Northward propagation of the 30–50 day mode in the Indian monsoon region

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Abstract. A highly simplified zonally symmetric system of equations is used to understand the basic mechanism of the northward movement of the 30–50 day oscillation in the Indian monsoon region. Zonally symmetric perturbations are used on the July mean basic flow in the Indian region. A two-level model is employed. A simple parametrization of cumulus heating is included. An expression is derived for the northward phase speed. A right order of magnitude for the phase speed is obtained. The northward movement appears to be controlled by the basic zonal wind profile and cumulus heating.

Keywords. Northward propagation; 30–50 day oscillation; Indian monsoon region.

1. Introduction

A 30–50 day oscillation in the equatorial region was first observed by Madden and Julian (1971). These were eastward-moving east-west overturnings in the longitude–height plane. Alexander *et al* (1978) composited active and break epochs and found slow northward movement. Yasunari (1979, 1980) and Sikka and Gadgil (1980) found similar northward propagation in satellite cloudiness. Krishnamurti and Subrahmanyam (1982) found a northward-propagating mode in the zonal component of wind over India. Krishnamurti *et al* (1985) also found a wave number one eastward-propagating mode in the velocity potential field at 200 mb. Lau and Lau (1986) found this oscillation in a Geophysical Fluid Dynamics Laboratory (GFDL) general circulation model. Lau and Peng (1987) have given a theory of the eastward movement attributing the latter to mobile wave conditional instability of the second kind (CISK).

The 30–50 day (oscillation) mode progresses slowly northwards with a phase speed of 0.7° latitude per day in the Indian monsoon region. In an earlier paper (Keshavamurty *et al* 1986), we studied some mechanisms for this oscillation and its northward movement. In this paper we present a highly simplified treatment with a view to understand the essential basic mechanism of the northward movement. The actual observed movement in nature is more complicated and other mechanisms may also be at work.

2. The model equations

We have zonally symmetric perturbations on a basic flow $U(y, p)$. The equations of

motion are

$$\frac{\partial u'}{\partial t} + \left(\frac{\partial U}{\partial y} - f \right) v' = -k u', \quad (1)$$

$$\frac{\partial v'}{\partial t} + f u' = -\frac{\partial \phi'}{\partial y} - k v'. \quad (2)$$

The thermodynamic energy equation is

$$\frac{\partial}{\partial t} \left(\frac{\partial \phi'}{\partial p} \right) + \sigma \omega' = -\frac{R}{C_p p} \dot{Q}' - k' \frac{\partial \phi'}{\partial p}. \quad (3)$$

In these equations u' and v' are the zonal and meridional components of perturbation wind respectively, ϕ' the geopotential, $\omega = dp/dt$, k the coefficient of Rayleigh friction and k' the Newtonian cooling coefficient.

The fact that we have used zonally symmetric perturbations is a limitation. As there is some zonal symmetry in the monsoon region, i.e. the zonal scale of the inter-tropical convergence zone (ITCZ) is much larger than the meridional scale, we believe this is a reasonable approximation. In our future (model) studies we shall retain x -dependence in the equations. We assume a highly simplified boundary layer CISK type of heating (Charney and Eliassen 1964)

$$-\frac{R}{C_p p} \dot{Q}' = K \frac{\partial u'_3}{\partial y}. \quad (4)$$

Here $k = k'$ and $K = H \sigma g \rho \sin(2\alpha) \sqrt{(K_e/2f_0)}$, where $H = 1.5$; $\alpha = 22\frac{1}{2}$ deg; K_e = coefficient of eddy viscosity; f_0 at 10°N ; ρ = density of air; σ = static stability. We use a two-level model:

We also use a drastically simplified thermodynamic energy equation

$$\sigma \omega'_2 = K \frac{\partial u'_3}{\partial y}. \quad (5)$$

The implication is that the cumulus heating goes to set up vertical circulations. As the temperature fluctuations and horizontal advection are small in the tropics this is not an

unreasonable approximation. It is not claimed that the temperature fluctuations are totally unimportant but we would like to see the result of this approximation.

