Gravity anomalies of a trapezoidal model with quadratic density function

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Abstract. The decrease of density contrast with depth in sedimentary strata is approximated by a quadratic function. The anomaly equation of a trapezoidal model with the quadratic density function is derived. Nonlinear optimization technique using the Marquardt algorithm has been developed and used to interpret a synthetic anomaly profile of the trapezoidal model. The exact values of the coefficients of the quadratic density function are assumed to be known. The convergence of the method is shown by plotting the values of the objective function λ and the various parameters with respect to iteration number. θ and half-width of the trapezoidal model are found to be correlated. The method is also applied to interpret the gravity anomalies over San Jacinto graben, California. Finally, the use of modelling with quadratic density function is discussed.

Keywords. Marquardt algorithm; quadratic density function; gravity anomalies; trapezoidal model.

1. Introduction

Density data obtained from bore-holes and the results of seismic surveys in sedimentary basins show that the density of the sedimentary rocks increases with depth. Athy (1930) observed that the density of sediments increases exponentially with depth. Cordell (1973) derived the anomaly equation for a prismatic model with exponential density function. The representation of density contrast by an exponential function, however, is not convenient in the analysis of gravity anomalies, as the anomaly equations are not obtained in closed form. The exponential decrease of density contrast over a small interval of depth is approximated by a linear function by Murthy and Rao (1979). They have divided the side of a polygon body into several segments and calculated the contribution of each segment with linear decrease of density contrast. It was earlier shown that the decrease of density contrast in many sedimentary basins could be approximated to a fair degree of accuracy by a quadratic function (Bhaskara Rao 1986).

Modelling with variable density contrast has recently gained importance because of the ready availability of density-depth data from borewells drilled for oil and other investigations. We felt the necessity of deriving the anomaly equations with quadratic density function for various structural models usually associated with sedimentary basins. In this connection, the gravity anomaly over a fault model was recently analyzed with a quadratic density function (Bhaskara Rao 1985). Anticlines and synclines are two important structures associated with sedimentary basins and are usually approximated by two-dimensional isosceles triangular prisms (Rao and Avasthi 1973). However, it is believed that a trapezoidal prism would be a better approximation to an anticline (Bhimasankaram *et al* 1977). Similarly, a better approximation to syncline would be a trapezoidal model. In this paper we have derived the anomaly equation for a

trapezoidal model approximating a syncline structure with quadratic density function and developed the methods of interpretation.

2. Anomaly equation

It was found that the decrease of density contrast with depth in many sedimentary strata could be approximated by a quadratic function (Bhaskara Rao 1986), as

$$\Delta\rho(Z) = a_0 + a_1 Z + a_2 Z^2, \quad (1)$$

where a_0 , a_1 and a_2 are constants of the quadratic function and a_0 represents the extrapolated value of density contrast at the surface.

The equation for gravity anomaly of a trapezoidal model is derived by integrating the gravity effect of an elementary mass (figure 1). Thus,

$$\Delta g(x) = 2\gamma \int_{Z=Z_1}^{Z_2} \int_{u=-T-(Z_1-Z)\cot\theta}^{T+(Z_1-Z)\cot\theta} \frac{\Delta\rho(Z)Z \, dZ \, du}{(u-x)^2 + Z^2}, \quad (2)$$

where Z_1 and Z_2 are the depths to top and bottom, T is the half-width of the lower surface, and θ is the inclination with the horizontal of the sides. Substituting the value of $\Delta\rho(Z)$ in (2), we have

$$\begin{aligned} \Delta g(x) = & 2\gamma a_0 \int_{Z=Z_1}^{Z_2} \int_{u=-T-(Z_1-Z)\cot\theta}^{T+(Z_1-Z)\cot\theta} \frac{Z \, dZ \, du}{(u-x)^2 + Z^2} \\ & + 2\gamma a_1 \int_{Z=Z_1}^{Z_2} \int_{u=-T-(Z_1-Z)\cot\theta}^{T+(Z_1-Z)\cot\theta} \frac{Z^2 \, dZ \, du}{(u-x)^2 + Z^2} \\ & + 2\gamma a_2 \int_{Z=Z_1}^{Z_2} \int_{u=-T-(Z_1-Z)\cot\theta}^{T+(Z_1-Z)\cot\theta} \frac{Z^3 \, dZ \, du}{(u-x)^2 + Z^2}. \end{aligned}$$

