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A study on the sensitivity of the radar scattering coefficient to oceanic winds

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Abstract. Sensitivity of the radar scattering coefficient to the oceanic wind vector in the mid-angular range for frequencies from L- to Ku- band is studied. This is based on computations of scattering coefficient via the two-scale scattering theory employing a semi-empirical model for the ocean spectrum suggested by Fung and Lee and the slope distribution by Cox and Munk. Higher frequency and incident angles of over 45 degrees seem to yield better wind sensitivity.

Keywords. Microwaves; radar scattering coefficient; ocean spectrum; sea surface wind speed.

1. Introduction

The scattering of microwaves from the sea surface has been explained reasonably well by the two-scale roughness model of the sea surface, where the small scale waves are assumed to satisfy the condition of the small perturbation method, while the large scale waves are assumed to satisfy the Kirchhoff approximation (Moore and Fung 1979). The two-scale roughness model used in conjunction with the wind sensitive sea spectrum has resulted in a relationship between the back scattering coefficient and the wind vector, which agrees with trends in the experimental scattering coefficient data (Fung and Lee 1982). The scattering coefficient data have been acquired by various airborne and spaceborne scatterometers, operating at different frequencies, polarization and observation configurations. The question as to what are the optimum system parameters so that the wind vector can be best inferred from the scattering coefficient measurements still remains to be satisfactorily answered. This paper makes a preliminary but systematic attempt towards that goal. The scattering coefficient (σ^0)-wind vector relationships for different frequencies are evaluated via computations of σ^0 based on the semi-empirical model of the ocean spectrum suggested by Fung and Lee (1982, 1983). To facilitate sensitivity study, this relationship has been first given a simple analytic form. The coefficients of this relationship have then been expressed in terms of polynomials of observation angles for different frequency bands and polarizations. This has enabled us to study the sensitivity of the scattering coefficient to wind speed and wind direction over the range of angles between 30 and 60 degrees for different frequency bands and polarizations. The sensitivity functions are expected to be useful in defining an optimum system for remote sensing of oceanic wind vector.

2. Scattering coefficient-wind relationship

The scattering coefficient σ^0 for an arbitrary wind vector is known to satisfy an expression of the type (Ulaby *et al* 1982)

$$\sigma^0 = A_0 + A_1 \cos\phi + A_2 \cos 2\phi \tag{1}$$

where

ϕ the angle between the incidence plane and the wind direction.

The coefficients A_0, A_1 and A_2 are generally related to surface wind speed. These coefficients can be expressed in terms of σ_u^0, σ_c^0 and σ_d^0, which represent scattering coefficients for the specific cases of upwind, crosswind and downwind directions respectively:

$$A_0 = \frac{\sigma_u^0 + 2\sigma_c^0 + \sigma_d^0}{4} \tag{2}$$

$$A_1 = \frac{\sigma_u^0 - \sigma_d^0}{2} \tag{3}$$

$$A_2 = \frac{\sigma_u^0 - 2\sigma_c^0 + \sigma_d^0}{4} \tag{4}$$

The specific values of σ_u^0, σ_c^0 and σ_d^0 can be expressed by relationships of the power law type:

$$\sigma^0 = aW^b, \tag{5}$$

where the coefficients a and b are functions of the incidence angle θ, polarization and frequency. The scattering coefficient σ^0 is computed by the two-scale roughness approach (Chan and Fung 1977) using the semi empirical two dimensional sea spectrum $S(K, \phi)$ suggested by Fung and Lee (1982, 1983) and the slope probability density function P_θ as given by Cox and Munk (1954). The geometry of scattering by a two-scale rough surface is illustrated in figure 1. The scattering coefficient σ_{pp}^0 by the above approach is given by

$$\sigma_{pp}^0 = \int_{-\infty}^{\infty} \int_{-\cot\theta}^{\infty} \sigma_{pp}^0 (\theta', \phi)\, P_\theta (Zx', Zy')\, \mathrm{d}Z_x \mathrm{d}Z_y \tag{6}$$

