Numerical hysteresis model for intermittent studies in unsaturated soils

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Abstract. In the present study, the use of one of the recent dependent domain models of capillary hysteresis in the numerical analysis of intermittent infiltration and redistribution of water in two types of soils (a sand and Rubicon Sandy Loam) has been shown. The numerical results for both the soils have been presented in terms of pressure head depth, moisture content depth and the pressure head-moisture content relationships. The capillary hysteresis model has been found to be very useful for the prediction of both wetting and drying scanning curves of various orders.

Keywords. Capillary hysteresis model; unsaturated soils; scanning curves; intermittent conditions.

1. Introduction

In groundwater recharge problems, the study of soil moisture-depth profile under rapid intermittent surface flux conditions requires the inclusion of a capillary hysteresis model in the numerical simulation of flow processes. Under intermittent conditions, the effect of hysteresis is very significant and therefore it is *a priori* necessary to have the complete knowledge of higher order wetting and drying scanning curves. Watson and Perrens (1973) developed an interpolative capillary hysteresis model for the evaluation of higher order scanning curves from experimentally measured primary scanning curves of soil-moisture. The method is limited in application since it is quite impossible to measure all the scanning curves (Everett 1967) in the laboratory. In this regard, the conceptually based physical models are quite suitable for prediction of scanning curves from a limited set of experimental data.

Among the conceptually based domain models, Mualem and Dagan (1975) developed a dependent domain model by accommodating the concept of dependency of the state of neighbouring pores (Everett 1967; Topp 1971b) in the independent domain model of Mualem (1974) by including parameters like blockage against air-entry, $P_a(\theta)$, and blockage against water-entry, $P_w(\theta)$. Mualem (1977) presented a model to evaluate the hysteresis curve from either the boundary drying or the boundary wetting curve. Parlange (1976) proposed a simple extrapolative conceptual domain model of capillary hysteresis using the Mualem diagram. The model requires only the knowledge of the boundary loop and its derivative to predict all primary and other higher order wetting and drying scanning curves.

The use of the non-independent domain model of Poulouvasilis and Childs (1971) in the numerical analysis of intermittent flow study is shown by Lees and Watson (1975).

The purpose of the present study is to show the use of the dependent domain model of Mualem and Dagan (1975) in the numerical study of intermittent infiltration and redistribution of water in two types of soil. The types of soil selected are a sand and Rubicon Sandy Loam which are subject to the assumptions that the medium is homogeneous, isotropic, isothermal and rigid in behaviour. It is also shown that, in the subsequent cycles of intermittent sequences of flow processes, how the necessary higher order scanning curves are evolved for the surface node.

2. Numerical method

The water transport equation for unsaturated soil may be written as

$$C(h) \frac{\partial h}{\partial t} = \frac{\partial}{\partial Z} \left[K(h) \frac{\partial h}{\partial Z} \right] + \frac{\partial K}{\partial Z}(h), \quad (1)$$

where C is the specific moisture capacity, h the pressure head and Z the depth with positive direction upwards, t the time, and K the unsaturated hydraulic conductivity.

The numerical solution of the above equation can be obtained by a finite difference technique following Crank-Nicholson's approach. The method involves the use of a three-point central difference grid (figure 1) with the current value being evaluated at the mid-point. The finite difference form of the (1) is given by

$$A_j h_{(j-1, k+1)} + B_j h_{(j, k+1)} + C_j h_{(j+1, k+1)} = D_j; \quad (1 \leq j \leq N) \quad (2)$$

where the equation coefficients A_j , B_j , C_j and D_j are given by

$$A_j = -K_{(j-\frac{1}{2}, k+\frac{1}{2})}, \quad (3)$$

$$B_j = \frac{2}{r} C_{(j, k+\frac{1}{2})} - A_j - C_j, \quad (4)$$

Figure 1. Finite difference grid setting.

$$C_j = K_{(j+\frac{1}{2}, k+\frac{1}{2})}, \quad (5)$$

$$D_j = A_j[h_{(j,k)} - h_{(j-1,k)}] + C_j[h_{(j,k)} - h_{(j+1,k)}] + 2\Delta Z(A_j - C_j) + 2C_{(j,k+\frac{1}{2})} \cdot [h_{(j,k)}/r], \quad (6)$$

where

$$r = \Delta t / \Delta Z^2. \quad (7)$$

Equation (2) represents a set of N linear equations for N nodes along the depth axis which is given by

$$\begin{aligned} A_1 h_0 + B_1 h_1 + C_1 h_2 &= D_1 \\ A_2 h_1 + B_2 h_2 + C_2 h_3 &= D_2 \\ A_3 h_1 + B_3 h_2 + C_3 h_3 &= D_3 \\ &\text{-----} \\ A_N h_{N-1} + B_N h_N + C_N h_{N+1} &= D_N \end{aligned} \quad (8)$$

The matrix obtained in (8) is known as the 'tridiagonal matrix'.