Thus, the equation for the gravity anomaly of a trapezoidal model is given by

$$\begin{aligned} \Delta g(x) = & 2\gamma a_0 [Z_2(\phi_4 - \phi_3) - Z_1(\phi_1 - \phi_2) + F_1 \{\sin \theta \ln(r_4/r_1) \\ & + \cos \theta(\phi_1 - \phi_4)\} - F_2 \{\sin \theta \ln(r_3/r_2) - \cos \theta(\phi_2 - \phi_3)\}] \\ & + \frac{2\gamma a_1}{2} [Z_2^2(\phi_4 - \phi_3) - Z_1^2(\phi_1 - \phi_2) + F_1 \{(Z_2 - Z_1) \sin \theta \\ & + F_1(\sin 2\theta \ln(r_4/r_1) + \cos 2\theta(\phi_1 - \phi_4)) \\ & - F_2 \{(Z_2 - Z_1) \sin \theta - F_2(\sin 2\theta \ln(r_3/r_2) - \cos 2\theta(\phi_2 - \phi_3))\}] \\ & + \frac{2\gamma a_2}{3} \left[Z_2^3(\phi_4 - \phi_3) - Z_1^3(\phi_1 - \phi_2) + F_1 \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \sin \theta \right. \right. \\ & + F_1(Z_2 - Z_1) \sin 2\theta + F_1^2(\sin 3\theta \ln(r_4/r_1) + \cos 3\theta(\phi_1 - \phi_4)) \\ & - F_2 \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \sin \theta - F_2(Z_2 - Z_1) \sin 2\theta \right. \\ & \left. \left. + F_2^2(\sin 3\theta \ln(r_3/r_2) - \cos 3\theta(\phi_2 - \phi_3)) \right\} \right], \quad (3) \end{aligned}$$

Figure 1. Trapezoidal model.

where

$$F_1 = (x + T) \sin \theta + Z_2 \cos \theta,$$

$$F_2 = (x - T) \sin \theta - Z_2 \cos \theta,$$

$$\phi_1 = \pi/2 + \arctan [x + T + (Z_2 - Z_1) \cot \theta]/Z_1,$$

$$\phi_2 = \pi/2 + \arctan [x - T - (Z_2 - Z_1) \cot \theta]/Z_1,$$

$$\phi_3 = \pi/2 + \arctan (x - T)/Z_2,$$

$$\phi_4 = \pi/2 + \arctan (x + T)/Z_2,$$

$$r_1^2 = (x + T + \overline{Z_2 - Z_1} \cot \theta)^2 + Z_1^2,$$

$$r_2^2 = (x - T - \overline{Z_2 - Z_1} \cot \theta)^2 + Z_1^2,$$

$$r_3^2 = (x - T)^2 + Z_2^2,$$

and $r_4^2 = (x + T)^2 + Z_2^2.$

The various symbols used in (3) are shown in figure 1. The first part in (3) represents the case of constant density contrast.

3. Method of interpretation

Apart from the constants a_0 , a_1 and a_2 , defining the density function with depth the parameters of the trapezoidal model to be interpreted are Z_1 , Z_2 , T and θ . The constants a_0 , a_1 and a_2 are found by the least squares fitting of the density-depth values to a quadratic function. The density depth values can be obtained from the measured density values in bore-holes, or densities deduced by seismic methods or densities deduced otherwise.

