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Is there an attractor for the Indian summer monsoon?

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Abstract. Aperiodicity in the time series of Indian summer rainfall for 116 years is analysed using the phase space approach. The question whether a low-dimensional strange attractor is associated with the chaotic behaviour of the monsoon system is investigated. It is found that a strange attractor of dimensionality around 5·1 exists and the system has 12 relevant degrees of freedom.

Keywords. Monsoon climate and variability; deterministic chaos; strange attractors; phase space analysis.

1. Introduction

The seasonal event of southwest summer monsoon over the Indian region is well known for its variability on a wide range of time scales, especially its large variability on the interannual time scale. Conventional periodicity analysis (Mooley and Parthasarathy 1984) of time series of monsoon rainfall during the last 100 years or so reveals two cycles (2·8-year and 14-year) but these are seen only during the last 30 years of the data record. Thus, there appears to be a large aperiodic component in monsoon fluctuations. It is important to know whether this non-periodic part is due to the intrinsic variability of the system (Lorenz 1984). We try to understand whether there is a strange attractor underlying the evolution of the monsoon and determine its topological characteristics. To examine this question, we have analysed a long-term monsoon rainfall record. This is the all-India summer monsoon rainfall for the period 1871–1986 (Mooley and Parthasarathy 1984). We have used the recently developed techniques in nonlinear dynamics (Packard *et al* 1980; Grassberger and Procaccia 1983). More precisely, we obtain the dimensionality d of the attractor for the monsoon system, and a quantitative estimate of the minimum number of variables n_c necessary to characterize a long-term monsoon rainfall record, and the long-term behaviour of the monsoon system that it represents.

2. Theory

For a multivariable nonlinear system such as the monsoon, an extremely useful approach is the description of its dynamics in phase space. Recent advances (Packard *et al* 1980; Grassberger and Procaccia 1983) in nonlinear dynamics enable us to obtain information about the behaviour of the system in phase space without explicit knowledge of the dynamical equations for the system, or their actual solutions. If one knows the behaviour of the system in phase space, it may be possible to model its nonlinear dynamics by an iterative map. Recent studies using the phase space approach have shown that strange attractors underlie the dynamics of global paleoclimate (Nicolis and Nicolis 1984), the southern oscillation (Hense 1987) and some mid-latitude systems (Fraedrich 1986).

For dissipative dynamical systems it is clear that there should be an attractor—that is a region in phase space, where the system spends all its time after the transient phenomena have died down. It is well-known that the attractor can be a fixed point (static situation), a limit cycle (periodic motion) or a strange attractor (chaotic motion). In terms of topological properties the attractors are characterized by their dimensionality. The dimensionality d is zero for a fixed point, $d = 1$ for a limit cycle while a strange attractor has non-integer dimensionality.

The crucial point for our purposes is that the dimensionality of the attractor as well as the long term phase space description of the multivariable system, can be deduced by analysing a time series in one variable. We summarize here the algorithm for the calculation of dimensionality proposed by Grassberger and Procaccia (1983).

Consider a time series in which some variable is measured as a function of time for a long period. For the summer monsoon system one such variable is a long record of seasonal rainfall averaged over the monsoon region. From such a time series one samples n points separated by a fixed time interval τ so that one obtains points $X(t)$, $X(t+\tau)$, $X(t+2\tau)\cdots X(t+(n-1)\tau)$. By an appropriate choice of τ, these points are expected to be linearly independent. We denote the n points by a vector $\mathbf{X}(t)$. Let us next consider M different instants of time t_i, $i = 1, M$ and for each t_i we obtain from the time series the vector $\mathbf{X}_i = \mathbf{X}(t_i)\, \mathrm{i} = 1 \cdots \mathrm{M}$. From these M vectors one calculates a correlation function of the attractor

$$C(r) = \frac{1}{M^2} \sum_{\substack{i,j=1\\ i\neq j}}^{M} \Theta(r - |\mathbf{X}_i - \mathbf{X}_j|), \tag{1}$$

where $|\mathbf{X}_i - \mathbf{X}_j|$ is the distance between the two points i and j in the n dimensional vector space and r is an arbitrary distance parameter. It should be noted that the points we have chosen on the attractor are random in time. However, they are spatially correlated as they all lie on the attractor. Thus $C(r)$ is a global measure of this spatial correlation. Θ is the Heaviside function, $\Theta(x) = 1$ if $x > 0$, $\Theta(x) = 0$ if $x < 0$. Thus $C(r)$ simply counts the number of pairs of points whose distance $|\mathbf{X}_i - \mathbf{X}_j|$ is less than r. It has been shown (Grassberger and Procaccia 1983) that the dimensionality d of the attractor is related to $C(r)$ by the relation

$$C(r) = r^d \text{ (for small } r\text{)}. \tag{2}$$

The basic idea here is that the number of points (and hence $C(r)$) of a d-dimensional attractor inside a n-dimensional ball of radius r ($d \leqslant n$) scales as r^d. Thus, the slope of the curve $\log C(r)$ plotted against $\log r$ gives the dimensionality d. In practice, one evaluates d for different number n of components for each vector $\mathbf{X}_i$. Then if the dynamics is governed by a low-dimensional attractor the slope d saturates beyond a critical value n_c. n_c then gives the minimum number of dynamic variables that would be necessary to characterize the motion in phase space. It should also be pointed out that if one analyses a series with "pure noise" then there would be no saturation in slope beyond reasonable values of n_c