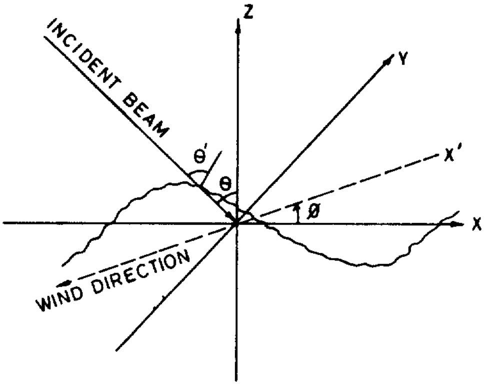

Figure 1. Geometry of two-scale rough surface scattering.

where

$$\sigma_{pp} = 8k^4 |\alpha_{pp}|^2 S(K, \phi)/K, \tag{7}$$

$pp = VV$ or HH polarization; θ' = local incidence angle; k = electromagnetic wavenumber; K = ocean wave number $= 2k \sin\theta$; θ = radar incidence angle; $Z'_x = Z_x \cos\phi + Z_y \sin\phi$, $Z'_y = Z_y \cos\phi - Z_x \sin\phi$, Z_x, Z_y = surface slopes along X and Ydirections respectively.

$$\alpha_{hh} = R_h \cos^2\theta \tag{8}$$

$$\alpha_{vv} = R_v \cos^2\theta + (1 - 1/\varepsilon_y)\, T_v^2 \sin^2\theta/2 \tag{9}$$

R_h is the Fresnel reflection coefficient for horizontal polarization, R_v, T_v are the Fresnel reflection and transmission coefficients for vertical polarization, ε_y is the relative permittivity of the sea surface.

The spectrum used here (Fung and Lee 1983) has the following form:

$$S(K, \phi) = S(K)[(2\pi)^{-1} + C(1 - \exp(-3K^2))\cos 2\phi] \tag{10}$$

$$S(K) = \begin{cases} S_1(K), & K \leqslant 0{\cdot}04 \text{ rad/cm} \\ S_2(K), & K > 0{\cdot}04 \text{ rad/cm} \end{cases} \tag{11}$$

$$S_1(K) = (0{\cdot}0014/K^3)\exp[-0{\cdot}74 g^2/K^2 W^4] \tag{12}$$

$$S_2(K) = 0{\cdot}875(2\pi)^{p-1} g^{(1-p)/2}(1 + K^2/4{\cdot}3923)(K + K^3/13{\cdot}1769) \tag{13}$$

where g = gravitational acceleration; $p = 5 - \log U_*$; U_* = friction velocity (cm/sec), and C is a wind dependent coefficient. The term $S(K)$ significantly depends on the wind speed. The probability slope distribution function P_θ has been modelled as a two dimensional Gaussian distribution function (Cox and Munk 1954).

Four representative frequencies, viz, 1, 5, 9 and 13 GHz (belonging to L-, C-, X- and Ku- bands respectively) were selected for σ^0 calculations. Computed σ^0 data for different incidence angles at an interval of half a degree were used for power law fits with wind speed i.e. for the equation $\sigma^0 = a(\theta) W^{b(\theta)}$. The coefficients $a(\theta)$ and $b(\theta)$, thus generated, were given the form of functions of polynomials by the method of numerical curve fitting. The analytical expressions for $a(\theta)$ and $b(\theta)$ have the following form

$$a(\theta) = \exp\left(\frac{1}{10}\sum_{i=0}^{4} a_i \theta^i\right) \tag{14}$$

$$b(\theta) = \tan\left(\log e \sum_{i=0}^{3} b_i \theta^i\right) \tag{15}$$

The polynomials for both $a(\theta)$ and $b(\theta)$ were found to give excellent fits with RMS errors always less than 3 %. Tables 1 and 2 give the numerical values of $a(\theta)$ and $b(\theta)$ for all the cases. Similar analytical functions were earlier generated for the case of scattering in the quasi-specular range of incidence angles (Sarkar and Bhaduri 1984).

3. Sensitivity of scattering coefficient to the wind vector

The sensitivity of scattering coefficient has earlier been studied for soil moisture determination (Ulaby *et al* 1979). In this paper it is done for the sea surface wind vector.

Table 1. Coefficients of polynomial for $a(\theta)$.