The nonlinear optimization technique using the Marquardt algorithm is adopted to find out the unknown parameters, as the anomaly equation is too lengthy to develop any simple method of interpretation. The basis of the method is that an initial model is assumed and the various parameters are successively improved so as to minimize the objective function defined by

$$F = \sum_{i=1}^{N_{\text{obs}}} [\Delta g_{\text{obs}}(x_i) - \Delta g_{\text{cal}}(x_i)]^2, \quad (4)$$

where Δg_{obs} and Δg_{cal} are observed and calculated anomalies respectively and N_{obs} is the total number of observations. The method of the Marquardt algorithm has been discussed by many workers (Radhakrishna Murthy and Bhaskara Rao 1982, Coles 1976). The necessary system of simultaneous equations is given by

$$\begin{aligned} & \sum_{k=1}^4 \sum_{i=1}^{N_{\text{obs}}} \frac{\partial \Delta g(i)}{\partial P_j} \frac{\partial \Delta g(i)}{\partial P_k} (1 + \delta_{jk} \lambda) dP_k \\ &= \sum_{i=1}^{N_{\text{obs}}} [\Delta g_{\text{obs}}(i) - \Delta g_{\text{cal}}(i)] \frac{\partial \Delta g(i)}{\partial P_j} \quad \text{for } j = 1 \text{ to } 4, \end{aligned} \quad (5)$$

where $\delta_{jk} = 1$ for $j = k$,
 $= 0$ for $j \neq k$,

and P_k represents one of the parameters Z_1 , Z_2 , T and θ . Using the notation,

$$\begin{aligned} F_3 &= \sin \theta \ln (r_4/r_1) + \cos \theta (\phi_1 - \phi_4), F_4 = \sin \theta \ln (r_3/r_2) \\ &\quad - \cos \theta (\phi_2 - \phi_3) \\ F_5 &= \sin 2\theta \ln (r_4/r_1) + \cos 2\theta (\phi_1 - \phi_4), F_6 = \sin 2\theta \ln (r_3/r_2) \\ &\quad - \cos 2\theta (\phi_2 - \phi_3), \\ F_7 &= \sin 3\theta \ln (r_4/r_1) + \cos 3\theta (\phi_1 - \phi_4), F_8 = \sin 3\theta \ln (r_3/r_2) \\ &\quad - \cos 3\theta (\phi_2 - \phi_3), \\ F_9 &= x + T + Z_2 \cot \theta, F_{10} = x - T - Z_2 \cot \theta, \\ F_{11} &= (Z_2 - Z_1) \operatorname{cosec}^2 \theta, F_{12} = x + T + (Z_2 - Z_1) \cot \theta, \\ F_{13} &= x - T - (Z_2 - Z_1) \cot \theta, F_{14} = (x + T) \cos \theta - Z_2 \sin \theta, \\ F_{15} &= (x - T) \cos \theta + Z_2 \sin \theta, \\ G_1 &= F_{10}/r_2^2 - F_9/r_1^2, G_3 = -F_9/r_1^2, \\ G_4 &= F_{10}/r_2^2, G_5 = (F_9 \cot \theta - Z_1 \operatorname{cosec}^2 \theta)/r_1^2, \\ G_6 &= -(Z_1 \operatorname{cosec}^2 \theta + F_{10} \cot \theta)/r_2^2, G_7 = Z_1 \cot \theta/r_1^2 + Z_1 \cot \theta/r_2^2, \\ G_8 &= (x - T)/r_3^2, G_9 = Z_1 \cot \theta/r_1^2 + (x + T)/r_4^2, \\ G_{10} &= (x - T)/r_3^2 - Z_1 \cot \theta/r_2^2, G_{11} = Z_2/r_4^2 - F_{12} \cot \theta/r_1^2, \\ G_{12} &= Z_2/r_3^2 + F_{13} \cot \theta/r_2^2, G_{13} = Z_1/r_1^2 + Z_1/r_2^2, \\ G_{14} &= Z_2/r_4^2 + Z_2/r_3^2, G_{15} = Z_1/r_1^2 - Z_2/r_4^2, \\ G_{16} &= Z_2/r_3^2 - Z_1/r_2^2, G_{17} = (x + T)/r_4^2 - F_{12}/r_2^2, \\ G_{18} &= F_{13}/r_2^2 - (x - T)/r_3^2, G_{19} = -(Z_1 F_{11}/r_1^2 + Z_1 F_{11}/r_2^2), \\ G_{21} &= -Z_1 F_{11}/r_1^2, G_{22} = Z_1 F_{11}/r_2^2, \\ G_{23} &= F_{11} F_{12}/r_1^2, \quad \text{and} \quad G_{24} = -F_{11} F_{13}/r_2^2, \end{aligned}$$