To facilitate the solution of (3) through (7), the 'auto-selection iteration technique' was employed in a computer program developed following the approach elaborated by Whisler and Watson (1968). In (1), the $K(h)$ relationship is hysteretic, it was desirable to obtain the capillary conductivity values, from a $K(\theta)$ relationship. The specific moisture capacity was obtained by taking a gradient along the hysteretic path of a particular node at a desired pressure head value from a $h(\theta)$ relationship. Moreover the $C(h)$ curve is very unstable and oscillatory in nature and becomes discontinuous on the boundary drying curve within the air-entry region. To avoid these difficulties it is assumed that $d\theta/dh$ should be constant within a $\Delta h = (h_i - h_{i-1})$ interval, but in doing so the resulting $C(h)$ histogram becomes discontinuous between successive h intervals. So, as a remedy the average $C(h)$ values were obtained in adjacent intervals and then interpolated at the required point. At no time should the $C(h)$ value be negative or zero. It should have always a finite positive value.

2.1 Mualem-Dagan model of capillary hysteresis

It is sufficient for continuity of discussion at this point to outline the general form of the equations.

2.1a *Entry into the scanning curve system from the boundary wetting curve:* The equation for drying scanning curve is given as:

$$\begin{aligned} \theta \left[\begin{array}{ccc} \psi_1 & \text{---} & \psi_N \\ \psi_{\min} & \psi_2 & \psi \end{array} \right] &= \theta_w(\psi_1) - \bar{P}_d(\theta) \cdot [1 - H(\psi)] \cdot [L(\psi_N) - L(\psi)] \\ &+ \sum_{j=1}^{(N-1)/2} \bar{P}_d(\theta_{2j}) \cdot [1 - H(\psi_{2j})] \cdot [L(\psi_{2j+1}) - L(\psi_{2j-1})]. \end{aligned} \quad (9)$$

In the above equation N should be odd and ≥ 1 .

The equation for wetting scanning curves is given as:

$$\theta \left[\begin{array}{ccc} \psi_1 & \text{---} & \psi \\ \psi_{\min} & \psi_2 & \psi_N \end{array} \right] = \theta_w(\psi_1) + \bar{P}_d(\theta_N) \cdot [1 - H(\psi_N)] \cdot [L(\psi) - L(\psi_N)] \\ - \sum_{j=1}^{N/2} \bar{P}_d(\theta_{2j}) \cdot [1 - H(\psi_{2j})] \cdot [L(\psi_{2j-1}) - L(\psi_{2j})] \\ + \sum_{j=1}^{(N/2)-1} \bar{P}_d(\theta_{2j+1}) \cdot [1 - H(\psi_{2j+1})] \cdot [L(\psi_{2j+1}) - L(\psi_{2j+1})]. \quad (10)$$

In (10) N should be even and ≥ 2 .

2.1b Entry into the scanning curve system from the boundary draining curve: The equation for the drying scanning curve is written as:

$$\theta \left[\begin{array}{ccc} \psi_{\max} & \psi_2 & \text{---} & \psi_N \\ \psi_1 & & & \psi \end{array} \right] = \theta_d(\psi_1) - \bar{P}_d(\theta) \cdot [1 - H(\psi)] \cdot [L(\psi_N) - L(\psi)] \\ + \sum_{j=1}^{N/2} \bar{P}_d(\theta_{2j-1}) \cdot [1 - H(\psi_{2j-1})] \\ \times [L(\psi_{2j}) - L(\psi_{2j-1})] \\ - \sum_{j=1}^{(N/2)-1} \bar{P}_d(\theta_{2j+1}) \cdot [1 - H(\psi_{2j+1})] \\ [L(\psi_{2j}) - L(\psi_{2j+1})]. \quad (11)$$

In (11) N should be even and ≥ 2 .