Frequency (GHz)	Polarization	ϕ (degree)	a_0	a_1	a_2	a_3	a_4
1	*HH*	0	123·2207	−13·9647	·3733	−4·7242E-03	2·15745E-05
		90	66·9564	−9·7345	·2502	−3·1697E-03	1·4345E-05
		180	30·0024	−6·0664	·1340	−·0159E-01	6·5992E-06
5	*HH*	0	−99·1490	7·1054	−·3281	·0524E-01	−2·9972E-05
		90	−3·4329	−1·8507	−·03206	·0106E-01	−8·5608E-06
		180	−2·5878	−2·1942	−·01208	6·8585E-04	−6·1646E-06
9	*HH*	0	−125·3527	9·2265	−·3866	·5876E-02	−3·2287E-05
		90	−37·9788	1·3275	−·1330	2·3937E-03	−1·4868E-05
		180	−25·8341	·0227	−·0857	·1694E-02	−1·1205E-05
13	*HH*	0	−188·9591	15·0596	−·5806	8·5886E-03	−4·6087E-04
		90	−93·7279	6·5711	−·3116	·4936E-02	−2·8001E-04
		180	−91·5667	6·1769	−·2947	·4680E-02	−·2672E-04
1	*VV*	0	126·1752	−14·3168	·4008	·5089E-02	·24079E-04
		90	72·9106	−10·0866	·2777	−·3534E-02	·16849E-04
		180	32·9566	−6·4185	·1615	−·1964E-02	·91034E-05
5	*VV*	0	−96·6940	6·8092	−·3033	·4923E-02	−·27787E-04
		90	−·9777	−2·1469	−·00733	·7483E-03	−·63759E-05
		180	−·13257	−2·4904	·01264	·3644E-03	−·39797E-05
9	*VV*	0	−122·9177	8·9326	−·3619	·5556E-02	−·30119E-04
		90	−35·5438	1·0337	−·1084	·2074E-02	−·12700E-04
		180	−23·3991	·2710	−·0611	·13749E-02	−·90374E-05
13	*VV*	0	−186·5084	14·7642	−·5560	·8268E-02	−·43917E-04
		90	−91·2772	6·2757	−·2869	·4616E-02	−·25831E-04
		180	−89·1160	5·8815	−·2701	·4360E-02	−·24551E-04

Since we are essentially concerned with two parameters namely wind speed W and wind direction ϕ, both $\partial\sigma^0/\partial w$ and $\partial\sigma^0/\partial\phi$ were computed. The sensitivities, as functions of incidence angle θ, for the L-, C-, X- and Ku-frequency bands and *HH* polarizations, are illustrated in figures 2–6. All of them confirm the expected behaviour of increase in sensitivity for higher frequencies. Fung and Lee (1982) have also shown qualitatively that the wind dependence of scattering coefficient increases with frequency and incidence angle. For wind speed higher than 15 m/sec, the increase in sensitivity is not significant. As far as sensitivity to wind speed alone is concerned, significant difference between X- and Ku- bands is not seen.

The curves for sensitivity to direction generally have double peaks, their levels depending upon the actual value of ϕ. In figure 2 are illustrated the curves for sensitivity of scattering coefficient to the wind direction for wind speeds of 10 and 15 m/sec. The behaviour of these curves for other wind speed values is found to be similar. The values of sensitivity to wind direction are generally found to be low.

Table 2. Coefficients of polynomials for $b(\theta)$.

Frequency (GHz)	Polarization	ϕ (degree)	b_0	b_1	b_2	b_3
1	*HH*	0	−2·79753	15·76619	−19·08023	7·4977
	and	90	−·567295	7·31430	−9·13235	3·65803
	VV	180	·88258	1·02593	−·82509	·30197
5	*HH*	0	−·67386	7·7006	−7·14803	2·21335
	and	90	−2·46066	13·75060	−15·03795	5·47711
	VV	180	−1·12791	10·54713	−11·6940	4·32868
9	*HH*	0	·35279	4·63796	−3·54326	·85391
	and	90	−1·84239	11·79946	−12·19866	4·25974
	VV	180	−·65313	9·39224	−10·19665	3·72259
13	*HH*	0	·99687	2·67938	−1·14607	−0·083213
	and	90	−1·10529	9·59061	−9·41842	3·14123
	VV	180	−·083323	7·62064	−7·9354	2·8039

Figure 6 shows the variation of directional sensitivity of the scattering coefficient over the range of ϕ between upwind and downwind for an incidence angle of 50 degrees. A periodic variation can be clearly discerned. In view of the original dependence of scattering coefficient on ϕ (1), the periodicity of the sensitivity curve can be easily understood. The pattern of the sensitivity curves for other incidence angles is similar. Keeping in mind sensitivities to speed and direction, it can be seen that incidence angles higher than 45 degree should be preferred from a sensitivity point of view.