the partial derivatives in (5) are given by

$$\begin{aligned}
\frac{\partial \Delta g}{\partial Z_1} &= 2\gamma a_0 [-(\phi_1 - \phi_2) - Z_1 G_1 + F_1 (G_5 \sin \theta + G_3 \cos \theta) \\
&\quad - F_2 (G_6 \sin \theta - G_4 \cos \theta)] + \frac{2\gamma a_1}{2} [-2Z_1 (\phi_1 - \phi_2) \\
&\quad - Z_1^2 G_1 + F_1 \{-\sin \theta + F_1 (G_5 \sin 2\theta + G_3 \cos 2\theta)\} \\
&\quad - F_2 \{-\sin \theta - F_2 (G_6 \sin 2\theta - G_4 \cos 2\theta)\}] \\
&\quad + \frac{2\gamma a_2}{3} [-3Z_1^2 (\phi_1 - \phi_2) - Z_1^3 G_1 + F_1 \{-Z_1 \sin \theta - F_1 \sin 2\theta \\
&\quad + F_1^2 (G_5 \sin 3\theta + G_3 \cos 3\theta)\} - F_2 \{-Z_1 \sin \theta + F_2 \sin 2\theta \\
&\quad + F_2^2 (G_6 \sin 3\theta - G_4 \cos 3\theta)\}] \\
\frac{\partial \Delta g}{\partial Z_2} &= 2\gamma a_0 [(\phi_4 - \phi_3) + Z_2 G_8 - Z_1 G_7 + F_3 \cos \theta \\
&\quad + F_1 (G_{11} \sin \theta + G_9 \cos \theta) + F_4 \cos \theta - F_2 (G_{12} \sin \theta - G_{10} \cos \theta)] \\
&\quad + \frac{2\gamma a_1}{2} [2Z_2 (\phi_4 - \phi_3) + Z_2^2 G_8 - Z_1^2 G_7 + \cos \theta \{(Z_2 - Z_1) \sin \theta \\
&\quad + F_1 F_5\} + F_1 \{\sin \theta + F_5 \cos \theta + F_1 (G_{11} \sin 2\theta + G_9 \cos 2\theta)\} \\
&\quad + \cos \theta \{(Z_2 - Z_1) \sin \theta - F_2 F_6\} - F_2 \{\sin \theta + F_6 \cos \theta \\
&\quad - F_2 (G_{12} \sin 2\theta - G_{10} \cos 2\theta)\}] + \frac{2\gamma a_2}{3} \left[3Z_2^2 (\phi_4 - \phi_3) \right. \\
&\quad + Z_2^3 G_8 - Z_1^3 G_7 + \cos \theta \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \sin \theta + F_1 (Z_2 - Z_1) \sin 2\theta \right. \\
&\quad + F_1^2 F_7 \left. \right\} + F_1 \{Z_2 \sin \theta + (Z_2 - Z_1) \cos \theta \sin 2\theta \\
&\quad + 2F_1 F_7 \cos \theta + F_1 \sin 2\theta + F_1^2 (G_{11} \sin 3\theta + G_9 \cos 3\theta)\} \\
&\quad + \cos \theta \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \sin \theta - F_2 (Z_2 - Z_1) \sin 2\theta + F_2^2 F_8 \right\} \\
&\quad - F_2 \{Z_2 \sin \theta + (Z_2 - Z_1) \cos \theta \sin 2\theta - F_2 \sin 2\theta \\
&\quad - 2F_2 F_8 \cos \theta + F_2^2 (G_{12} \sin 3\theta - G_{10} \cos 3\theta)\} \left. \right] \\
\frac{\partial \Delta g}{\partial T} &= 2\gamma a_0 [Z_2 G_{14} - Z_1 G_{13} + F_3 \sin \theta + F_1 (G_{17} \sin \theta + G_{15} \cos \theta) \\
&\quad + F_4 \sin \theta - F_2 (G_{18} \sin \theta - G_{16} \cos \theta)] \\
&\quad + \frac{2\gamma a_1}{2} [G_{14} Z_2^2 - G_{13} Z_1^2 + \sin \theta \{(Z_2 - Z_1) \sin \theta + F_1 F_5\}
\end{aligned}$$