The equation for the wetting scanning curve is given as:

$$\theta \left[\begin{array}{ccc} \psi_{\max} & \psi_2 & \text{---} & \psi \\ \psi_1 & & & \psi_N \end{array} \right] = \theta_d(\psi_1) + \bar{P}_d(\theta_N) \cdot [1 - H(\psi_N)] \cdot [L(\psi) - L(\psi_N)] \\ + \sum_{j=1}^{(N-1)/2} \bar{P}_d(\theta_{2j-1}) \cdot [1 - H(\psi_{2j-1})] \cdot [L(\psi_{2j}) - L(\psi_{2j-1})] \\ - \sum_{j=1}^{(N-1)/2} \bar{P}_d(\theta_{2j+1}) \cdot [1 - H(\psi_{2j+1})] \cdot [L(\psi_{2j}) - L(\psi_{2j+1})]. \quad (12)$$

In (12) N should be odd and ≥ 1 .

2.2 Description of the physical system

The porous materials selected for the present study are a sand (data obtained from Perrens and Watson 1977, who called it no. 17 sand) and Rubicon Sandy Loam (from Topp 1969). At a saturated moisture content of $0.292 \text{ cm}^3/\text{cm}^3$, No. 17 sand has a capillary conductivity of 0.8 cm/min and an air-entry value (Childs 1960) of -34.5 cm .

Figure 2. The hydrologic characteristics of the porous media. a. $K(\theta)$ for sand, b. $K(\theta)$ for sand, c. $h(\theta)$ for Rubicon Sandy Loam, and d. $h(\theta)$ for Rubicon Sandy Loam.

Similarly Rubicon Sandy Loam has a capillary conductivity of 0.03 cm/min at a saturated moisture content of $0.38 \text{ cm}^3/\text{cm}^3$ with an air-entry value of -30.0 cm .

The analysis presented is the unsteady flow to a semi-infinite and initially uniformly dry profile for a 70.0 cm column of No. 17 sand, whereas for the Rubicon Sandy Loam a water table was set at a depth of 120 cm below the surface of an initially partially dry profile. In each case the entire column was divided into equal intervals in the vertical Z-direction with each node separated by 1 and 1.2 cm for No. 17 sand and Rubicon Sandy Loam respectively. This yields 71 nodes for the sand and 101 nodes for Sandy Loam. The hydrologic characteristics $K(\theta)$ and $h(\theta)$ for the sand and Rubicon Sandy Loam are shown in figures 2a and b (Watson and Perrens 1973) and figures 2c and d (Topp 1969).

2.3 Evaluation of the domain functions

A capillary hysteresis subprogram has been developed which is based on the domain model of Muallem and Dagan (1975). This subprogram generates the various primary, secondary and other higher order redrying and rewetting scanning curves. To effect this a separate subroutine is used for the evaluation of distribution functions $H(h)$, $P_d(\theta)$ and $L(h)$ for both sand and Rubicon Sandy Loam. For the evaluation of these functions apart from the boundary loop, one primary drying curve for each soil originating from $(h_d = -20.2 \text{ cm}; \theta_d = 0.27 \text{ cm}^3/\text{cm}^3)$ for the sand and $(h_d = -6.0 \text{ cm}; \theta_d = 0.36 \text{ cm}^3/\text{cm}^3)$ for the Rubicon Sandy Loam are also selected. In addition the distribution function $P_w(\theta)$ is also derived for Rubicon Sandy Loam using one primary wetting curve originating from $(h_w = -141.23 \text{ cm}; \theta_w = 0.20 \text{ cm}^3/\text{cm}^3)$. The distribution functions $H(h)$, $P_d(\theta)$ and $P_w(\theta)$ are shown in figures 3 through 6. The subsequent predictions of various order of scanning curves during the solution process are made by these functions.

Figure 3. The distribution function, $H(h)$, for sand.

Figure 4. The distribution function, $H(h)$, for Rubicon Sandy Loam.

2.4 Flux pattern

2.4a Sand: In the case of sand, the surface flux (F_s) applied and the time of change of boundary conditions constrained during the $4\frac{1}{2}$ cycles of intermittence are as follows:

$$\begin{array}{lll}
 F_s = -0.5 \text{ cm/min}, & 0 \leq t \leq 10 \text{ min}, & Z = 0. \\
 F_s = 0, & 10 \text{ min} \leq t \leq 20 \text{ min}, & Z = 0. \\
 F_s = -0.5 \text{ cm/min}, & 20 \text{ min} \leq t \leq 21 \text{ min}, & Z = 0.
 \end{array}$$

Figure 5. The function for pore blockage against air-entry, $P_d(\theta)$, for sand.