4. Conclusions

1) It is seen that higher frequencies are more sensitive to wind speed as well as to wind direction. However for the X-band and the Ku-band curves are not significantly different. Hence attenuation and other considerations are important in opting for a particular frequency band among them.

2) The ϕ dependence of sensitivity curves rises steeply upto the incidence angle of about 45 degrees and thereafter the increment is not significant.

3) It can be also concluded that sensitivity is higher for lower wind speed. Hence there is a possibility of more accurate measurements in cases of low wind speeds in absolute terms.

4) The wind direction sensitivity of the scattering coefficient varies periodically with ϕ, with peaks around 60 degrees of ϕ.

5) There is no significant difference between the sensitivity curves for *HH* and *VV* polarization.

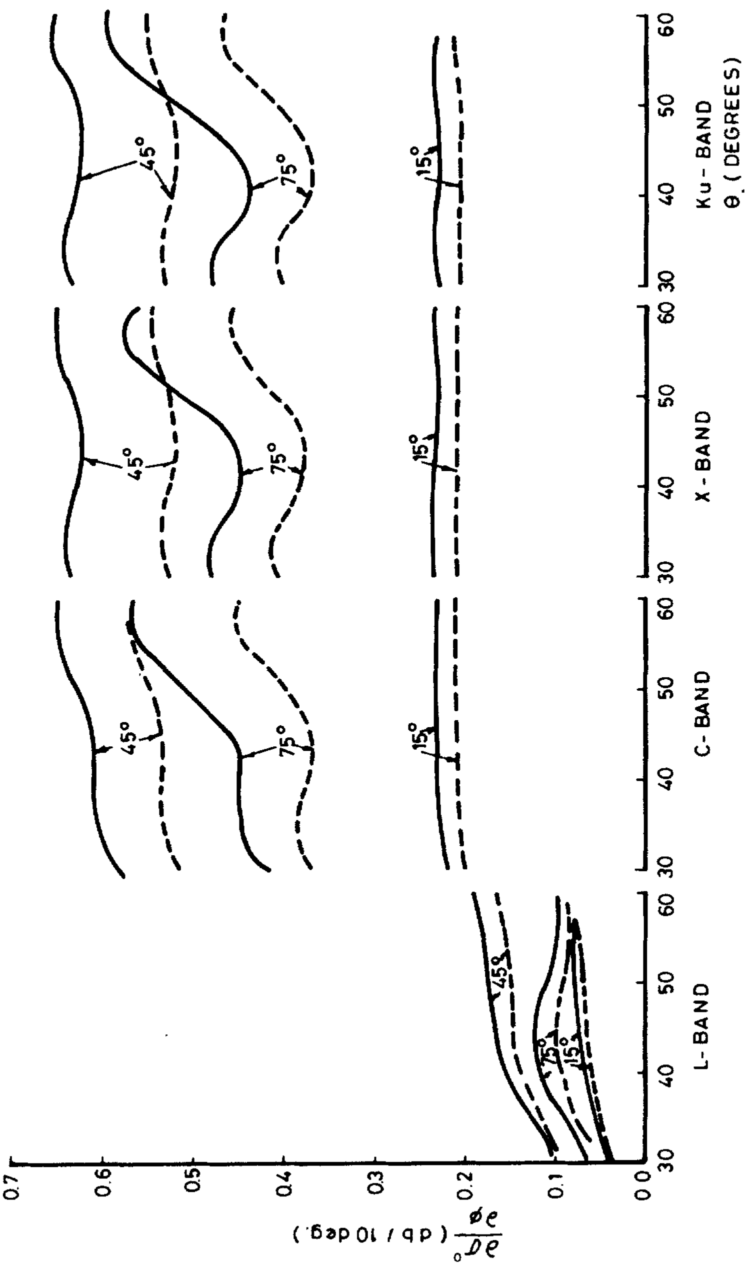

Figure 2. Sensitivity of scattering coefficient to wind direction versus incidence angle for L-, C-, X- and Ku-bands for different values of ϕ and wind speed values (——— $W = 15$ m/s, - - - - $W = 10$ m/s).