$$\begin{aligned}
& + F_1 \{F_5 \sin \theta + F_1 (G_{17} \sin 2\theta + G_{15} \cos 2\theta)\} \\
& + \sin \theta \{ (Z_2 - Z_1) \sin \theta - F_2 F_6 \} - F_2 \{F_6 \sin \theta - F_2 (G_{18} \sin 2\theta \\
& - G_{16} \cos 2\theta)\} + \frac{2\gamma a_2}{3} \left[G_{14} Z_2^3 - G_{13} Z_1^3 + \sin \theta \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \sin \theta \right. \right. \\
& \left. \left. + F_1 (Z_2 - Z_1) \sin 2\theta + F_1^2 F_7 \right\} + F_1 \{ (Z_2 - Z_1) \sin \theta \sin 2\theta \right. \\
& \left. + 2F_1 F_7 \sin \theta + F_1^2 (G_{17} \sin 3\theta + G_{15} \cos 3\theta) \right\} \\
& + \sin \theta \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \sin \theta - F_2 (Z_2 - Z_1) \sin 2\theta + F_2^2 F_8 \right\} \\
& - F_2 \{ (Z_2 - Z_1) \sin \theta \sin 2\theta - 2F_2 F_8 \sin \theta \\
& \left. + F_2^2 (G_{18} \sin 3\theta - G_{16} \cos 3\theta) \right\} \Big], \text{ and}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Delta g}{\partial \theta} = & 2\gamma a_0 [-G_{19} Z_1 + F_3 F_{14} + F_1 \{ \cos \theta \ln (r_4/r_1) + G_{23} \sin \theta \\
& - \sin \theta (\phi_1 - \phi_4) + G_{21} \cos \theta \} - F_4 F_{15} - F_2 \{ \cos \theta \ln (r_3/r_2) \\
& + G_{24} \sin \theta + \sin \theta (\phi_2 - \phi_3) - G_{22} \cos \theta \}] \\
& + \frac{2\gamma a_1}{2} [-G_{19} Z_1^2 + F_{14} \{ (Z_2 - Z_1) \sin \theta + F_1 F_5 \} \\
& + F_1 \{ (Z_2 - Z_1) \cos \theta + F_{14} F_5 + F_1 (2 \cos 2\theta \ln (r_4/r_1) \\
& + G_{23} \sin 2\theta - 2 \sin 2\theta (\phi_1 - \phi_4) + G_{21} \cos 2\theta) \} \\
& - F_{15} \{ (Z_2 - Z_1) \sin \theta - F_2 F_6 \} - F_2 \{ (Z_2 - Z_1) \cos \theta \\
& - F_6 F_{15} - F_2 (2 \cos 2\theta \ln (r_3/r_2) + G_{24} \sin 2\theta \\
& + 2 \sin 2\theta (\phi_2 - \phi_3) - G_{22} \cos 2\theta) \}] \\
& + \frac{2\gamma a_2}{3} \left[-G_{19} Z_1^3 + F_{14} \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \sin \theta + F_1 (Z_2 - Z_1) \sin 2\theta \right. \right. \\
& \left. \left. + F_1^2 F_7 \right\} + F_1 \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \cos \theta + F_{14} (Z_2 - Z_1) \sin 2\theta \right. \right. \\
& \left. \left. + 2F_1 (Z_2 - Z_1) \cos 2\theta + 2F_1 F_7 F_{14} + F_1^2 (3 \cos 3\theta \ln (r_4/r_1) \right. \right. \\
& \left. \left. + G_{23} \sin 3\theta - 3 \sin 3\theta (\phi_1 - \phi_4) + G_{21} \cos 3\theta) \right\} \right. \\
& \left. - F_{15} \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \sin \theta - F_2 (Z_2 - Z_1) \sin 2\theta + F_2^2 F_8 \right\} \right. \\
& \left. - F_2 \left\{ \frac{(Z_2^2 - Z_1^2)}{2} \cos \theta - F_{15} (Z_2 - Z_1) \sin 2\theta - 2F_2 (Z_2 - Z_1) \cos 2\theta \right. \right.
\end{aligned}$$