Figure 6. The function for pore blockage against air-entry, $P_d(\theta)$, and water-entry, $P_w(\theta)$, for Rubicon Sandy Loam.

$F_s = 0$,	$21 \text{ min} \leq t \leq 25 \text{ min}$,	$Z = 0$.
$F_s = -0.5 \text{ cm/min}$,	$25 \text{ min} \leq t \leq 25.3 \text{ min}$,	$Z = 0$.
$F_s = 0$,	$25.3 \text{ min} \leq t \leq 26.3 \text{ min}$,	$Z = 0$.
$F_s = -0.5 \text{ cm/min}$,	$26.3 \text{ min} \leq t \leq 26.4 \text{ min}$,	$Z = 0$.
$F_s = 0$,	$26.4 \text{ min} \leq t \leq 26.7 \text{ min}$,	$Z = 0$.
$F_s = -0.5 \text{ cm/min}$,	$26.7 \text{ min} \leq t \leq 36.7 \text{ min}$,	$Z = 0$.

2.4b *Rubicon Sandy Loam*: The surface flux applied during the $2\frac{1}{2}$ cycles of intermittence in Rubicon Sandy Loam is:

$F_s = -0.02$ cm/min,	$0 \leq t \leq 100$ min,	$Z = 0$.
$F_s = 0$,	$100 \text{ min} \leq t \leq 200$ min,	$Z = 0$.
$F_s = -0.015$ cm/min,	$200 \text{ min} \leq t \leq 250$ min,	$Z = 0$.
$F_s = 0$,	$250 \text{ min} \leq t \leq 300$ min,	$Z = 0$.
$F_s = -0.02$ cm/min,	$300 \text{ min} \leq t \leq 350$ min,	$Z = 0$.

3. Numerical results and discussions

3.1 Sand

The numerical results for pressure head-depth, moisture content-depth and pressure head-moisture content relationships have been presented in figures, 7, 8 and 9, respectively. The numerals on figures 7 and 8 represent the infiltration profiles (the solid lines) at the end of 10 min (for the 1st cycle), 21 min (2nd cycle), 25.3 min (3rd cycle), 26.4 min (4th cycle) and 36.7 min ($4\frac{1}{2}$ cycles), and the redistribution profiles (the dashed lines) at the end of 20 min (for 1st cycle), 25 min (2nd cycle), 26.3 min (3rd cycle), 26.7 min (4th cycle) for both pressure head-depth and moisture content-depth relationships.

In the 1st cycle of intermittence, the 10 min pressure head and moisture content profiles in figures 7 and 8 show that during the 1st infiltration event of 10 min the soil has got wet up to a depth of -26.0 cm with an entry of 4.32 cm³ of water. The pressure head and moisture content were -20.63 cm and 0.26 cm³/cm³, respectively. During the 10 min redistribution event of this cycle (see the 20 min profiles in figures 7 and 8), the soil drains under the influence of gravity and causes the near surface region to lose water. The draining water accumulates below the wet front to a depth of about -40 cm (figure 8). At the end of this event the pressure head at the surface decreased to a value of -42.1 cm with moisture content as 0.1 cm³/cm³.

It has been observed that during the redistribution event of the 1st cycle there is a development of negative moisture content gradient ($d\theta/dZ$) at $Z = 0$ (figure 8). If Z_{wf} is considered to be the wet front depth at the end of the 1st infiltration event, then it would be found that the reduced gradient ($\bar{\theta}_z$) will be negative on the draining part of the profile and zero near but above Z_{wf} . Below Z_{wf} it develops a positive gradient, which shows that the drained water is accumulating below Z_{wf} . At the maximum moisture content (θ_m) in the profile, $d\theta/dZ = 0$. Further as Z_{wf} increases, $\bar{\theta}_z$ decreases, the downward movement of water being greater. Also the redistribution rate is faster for a greater value of Z_{wf} .

The 21 and 25 min profiles for pressure head and moisture content are shown in figures 7 and 8, respectively, which developed during the infiltration and redistribution events of the 2nd cycle of intermittence. The pressure head and moisture content values for the surface node during the infiltration event were -21.6 cm and about 0.23 cm³/cm³, respectively, and those during the redistribution event were -40.6 cm and 0.12 cm³/cm³. The total volume of water which entered the soil is 5.26 cm³. The depth to which the wet front reached during the infiltration and redistribution events are -40.0 and -41.5 cm, respectively (see figure 8).

