere model developed by the Oregon State University in the United States. The model was able to reproduce the Chandler wobble with a frequency of 0·068 cpm (period 14·7 months) and another peak at 0·150 cpm (period 6.67 months). The latter is the sum of the frequencies of the Chandler wobble and the annual cycle (0·068 + 0·083 = 0·151 cpm). The model generated another peak at a frequency of 0·099 cpm (period of 9·6 months). This was close to the difference between the Chandler wobble (0·068 cpm) and the first harmonic of the annual cycle (0·167 cpm). Apparently, the model was not run long enough to generate a 5–6 year cycle. But, this cycle was observed in our tide-gauge records. In a subsequent paper Currie and Hameed (1990) find further evidence of interactions between the annual cycle and other atmospheric signals, such as, a 40-month cycle and 26-month quasi-biennial oscillation. These interactions create a wide spectrum of peaks whose periods range from 2 to 68 months. As each of the major peaks were statistically significant and explained a large percentage (51%) of the total variance, they indicate that the spectrum of atmospheric variations was not entirely chaotic.

To find evidence of the Chandler wobble the monthly mean tide-gauge records of the four Indian stations were analysed. The principal peaks are summarized in table 7. A 2-point moving average was used as before to filter out the high-frequency noise. The annual cycle was removed before computing the power spectra by a 12-month step filter (Priestley 1981). As data for Madras (ii) were not available, the Madras peaks refer to the earlier period.

Applying a false alarm probability test it was found that only the first spectral peak, namely, one with a frequency of 0·075 cpm (13·4-month period) was significant at 95% level. The period of this cycle was close to that of the Chandler wobble (period of 14·7 months). It was also found that the average power in the 13·4-month cycle

Table 7. Mean monthly records.

	Frequency (cpm)			Period (months)		
Station	(i)	(ii)	(iii)	(i)	(ii)	(iii)
Bombay	0·075	0·162	0·252	13·4	6·2	4·0
Madras (i)	0·075	0·166	0·240	13·4	6·0	4·2
Cochin	0·075	0·166	0·240	13·4	6·0	4·2
Vizag	0·075	0·174	0·255	13·4	5·8	3·9

for all stations was comparable to the power in the 4·5–5·7-year cycle. But, at this stage it is difficult to categorically state that the 4·5–5·7-year cycle in Indian tide-gauge records was an interaction between the Chandler wobble and the annual cycle, because the difference between a 13·4 month and an annual cycle should generate an approximately 10-year cycle.

8. Tide-gauge records and monsoon rains

To examine the association, if any, between the tide-gauge records and monsoon rainfall the power spectrum of annual monsoon rainfall at Bombay was computed. The monthly rainfall recorded at Bombay during the monsoon months of June to September was averaged for each year from 1886 to 1986. This gave a seasonal value of the monsoon rain of every year. Tide-gauge records for the same period (1886 to 1986) were used.

The Mann-Kendall test was applied to rainfall data. This is shown in figure 7. It will be noted that variations of Z with time were a little similar to sea level fluctuations (figure 7). A rising trend was noticed from around 1945, but this was statistically significant only after 1962. The sealevel records at Bombay showed a statistically significant trend only after 1940, that is, about 22 years earlier.

The power spectrum of monsoon rainfall after first differencing showed a peak at 4·8 years. This satisfied the test for false alarm probability. The period of the cycle was very close to the 4·5-year cycle that was found for relative sea level (table 4). In view of this similarity, we analysed the cross-spectrum between monsoon rainfall and the tide-gauge records of Bombay.

Cross-correlations were computed between the tide-gauge records and monsoon rainfall for different lags. This is shown in figure 8. A Portmanteau test was used to