The continuity equation is

$$\frac{\partial v'_3}{\partial y} + \frac{0 - \omega'_2}{\Delta p} = 0 \quad (6)$$

$$\omega'_2 = \Delta p \frac{\partial v'_3}{\partial y}. \quad (7)$$

Substituting in the thermodynamic energy equation (at level 2) we get

$$\sigma \Delta p \frac{\partial v'_3}{\partial y} = K \frac{\partial u'_3}{\partial y}. \quad (8)$$

The equations of motion at level 3 are

$$\frac{\partial u'_3}{\partial t} + \left(\frac{\partial U_3}{\partial y} - f \right) v'_3 = -k u'_3 \quad (9)$$

and

$$\frac{\partial v'_3}{\partial t} + f u'_3 + \frac{\partial \phi'_3}{\partial y} = -k v'_3. \quad (10)$$

Taking $\partial/\partial t$ of (8) and substituting for $\partial u'_3/\partial t$ from (9) we get

$$\sigma \Delta p \left(\frac{\partial}{\partial t} + k \right) \frac{\partial v'_3}{\partial y} = -K \frac{\partial}{\partial y} \left\{ \left(\frac{\partial U_3}{\partial y} - f \right) v'_3 \right\}. \quad (11)$$

Assuming oscillatory solution

$$v'_3 = V_3 \exp(ivt),$$

where v is the frequency

$$\begin{aligned} \sigma \Delta p (iv + k) \frac{dV_3}{dy} &= -K \left\{ \left(\frac{\partial^2 U_3}{\partial y^2} - \beta \right) V_3 + \left(\frac{\partial U_3}{\partial y} - f \right) \frac{dV_3}{dy} \right\} \\ &\times \left\{ K \left(\frac{\partial U_3}{\partial y} - f \right) + (iv + k) \sigma \Delta p \right\} \frac{dV_3}{dy} = K \left(\beta - \frac{\partial^2 U_3}{\partial y^2} \right) V_3 \end{aligned} \quad (12)$$

$$\frac{dV_3}{V_3} = \frac{K \left(\beta - \frac{\partial^2 U_3}{\partial y^2} \right)}{\left\{ K \left(\frac{\partial U_3}{\partial y} - f \right) + k \sigma \Delta p + iv \sigma \Delta p \right\}} dy. \quad (13)$$

Therefore

$$V_3 = V_{30} \exp \left[\int \frac{K \left(\beta - \frac{\partial^2 U_3}{\partial y^2} \right)}{\left\{ K \left(\frac{\partial U_3}{\partial y} - f \right) + k \sigma \Delta p + iv \sigma \Delta p \right\}} dy \right].$$

$$= \frac{V_{30}}{\left[\sqrt{\left(K \left(\frac{\partial U_3}{\partial y} - f \right) + k\sigma\Delta p \right)^2 + v^2\sigma^2\Delta p^2} \right]} \times \exp \left[i \tan^{-1} \left\{ \frac{K \left(\frac{\partial U_3}{\partial y} - f \right) + k\sigma\Delta p}{v\sigma\Delta p} \right\} \right]. \quad (14)$$

Writing the exponent as

$$\frac{i}{y} \tan^{-1} \left\{ \frac{K \left(\frac{\partial U_3}{\partial y} - f \right) + k\sigma\Delta p}{v\sigma\Delta p} \right\} y$$

we can approximately write

$$C_y = -\frac{v}{l} = \frac{-vy}{\tan^{-1} \left\{ \frac{K \left(\frac{\partial U_3}{\partial y} - f \right) + k\sigma\Delta p}{v\sigma\Delta p} \right\}} \quad (15)$$

where l is the meridional wave number.

3. Calculation

We have used the smoothed profiles of zonal wind at 700 mb (U_3) and 200 mb (U_1) given in (Keshavamurty *et al* 1986). We have taken the period of oscillation as 40 days and have calculated the meridional phase speed C_y from (15) with and without friction. A value for friction corresponding to a decay time of 5 days was used. The meridional phase speed values are given in table 1 for the latitudes equator to 29°N. A phase speed of a few metres per second was obtained. In the case with friction there is southward movement in the region from the equator to 9°N. Considering the simplifying assumptions in the theory it is interesting that the theory predicts a reasonable speed.

Table 1. Calculated meridional phase speeds for latitudes equator to 29°N.