Figure 7. The pressure head depth profiles for $4\frac{1}{2}$ cycles of intermittence in sand. Solid lines represent the infiltration and the broken lines the redistribution.

It has been found that during the 2nd cycle of intermittence the top — 10 cm of soil wets up quickly with positive moisture content gradient and it can be seen in the moisture depth profile of 21 min (figure 8) that a 'peak' forms around — 10 and 30 cm of depth. The gradient (θ_z) is sufficient to transmit the water below this peak. Further, a feature to be noted in the moisture depth profile of 25 min (figure 8) is that though the gradient (θ_z) has become negative in the top few centimetres, yet below this depth the water drains along the 20 min profile, which is indicative of the fact that in sand the transmission zone is marked by this peak (see figure 8).

The pressure head and moisture content profiles of 25.3 and 26.3 min, in figures 7 and 8, developed during the infiltration and redistribution events of the 3rd cycle of intermittence; and those developed during the infiltration and redistribution events of the 4th cycle of intermittence are shown by the 26.4 and 26.7 min profiles also in figures 7 and 8. The pressure head and moisture content values for the surface node during the infiltration event of the 4th cycle were — 26.4 cm and $0.17 \text{ cm}^3/\text{cm}^3$, respectively and those during the redistribution event were — 38.0 cm and $0.12 \text{ cm}^3/\text{cm}^3$. The depth to which the wet front moved down at the end of the 4th cycle is — 46.0 cm (see 26.7 min profile in figure 8), and 5.6 cm^3 of water had entered the soil column. The collation of

Figure 8. The moisture content-depth profiles for $4\frac{1}{2}$ cycles of intermittence in sand. Solid lines represent the infiltration and the broken lines the distribution.

Figure 9. The $h(\theta)$ path of the surface node during the $4\frac{1}{2}$ cycles of intermittence of sand. The numerals 1, 2, 3, ..., 11 correspond to cumulative times, $t = 0, 10, 20, 21, 25, 25.3, 26.3, 26.4, 26.7, 31.7$ and 36.7 min, respectively.

the 26.3 and 26.7 min profiles of figure 8 indicates that during the 4th cycle the peak still persists and remains in its position whereas the wet front moves down slowly.

In the final half cycle of intermittence, only the infiltration event has been considered. The 31.7 and 36.7 min profiles of this event in figures 7 and 8 show that the peak disappears with the development of a positive moisture content gradient. In fact, the 36.7 min profile is in equilibrium with constant pressure head and moisture content values, and water infiltrates under unit hydraulic gradient with a capillary conductivity value equal to the surface flux. The values of pressure head and moisture content at the end of this event were -20.63 cm and $0.26 \text{ cm}^3/\text{cm}^3$, respectively. The wet front reached a depth of about -56.0 cm with 7.65 cm^3 of water entering the soil column.

Figure 9 shows the history of imbibition and drainage of the surface node after a number of intermittent infiltration and redistribution sequences. The numerals 1, 2, 3, ———, 9 correspond to the time $t = 0, 10, 20, 21, 25, 25.3, 26.3, 26.4$ and 26.7 min respectively at which pressure head and moisture content reversals have occurred. It can be ascertained from figure 9 that upon continued infiltration during the final infiltration event the current quarternary rewetting scanning curve closes the inner and outer loops at points 8, 6, 4, and 2 and advances to the points 10 and 11 which correspond to the times 31.7 and 36.7 min respectively. Moreover as the infiltration continues, the surface node remains at the point 11 and the bottom nodes start to come to equilibrium with the surface node. Thus the hysteresis model is quite efficient in generating the various order of scanning curves of either wetting or draining (figure 9).

3.2 *Rubicon Sandy Loam*

The model is also tested with Rubicon Sandy Loam for intermittent conditions. The flux applied at the surface of the soil column has always been less than the saturated value of hydraulic conductivity (see flux pattern). The numerical results for pressure head-depth, moisture content-depth and pressure head-moisture content relationships have been given in figures 10, 11 and 12, respectively. The numerals on figures 10 and 11 represent the infiltration profiles (solid lines) at the end of 100 min (for 1st cycle), 250 min (2nd cycle) and 350 min ($2\frac{1}{2}$ cycles), and the redistribution profiles (dashed lines) at the end of 200 min (for 1st cycle) and 300 min (2nd cycle) for both pressure head-depth and moisture content-depth relationships.