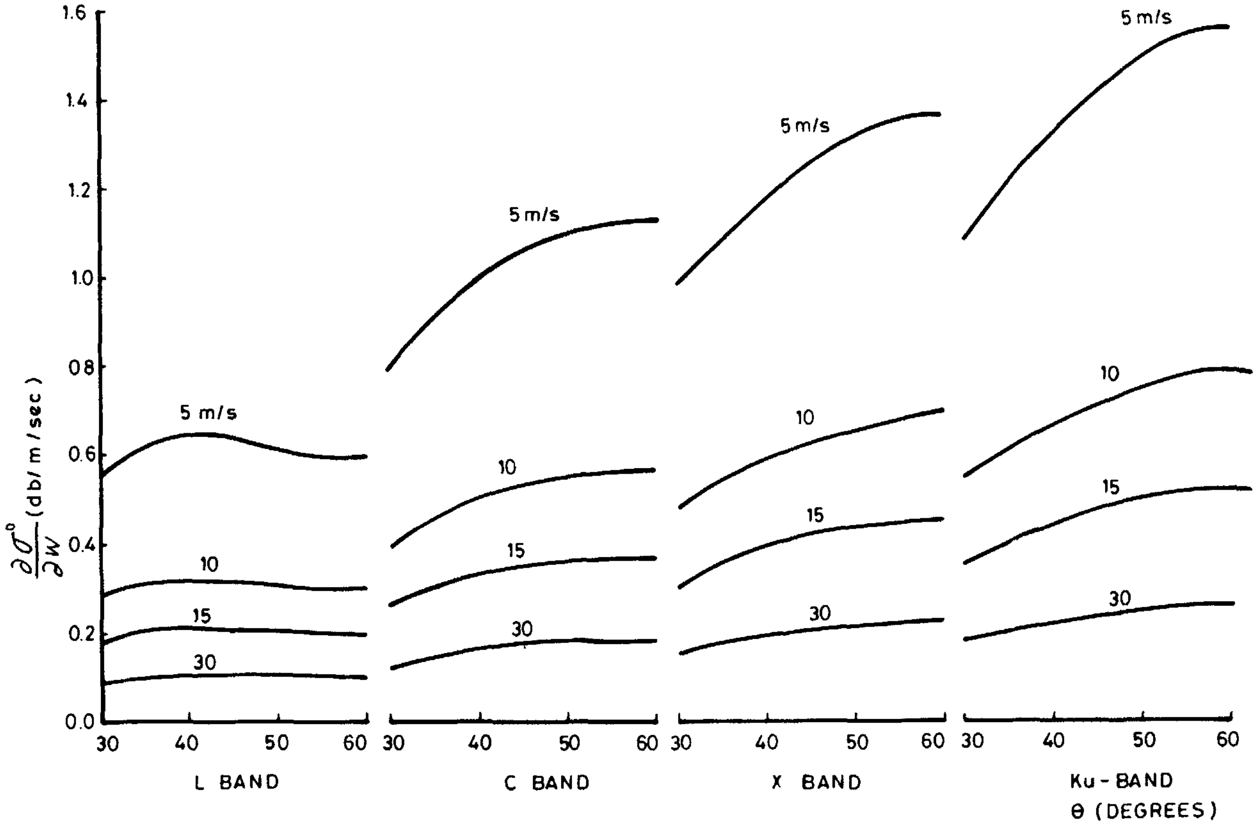

Figure 3. Sensitivity of scattering coefficient to wind speed versus incidence angle for L-, C-, X- and Ku- bands for upwind ($\phi = 0$ degree).