$$\begin{aligned}
& + 2F_2 F_8 F_{15} + F_2^2 (3 \cos 3\theta \ln(r_3/r_2) + G_{24} \sin 3\theta \\
& + 3 \sin 3\theta (\phi_2 - \phi_3) - G_{22} \cos 3\theta) \Big\} \Big]. \quad (6)
\end{aligned}$$

To start with, the program requires the initial values of Z_1 , Z_2 , T and θ to be known. In the interpretation of real anomalies, the initial values of various parameters may be taken from seismic or bore hole data. The objective function $F(1)$ as defined by (4) is found for the initial model. λ is initially given a value of 0.5. The derivatives in (5) are calculated using (6). Equations (5) are solved for the increments dZ_1 , dZ_2 , dT and $d\theta$. If Z'_1 , Z'_2 , T' and θ' are the initial values of Z_1 , Z_2 , T and θ , then the improved values Z_1^* , Z_2^* , T^* and θ^* are given by

$$Z_1^* = Z'_1 + dZ_1, Z_2^* = Z'_2 + dZ_2, T^* = T' + dT, \text{ and } \theta^* = \theta' + d\theta.$$

The objective function $F(2)$ is again calculated using these values. If $F(2) \leq F(1)$, then the step is a success and the value of λ is decreased by a factor of 1/2 and the value of $F(2)$ is assigned to $F(1)$. The method is iterated until some convergence criterion is met. If $F(2) > F(1)$ at any stage, the value of λ is doubled and the normal equations (4) are again framed and solved for the increments dZ_1 , dZ_2 , dT and $d\theta$. These are added to the values of Z_1 , Z_2 , T and θ at the previous successful step and the objective function $F(2)$ is calculated. This procedure is repeated until $F(2) \leq F(1)$. The usual convergence criterion is that the value of the objective function is below a certain tolerance limit or the increments of various parameters are negligible.

4. Examples

A synthetic anomaly of 20 km length (figure 2a) sampled at an interval of 1 km for $Z_1 = 1$ km, $Z_2 = 4$ km, $T = 2$ km, $\theta = 45^\circ$, $a_0 = -0.4$, $a_1 = 0.08$ and $a_2 = -0.005$ is interpreted by this method. To interpret this anomaly, the initial values of the parameters are taken as follows: $Z_1 = 0.5$ km, $Z_2 = 3.5$ km, $T = 1.5$ km and $\theta = 40^\circ$. The exact values of a_0 , a_1 and a_2 are assumed. The initial model and the corresponding gravity anomaly are also shown by dotted line in figure 2a. The convergence of the method is illustrated in figure 2b by plotting the values of the objective function, λ , and the parameters of the model with respect to iteration number. The values of various parameters at 20th iteration are as follows: $Z_1 = 1.00$ km, $Z_2 = 3.99$ km, $T = 2.07$ and $\theta = 45.72^\circ$.

To study the correlation between various parameters, the gravity anomalies are interpreted for different initial models. Though the interpreted values of Z_1 and Z_2 are nearly equal to the actual values in many cases, the values of T and θ are quite different from their actual values (table 1). This shows a strong correlation between the parameters T and θ . In other words, many values of T and θ could satisfy the interpretation within a given tolerance of error. Thus there is a trade-off between the errors and the interpreted values of the parameters T and θ . In the interpretation of real anomaly, it is therefore necessary to fix one of these parameters to deduce the other accurately.

The method is also applied to interpret the gravity anomalies over graben-type sedimentary basin. Figure 3 shows the interpretation of gravity anomaly profile over

Figure 2. Interpretation of gravity anomalies over a trapezoidal model with quadratic density function.