The zero time profiles in figures 10 and 11 are for the initial equilibrium state. In these figures the 100 min and 200 min profiles are those developed during the infiltration and redistribution events respectively, of the 1st cycle of intermittence. During these events the changes in pressure head and moisture content profiles occurred to a depth of about -50.0 cm. The 200 min moisture content depth profile shows that the drained water has redistributed below the depth of -27 cm (see figure 11).

In the second cycle of intermittence, the infiltration and redistribution were set for 50 min each. The pressure head profiles for this cycle have been shown by the 250 min for infiltration and 300 min for redistribution events (see figure 10); and corresponding moisture content profiles are shown in figure 11. The 300 min moisture content profile shows that the redistribution is not so marked as in the case of the previous redistribution event (figure 11). The reason is that the soil has reached that range of pressure head where the water content change occurs very little.

The final cycle of intermittence comprises of an infiltration event only (see 350 min

Figure 10. The $h(Z)$ profiles for $2\frac{1}{2}$ cycles of intermittence in Rubicon Sandy Loam. Solid lines represent the infiltration and the broken lines the redistribution.

profiles in figures 10 and 11), and during 50 min of infiltration the wetting has occurred mainly within -46.0 cm of the soil. The pressure head and moisture content values for the surface node during each of the above mentioned events can be read from figures 10 and 11.

In figure 12, the pressure head–moisture content relationship for the surface node is shown. The numerals 1, 2, ---, 5 on the diagram correspond to the time 0, 100, 200, 250 and 300 min, respectively, at which reversals have occurred. Upon continued infiltration the current rewetting curve (between 5 and 4 in figure 12) closes the tertiary (between 5 and 4) and secondary (between 3 and 2) loops at points 4 and 2 respectively, and advances beyond to the point 6 on the primary wetting curve (between 1 and 6) because the surface flux is greater now than during the initial infiltration.

Finally, it should be noted that for the prescribed surface flux and timings of the events, the steady state condition is not reached in Rubicon Sandy Loam.

4. Conclusions

The water movement in sand is found to be very rapid with the development of a distinct zone of transmission which is generally marked by a peak in moisture content

Figure 11. The $\theta(Z)$ profiles for $2\frac{1}{2}$ cycles of intermittence in Rubicon Sandy Loam. Solid lines represent the infiltration and the broken lines the redistribution.

Figure 12. The $h(\theta)$ path of the surface node during the $2\frac{1}{2}$ cycles of intermittence in Rubicon Sandy Loam. The numerals 1, 2, ---, 6 correspond to the cumulative time, $t = 0, 100, 200, 250, 300$ and 350 min, respectively.

profile. The peak, however, disappears when infiltration is continued for a longer period; whereas the water movement in Rubicon Sandy Loam is found to be very slow. In studies of intermittent infiltration and redistribution sequences in a sand and Rubicon Sandy Loam, the Mualem-Dagan model of capillary hysteresis is found to be quite satisfactory with regard to the evaluation of primary and other higher order scanning curves. This prompts one to use further the Mualem and Dagan (1975) model for the problems of rapid intermittent flow situations.

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List of symbols

A_j, B_j, C_j and D_j	= the equation coefficients,
C	= the specific moisture capacity,
F_s	= the surface flux,
$h, h(j, k)$	= the pressure head,
$h(\theta)$	= the moisture content-pressure head relationship,
$L(h)$	= the distribution function,
$H(h)$	= the distribution function,
K	= the unsaturated hydraulic conductivity,
${}_r h_d$	= the pressure head value at the point of reversal to draining,
${}_r h_w$	= the pressure head value at the point of reversal to wetting,
$P_d(\theta)$	= the blockage against air-entry (the distribution function),
$\bar{P}_d(\quad)$	= $P_d(\quad) \cdot P_w(\quad)$
$P_w(\theta)$	= the blockage against water-entry (the distribution function),
${}_r \theta_d$	= the moisture content value at the point of reversal to draining,
${}_r \theta_w$	= the moisture content value at the point of reversal to wetting,
θ_m	= the maximum moisture content,
$\bar{\theta}_z$	= $d\theta/d(Z/Z_{wf})$, the reduced gradient,
$d\theta/dh$	= the gradient of $h(\theta)$ relationship,
t	= time,
Z	= the depth,
Z_{wf}	= the wet front depth.

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