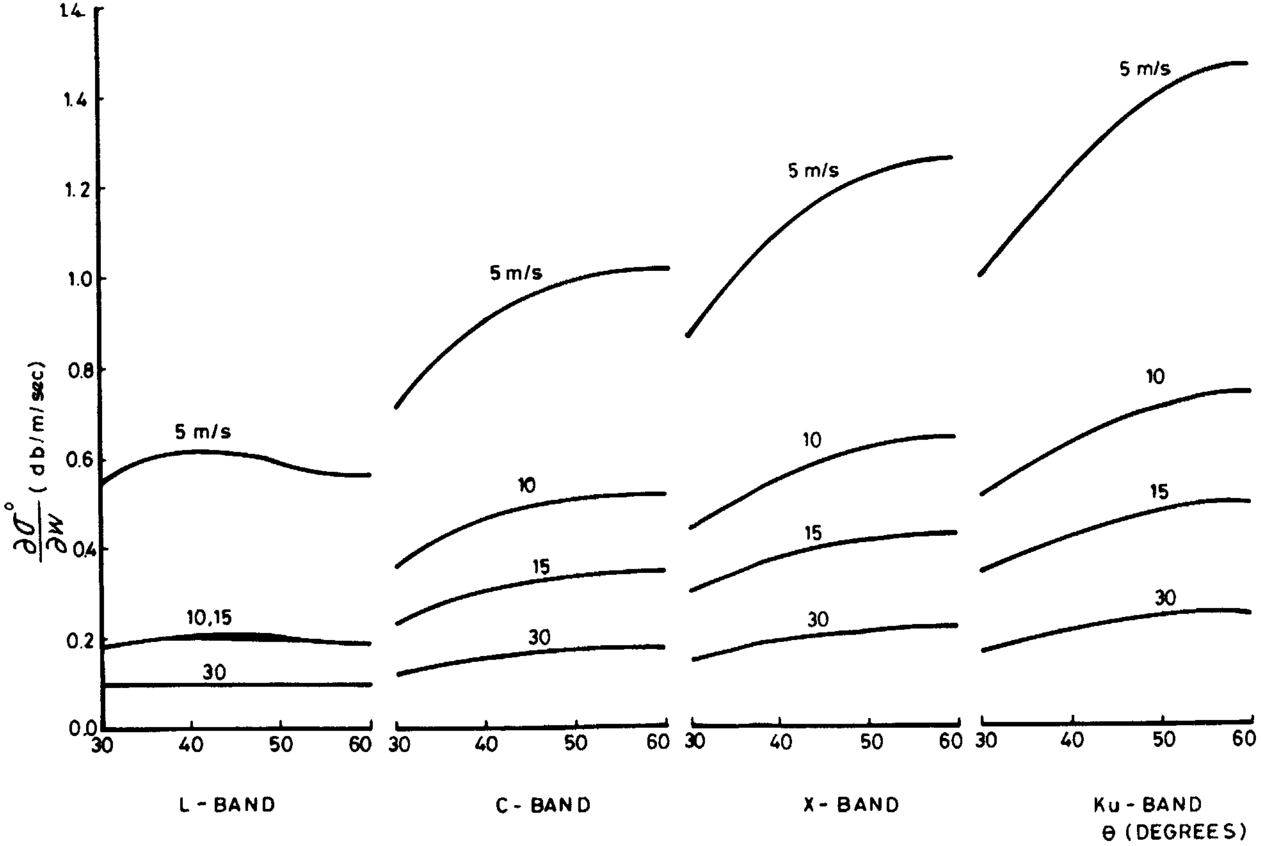

Figure 4. Sensitivity of scattering coefficient to wind speed versus incidence angle for L-, C-, X- and Ku- bands for ($\phi = 45$ degrees).