Latitude °N	Phase speed (m s ⁻¹)	
	Without friction	With friction
0-4	4.66	-0.53
5-9	1.76	-8.99
10-14	2.18	4.31
15-19	2.74	3.34
20-24	3.32	3.62
25-29	4.06	4.42

4. Wave-CISK

We shall now examine the result of introducing another simple type of cumulus heating, i.e. wave-CISK. Here we shall use the same equations of motion but the thermodynamic energy equation is

$$\begin{aligned}\frac{\partial}{\partial t}\left(\frac{\partial\phi'}{\partial p}\right) + \sigma\omega' &= -\frac{R}{C_p p}\dot{Q}' - k\frac{\partial\phi'}{\partial p} \\ &= K'\frac{\partial v'_3}{\partial y} - k\frac{\partial\phi'}{\partial p}.\end{aligned}\quad (16)$$

The cumulus heating is proportional to the divergence at the lower level. Following Lau and Peng (1987), the value of K' is given by

$$K' = mrLq_{\text{sat}}R/C_p p, \quad (17)$$

where m is the moisture availability factor ($=0.8$), r the relative humidity ($=0.75$), $q_{\text{sat}}(T_3)$ the saturation specific humidity at temperature T_3 at the lowest model level, L the latent heat of condensation, R the specific gas constant and C_p the specific heat at constant pressure.

Taking $(\partial/\partial t + k)$ of (2) and eliminating $(\partial/\partial t + k)u'$ using (1), we get

$$\begin{aligned}\left(\frac{\partial}{\partial t} + k\right)^2 v' + f\left(\frac{\partial}{\partial t} + k\right)u' + \left(\frac{\partial}{\partial t} + k\right)\frac{\partial\phi'}{\partial y} &= 0. \\ \left(\frac{\partial}{\partial t} + k\right)^2 v' - f\left(\frac{\partial U}{\partial y} - f\right)v' + \left(\frac{\partial}{\partial t} + k\right)\frac{\partial\phi'}{\partial y} &= 0.\end{aligned}\quad (18)$$

Taking $\partial/\partial y$ of the thermodynamic energy equation

$$\frac{1}{\Delta p}\left(\frac{\partial}{\partial t} + k\right)\left(\frac{\partial\phi'_3}{\partial y} - \frac{\partial\phi'_1}{\partial y}\right) + \sigma\frac{\partial\omega'_2}{\partial y} = K'\frac{\partial^2 v'_3}{\partial y^2}. \quad (19)$$

However, by the continuity equation

$$\omega'_2 = \Delta p \frac{\partial v'_3}{\partial y}$$

and $v'_1 = -v'_3$.

Therefore by eliminating $(\partial/\partial t)(\partial\phi'/\partial y)$ eq. (19) reduces to

$$-\frac{2}{\Delta p}\left(\frac{\partial}{\partial t} + k\right)^2 v'_3 + \frac{f}{\Delta p}\left[\frac{\partial}{\partial y}(U_1 + U_3) - 2f\right]v'_3 + (\sigma\Delta p - K')\frac{\partial^2 v'_3}{\partial y^2} = 0. \quad (20)$$

Assuming oscillatory solution of the type $v'_3 = V_3 \exp(i\omega t)$,

$$\left[-\frac{2}{\Delta p}(i\omega + k)^2 + \frac{f}{\Delta p}\left\{\frac{\partial}{\partial y}(U_1 + U_3) - 2f\right\}\right]V_3 + (\sigma\Delta p - K')\frac{d^2 V_3}{dy^2} = 0, \quad (21)$$

or

$$\frac{d^2 V_3}{dy^2} + n^2(y) V_3 = 0,$$

where

$$n^2(y) = \frac{f \left(\frac{\partial}{\partial y} (U_1 + U_3) - 2f \right) + 2(v^2 - k^2) - 4ikv}{\Delta p(\sigma \Delta p - K')} . \quad (22)$$

Using the same profiles of U_1 and U_3 and considering a period of 40 days, we have calculated n^2 . The values are presented in table 2. If linear friction is not included n^2 is negative everywhere and meridional propagation is not possible in this case, as shown in table 3.