Table 1. Interpretation of gravity anomalies for different initial models

Model	Function	λ	Z_1	Z_2	θ	T
Initial model	78.94	—	0.5	3.50	40.0	1.5
Interpreted model	0.001	0.0003	1.0	3.99	45.72	2.07
Initial model	431.97	—	0.5	3.50	40.0	2.5
Interpreted model	0.1026	0.01562	0.992	3.784	48.21	2.441
Initial model	47.79	—	0.5	3.0	55.0	2.5
Interpreted model	0.082	0.0009	1.012	3.77	56.35	2.905
Initial model	493.28	—	0.5	2.5	55.0	1.5
Interpreted model	0.0309	0.0002	1.009	3.874	50.63	2.508
Initial model	373.8	—	0.5	3.5	30.0	1.5
Interpreted model	0.1978	0.0312	0.969	3.848	41.75	1.907

Figure 3. Interpretation of gravity profile over San Jacinto graben structure, California, with quadratic density function (after Cordell 1973).

San Jacinto graben structure, California, with quadratic density function. This profile was originally interpreted by Cordell (1973) with exponential decrease of density contrast with depth. The profile is 9.5 km in length, sampled at 0.5 km interval. The origin of a trapezoidal model is taken at the minimum anomaly point. The constants of the quadratic density function obtained by least squares fitting of density-depth values corresponding to the case of $\lambda = 0.5 \text{ km}^{-1}$ (Cordell 1973) are as follows: $a_0 = -0.54$, $a_1 = 0.24$ and $a_2 = -0.035$. The initial values of an outcropping trapezoidal model are taken to be $Z_2 = 2.5 \text{ km}$, $T = 2.0 \text{ km}$, $\theta = 60^\circ$. In order to use the program directly, Z_1 is fixed and assigned a value of 10^{-5} km to avoid any possible singularities. For these values of the parameters, the value of the objective function is 679.4. The value of λ , which is 0.5 at the beginning, continuously decreased upto 0.0625 at the 4th iteration and again increased to 0.125 at the 5th iteration and remained constant thereafter. The program is terminated at 15th iteration as there are no significant changes in the values of the parameters. The values of various parameters at the end of 15th iteration are as follows: $Z_1 = 10^{-5} \text{ km}$, $Z_2 = 2.59 \text{ km}$, $T = 1.29 \text{ km}$, $\theta = 64.3^\circ$, $\lambda = 0.125$ and $F(1) = 29.45$. The trapezoidal model so obtained and the Cordell's model are drawn in figure 3. These are in close agreement. The residuals at the end of 15th iteration are also shown in figure 3. The residuals remain to be 2 to 3 mgals at $\pm 2 \text{ km}$ from the origin.

These are not quite unexpected in the interpretation of real anomaly, as we are simplifying the interpretation by neglecting local inhomogeneities in density and also the fault plane is approximated by a straight line. Nonetheless, the interpretation is quite stable and acceptable as it is carried out by a model defined by a minimum number of parameters.

5. Conclusions

A synthetic and a real anomaly profile over a trapezoidal model wherein the density contrast decreases with depth is interpreted. The decrease of density contrast is approximated by a quadratic function. The density values known from borewells or by seismic methods might be used to construct a quadratic density function. This is a simple and generalized representation of decrease of density contrast with depth. Thus, the cases of decrease of density contrast linearly and exponentially with depth could easily be accounted by the quadratic function, because the former is a particular case of quadratic function, and the first few terms in the series expansion of the latter constitute a quadratic function. Thus, the choice of quadratic function to represent the decrease of density contrast with depth is justified. It has also a further advantage in that the anomaly equation is derived in closed form for quite different geometrical models.

The anomaly equation could easily be programmed as it contains simple arctan and logarithmic terms. The method converges readily and gives fairly accurate values for Z_1 and Z_2 . However, the parameters T and θ are correlated. Different interpretations could be arrived at for these parameters within a given tolerance of error. It is, therefore, necessary to fix one of these parameters by geological or other considerations to find the other accurately. The method proposed here is useful to interpret gravity anomalies of various structures approximating trapezoidal models, particularly graben-type sedimentary basins, where the densities of the sediments at different depths are reasonably known.

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