3. Analysis and results

We closely follow these ideas and obtain d and n_c from a time series of monsoon rainfall. For time series we have taken the 116-year record (Mooley and Parthasarathy 1984) of the all-India summer monsoon (June to September) rainfall for the period 1871 to 1986. We chose a range of values of the shift parameter τ and carried out the following computations for each τ. The correlation function for the time series was computed according to (1) beginning in a two-dimensional phase space (i.e. $n = 2$) and for a range of values of the prescribed distance r. This procedure was repeated for increasing values of the embedding phase space dimension ($n = 2$ to 20). Figure 1 shows the variation of $\ln(N(r))$ with variation in $\ln(r)$ in each phase space. Here we show the results for $n = 2$ to 10 only. For convenience we have displayed the plots of $\ln(N(r))$ vs $\ln(r)$ instead of $\ln(C(r))$ vs $\ln(r)$, where $N(r)$ is M^2 times $C(r)$. Next, for each curve with a particular value of n, we determine its slope in the linear region of the curve using a regression procedure. This slope gives the dimensionality of the attractor according to (2). The variation of this dimensionality as n is varied is shown in figure 2. The above results are not sensitive to the choice of τ. We see that the dimensionality saturates to a limiting value. This saturation is seen to be around 5·1 which implies that the dynamics represented by the rainfall time series can be characterized by a strange attractor having a fractal dimensionality 5·1. The fractal dimensionality points to the fact that the observed aperiodicity comes essentially from the intrinsic variability of the system. Furthermore we see that the critical value of the embedding phase dimension n beyond which d saturates is around 12. This implies that at least twelve collective variables would be involved in a dynamical description of the monsoon system whose behaviour is represented by the rainfall time series.

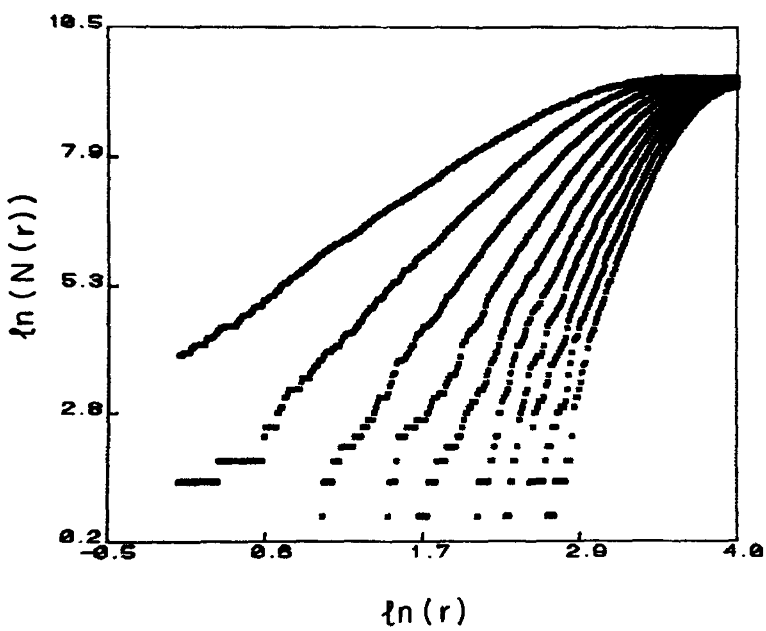

Figure 1. Dependence of $\ln(N(r))$ on $\ln(r)$ for embedding dimensions 2–10. Curves on the extreme left and extreme right correspond to dimensions 2 and 10 respectively.

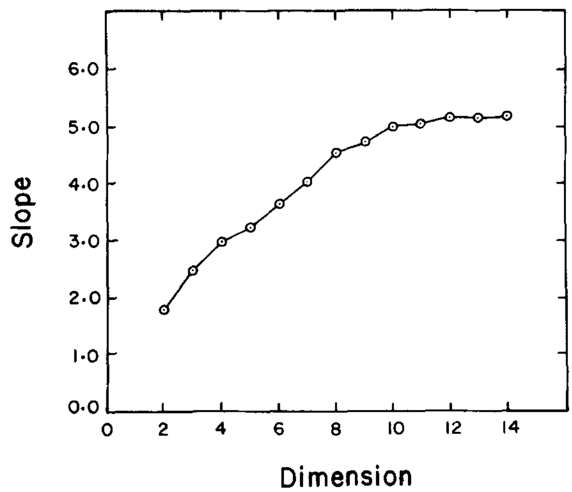

Figure 2. Slope of $\ln(N(r))$ vs $\ln(r)$ curves for embedding dimensions 2–14.

4. Conclusions

We have investigated the question whether there is an attractor underlying the evolution of monsoon climate over the last 116 years. For this purpose we have analysed the corresponding monsoon rainfall time series in the system phase space and evaluated a correlation function for the attractor. We find that a strange attractor of dimensionality around 5·1 exists and the system has 12 relevant degrees of freedom.

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