Table 2. Calculated values of n^2 and refractive index n (with friction).

Latitude	$n^2 \times 10^{12} \text{ m}^{-2}$		$n \times 10^6 \text{ m}^{-1}$	
	RP	IP	RP	IP
30S	-1.49387	-0.00261	0.00107	-1.22224
29	-1.38461	-0.00261	0.00111	-1.17669
28	-1.28819	-0.00261	0.00115	-1.13499
27	-1.20380	-0.00261	0.00119	-1.09718
26	-1.12604	-0.00261	0.00123	-1.06115
25	-1.04143	-0.00261	0.00128	-1.02050
24	-0.93422	-0.00261	0.00135	-0.96655
23	-0.85599	-0.00261	0.00141	-0.92520
22	-0.76900	-0.00261	0.00149	-0.87693
21	-0.70078	-0.00261	0.00156	-0.83713
20	-0.63882	-0.00261	0.00163	-0.79926
19	-0.57932	-0.00261	0.00171	-0.76113
18	-0.52235	-0.00261	0.00180	-0.72274
17	-0.44987	-0.00261	0.00194	-0.67073
16	-0.39640	-0.00261	0.00207	-0.62961
15	-0.34605	-0.00261	0.00222	-0.58826
14	-0.28637	-0.00261	0.00244	-0.53514
13	-0.24565	-0.00261	0.00263	-0.49564
12	-0.18639	-0.00261	0.00302	-0.43173
11	-0.14353	-0.00261	0.00344	-0.37888
10	-0.09296	-0.00261	0.00428	-0.30492
9	-0.05623	-0.00261	0.00550	-0.23720
8	-0.06294	-0.00261	0.00520	-0.25092
7	-0.06388	-0.00261	0.00516	-0.25279
6	-0.05474	-0.00261	0.00557	-0.23403
5	-0.03660	-0.00261	0.00681	-0.19143
4	-0.02353	-0.00261	0.00849	-0.15362
3	-0.04260	-0.00261	0.00632	-0.20650
2	-0.02855	-0.00261	0.00771	-0.16913
1	-0.00228	-0.00261	0.02434	-0.05359
0	-0.00062	-0.00261	0.03211	-0.04062
1	-0.00228	-0.00261	0.02434	-0.05359
2	-0.01068	-0.00261	0.01253	-0.10412
3	-0.02637	-0.00261	0.00802	-0.16258

(Continued)

(Table 2 continued)

Latitude	$n^2 \times 10^{12} \text{ m}^{-2}$		$n \times 10^6 \text{ m}^{-1}$	
	RP	IP	RP	IP
4	–0.03860	–0.00261	0.00664	–0.19658
5	–0.05075	–0.00261	0.00579	–0.22535
6	–0.06922	–0.00261	0.00496	–0.26315
7	–0.08001	–0.00261	0.00461	–0.28289
8	–0.10136	–0.00261	0.00410	–0.31839
9	–0.13419	–0.00261	0.00356	–0.36634
10	–0.17267	–0.00261	0.00314	–0.41555
11	–0.21318	–0.00261	0.00283	–0.46172
12	–0.26270	–0.00261	0.00254	–0.51255
13	–0.31961	–0.00261	0.00231	–0.56535
14	–0.37892	–0.00261	0.00212	–0.61557
15	–0.44827	–0.00261	0.00195	–0.66954
16	–0.51781	–0.00261	0.00181	–0.71959
17	–0.59799	–0.00261	0.00169	–0.77330
18	–0.70128	–0.00261	0.00156	–0.83742
19	–0.78129	–0.00261	0.00148	–0.88391
20	–0.82624	–0.00261	0.00143	–0.90898
21	–0.90458	–0.00261	0.00137	–0.95109
22	–0.93555	–0.00261	0.00135	–0.96724
23	–0.99739	–0.00261	0.00131	–0.99870
24	–1.02254	–0.00261	0.00129	–1.01121
25	–1.10260	–0.00261	0.00124	–1.05005
26	–1.21216	–0.00261	0.00118	–1.10098
27	–1.11461	–0.00261	0.00124	–1.05575
28	–1.06976	–0.00261	0.00126	–1.03429
29	–1.17057	–0.00261	0.00121	–1.08193
30N	–1.37497	–0.00261	0.00111	–1.17259

Table 3. Calculated values of n^2 and refractive index n (without friction).