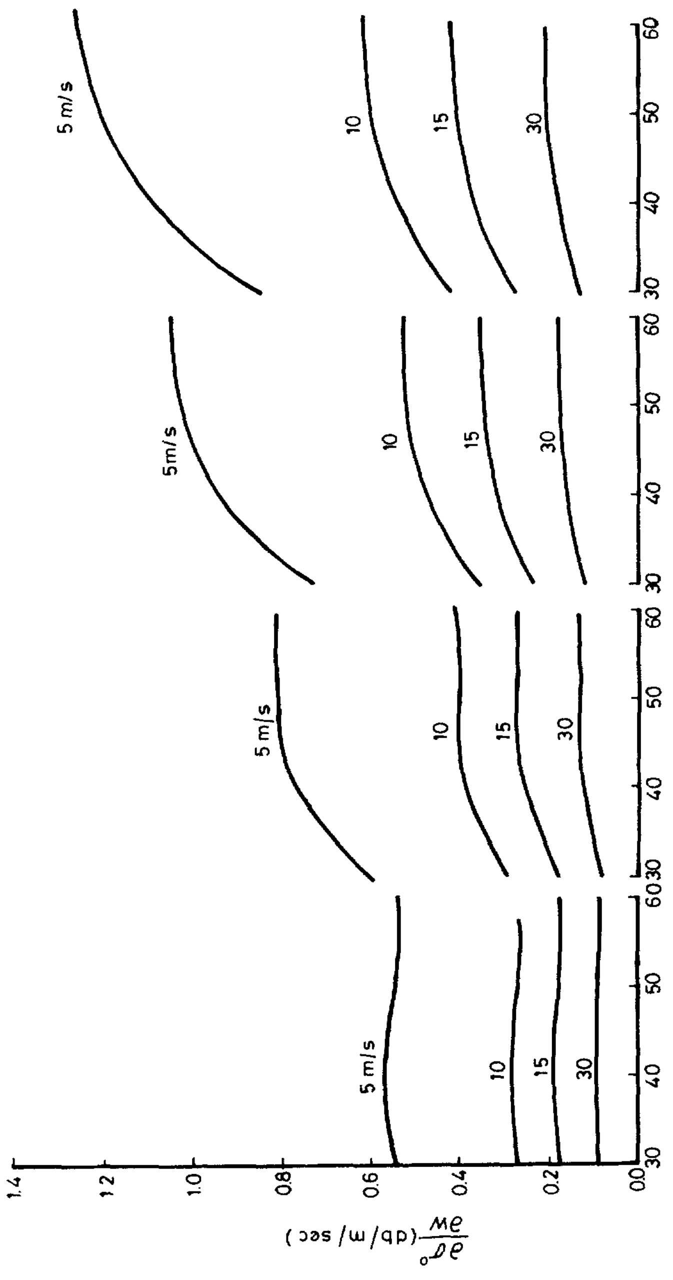

Figure 5. Sensitivity of scattering coefficient to wind speed versus incidence angle for L-, C-, X- and Ku-bands for crosswind (ϕ = 90 degrees).

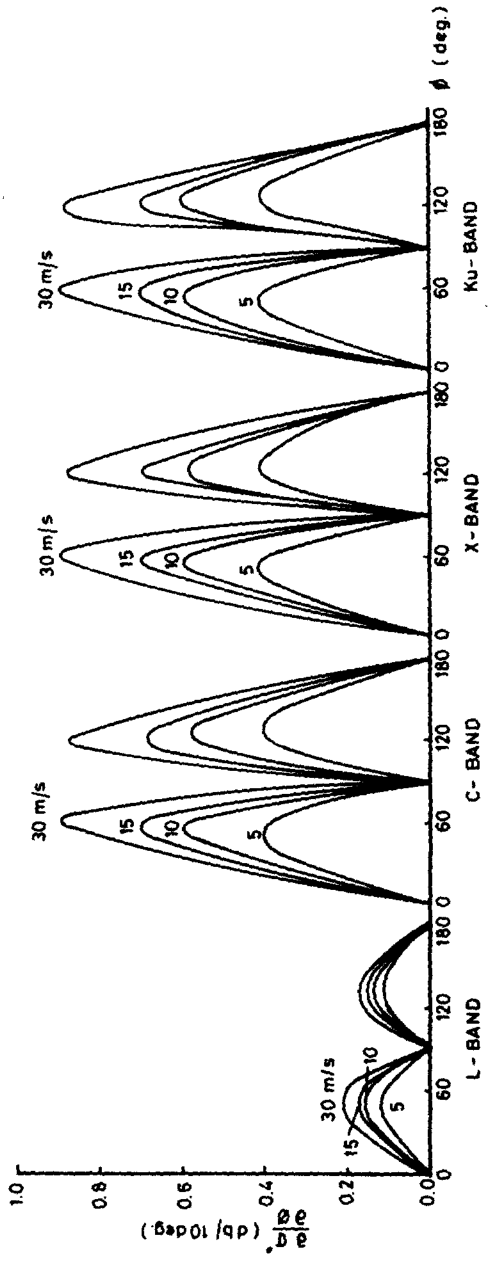

Figure 6. Sensitivity of scattering coefficient to wind direction versus ϕ for L-, C-, X- and Ku-bands for an incidence angle of 50 degrees.

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