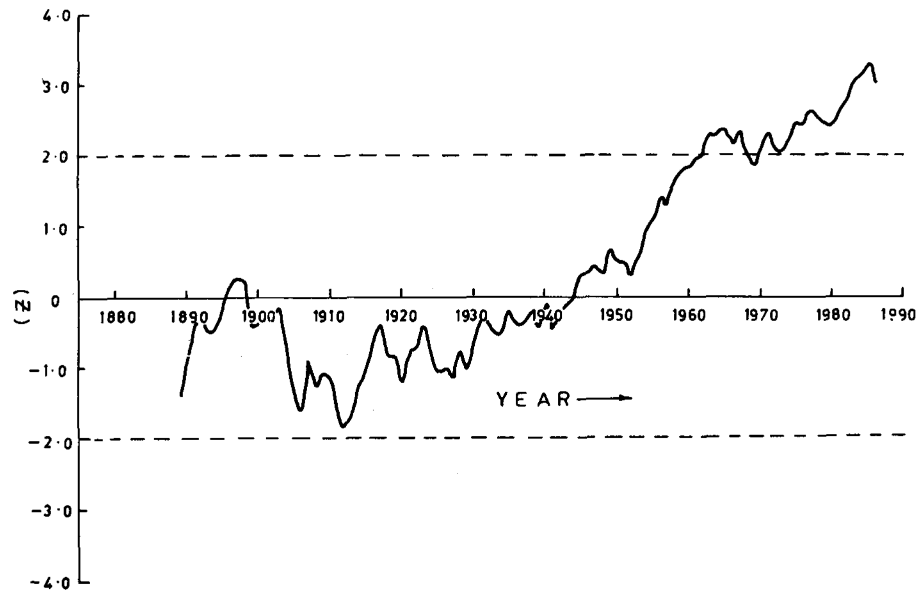


Figure 7. Mann-Kendall test: Bombay rainfall.

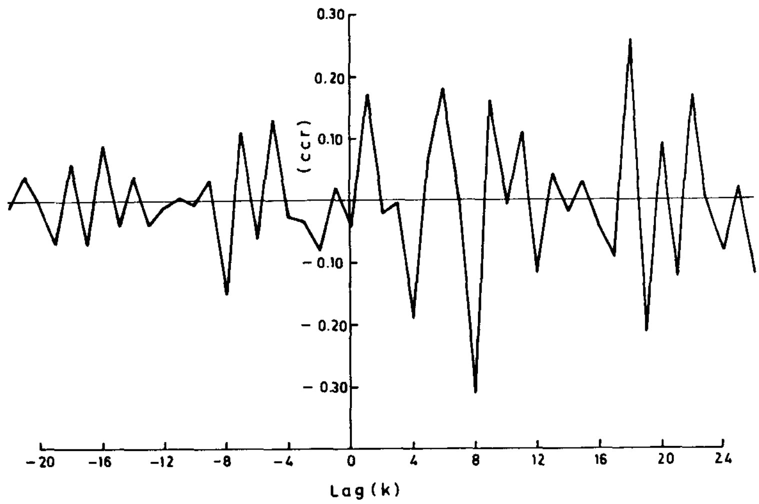


Figure 8. Cross-correlations: Monsoon rainfall and tide gauge records at Bombay.

find out if the cross-correlations indicate the existence of a pattern. It is also known as a Q test (Pierce 1972; Ljung and Box 1978). The statistic Q is defined by

$$Q(k) = n(n+2)\sum_{1}^{K}(1/n-k)r_{xy}^{2}(k) \tag{26}$$

The statistic is approximately distributed as the χ^2 variable with K degrees of freedom. n represents the number of data points used to calculate r_{xy}, the cross-correlation between two variables x and y.

The computed value of Q for 30 degrees of freedom was 65·4, while the χ^2 value for the same degree of freedom at 99·5% level of significance was 53·7. As the computed Q statistic was larger than the χ^2 value, we infer that a pattern does exist. The possibility appears to exist that one series leads another by a certain period of time. This is now being investigated.

9. Summary

The main results of the present investigation may be summarized as follows:

(i) The tide-gauge records of Bombay, Madras (i and ii), Cochin and Vishakhapatnam do not show a uni-directional trend. The Mann-Kendall test shows some evidence of a rising trend at Bombay from 1940–1986 and in Madras from 1910 to 1933. The trend at Bombay was estimated to be 0·8 mm/yr while the trend at Madras was 0·4 mm/yr for a short period. There was no other evidence of a statistically significant trend.

(ii) Correlograms of the tide-gauge records suggest that each time-series was made up of harmonic terms with tendency for an autoregressive nature.

(iii) Computation of periodograms indicates the existence of long period (50–60 years) cycles. Statistical tests show that first differencing was adequate for the removal of long-period cycles.

(iv) A spectral analysis after removal of long-period cycles revealed 4·5–5·7 year cycles riding on the longer cycles. This was confirmed by the computation of periodograms and by the maximum entropy approach. The spectral peaks passed the false alarm probability test.
(v) ARIMA models were fitted to the records of relative sealevel. The records of Bombay, Madras (ii) and Vizag suggest an autoregressive (AR) model of third order, while the best fit for Madras (i) and Cochin was a moving average (MA) model of third order. The adequacy of each model was ensured by a test for white noise for the cumulative spectrum of errors or deviations from the model-generated data.
(vi) Periodograms of monthly tide-gauge data for all four stations revealed a 13·5-month cycle, which was close to the Chandler wobble. This peak was statistically significant. But, the observed 4·5–5·7-year cycle could not be entirely explained as an interaction between the annual cycle and the Chandler wobble.
(vii) Annual monsoon rainfall at Bombay showed a rising trend that was significant only after 1962. The periodogram revealed a 4·8-year cycle after the trend was removed by first differences. This was similar to the cycle observed in the tide-gauge records. A Portmanteau test suggests the existence of a pattern between monsoon rainfall and tide-gauge records at Bombay.

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