Latitude	$n^2 \times 10^{12} \text{ m}^{-2}$	$n \times 10^6 \text{ m}^{-1}$
30S	–1.49	1.22i
29	–1.38	1.18i
28	–1.29	1.13i
27	–1.20	1.10i
26	–1.12	1.06i
25	–1.04	1.02i
24	–0.93	0.97i
23	–0.85	0.92i
22	–0.77	0.88i
21	–0.70	0.84i
20	–0.64	0.80i
19	–0.58	0.76i
18	–0.52	0.72i
17	–0.45	0.67i
16	–0.39	0.63i
15	–0.34	0.59i
14	–0.28	0.53i
13	–0.24	0.49i

(Continued)

(Table 3 continued)

Latitude	$n^2 \times 10^{12} \text{ m}^{-2}$	$n \times 10^6 \text{ m}^{-1}$
12	-0.18	0.43i
11	-0.14	0.38i
10	-0.09	0.30i
9	-0.05	0.23i
8	-0.06	0.25i
7	-0.06	0.25i
6	-0.05	0.23i
5	-0.03	0.19i
4	-0.02	0.15i
3	-0.04	0.20i
2	-0.03	0.16i
1	0.00	0.03
0	0.00	0.03
1	0.00	0.03
2	-0.01	0.10i
3	-0.02	0.16i
4	-0.04	0.19i
5	-0.05	0.22i
6	-0.07	0.26i
7	-0.08	0.28i
8	-0.10	0.32i
9	-0.13	0.36i
10	-0.17	0.41i
11	-0.21	0.46i
12	-0.26	0.51i
13	-0.32	0.56i
14	-0.38	0.61i
15	-0.45	0.67i
16	-0.52	0.72i
17	-0.60	0.77i
18	-0.70	0.84i
19	-0.78	0.88i
20	-0.82	0.91i
21	-0.90	0.95i
22	-0.93	0.97i
23	-1.00	1.00i
24	-1.02	1.01i
25	-1.10	1.05i
26	-1.21	1.10i
27	-1.11	1.05i
28	-1.07	1.03i
29	-1.17	1.08i
30N	-1.37	1.17i

5. Necessary condition for complex refractive index

$$\frac{d^2 V}{dy^2} + n^2 V = 0$$

$$\frac{d^2}{dy^2}(V_r + iV_i) + (n_r^2 + in_i^2)(V_r + iV_i) = 0.$$

We get

$$\frac{d^2 V_r}{dy^2} + n_r^2 V_r - n_i^2 V_i = 0 \quad (23)$$

$$\frac{d^2 V_i}{dy^2} + n_i^2 V_r + n_r^2 V_i = 0. \quad (24)$$

Multiplying (23) by V_i and (24) by $-V_r$ and adding, we get

$$\frac{d}{dy} \left(V_i \frac{dV_r}{dy} - V_r \frac{dV_i}{dy} \right) - n_i^2 (V_i^2 + V_r^2) = 0. \quad (25)$$

Integrating over a channel y_1 to y_2 and using the boundary condition

$$V_i = V_r = 0 \quad \text{at} \quad y = y_1 \quad \text{and} \quad y = y_2$$

we get

$$\int_{y_1}^{y_2} n_i^2 (V_i^2 + V_r^2) dy = 0. \quad (26)$$

This is possible only in the case where n_i^2 passes through zero, i.e. changes sign somewhere in the channel. It is seen that in the case of boundary layer CISK (Keshavamurty *et al* 1986 table 2) n_i^2 changes sign at the equator. This is mainly due to the term $(\partial U_3 / \partial y - f)$. Since the absolute vorticity changes sign at the equator there is a possibility of n^2 being complex and thus of meridional propagation of the 30–50-day mode.

6. Summary and conclusions

Using a very highly simplified system of equations we have derived an expression for the speed of the northward propagation of the 30–50-day mode. Using July mean wind profile, a calculation of this phase speed yields reasonable values. The northward movement is determined by the basic wind profile and cumulus heating. A necessary condition for the northward propagation is also derived.

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