

## Effect of parallel electric field and geomagnetic field distortion on precipitation of charged particles into the lower ionosphere

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**Abstract.** Mead axisymmetric distortions in the geomagnetic field and uniform electrostatic field parallel to geomagnetic field have been assumed to derive an expression for the inhomogeneity parameter,  $\alpha_d$ . Consequent change in the pitch angle diffusion of charged particles has been obtained. Using these parameters, the variations in the precipitating electron influx with varying stand-off distances and parallel electrostatic fields have been computed.

**Keywords.** Wave-particle interaction; geomagnetic field distortion; parallel electric field; charged particle precipitation.

### 1. Introduction

The solar wind interaction with the magnetospheric plasma results in the formation of bow shock, magnetosheath and plasma flow into the nightside of the earth. The flow pattern of energetic solar wind plasma and its entry into the nightside magnetosphere is responsible for injection of energetic particles in the inner magnetosphere. Varying fraction of injected energetic charged particles is trapped in different regions of the magnetosphere. The energy of the charged particles and direction of geomagnetic field determine the initial pitch angle distribution of trapped particles. The low pitch angle charged particles gyrating along the geomagnetic lines precipitate in the lower ionosphere. Very low frequency waves are generated and triggered by the gyrating or guided charged particles (Dowden 1962; Helliwell 1967; Dysthe 1971; Nunn 1974; Curtis and Wu 1979). Lightning generated VLF waves also propagate along the geomagnetic field lines with much lower phase velocity and nominal attenuation but suffer significant dispersion. These waves are known as whistlers (Storey 1953). The *in situ* measurements have established the existence of finite electrostatic field parallel to geomagnetic field lines (Mozer and Fahleson 1970; Mozer *et al* 1977). Effects of parallel electrostatic field on chorus, changing chorus spectrograms, whistler propagation and wave-particle interactions have been studied by many workers (Hsieh 1968; Gupta and Singh 1975, 1979; Misra and Singh 1977; Misra *et al* 1979; Brinca 1980, 1981). In this paper, we consider the whistler waves which are generated by lightning sources and propagating along the geomagnetic field lines with negligible attenuation. The whistler wave interactions with charged particles have been studied and it is shown that the wave-particle interaction may enhance the flux of precipitating electrons which, in addition to other sources, may form diffuse aurora.

In the study of wave-particle interaction, we ignore the variation of electric field along the geomagnetic field and consider the uniform electric field as assumed by Brinca (1980, 1981). This uniform parallel electric field may have its origin in the current driven instabilities. These instabilities are excited for electrons drifting with respect to

ions with an average speed larger than some threshold value. These instabilities cause anomalous resistivity which establishes electrostatic fields parallel to geomagnetic field lines (Block and Fälthammar 1976). The presence of localized potential double layers in the high latitude regions has been argued to be a competing process for the generation of parallel electric field in the magnetosphere (Block 1972, 1978). The parallel electric fields accelerate or decelerate the charged particles and give rise to inverted- $V$  formation reported and studied by many workers (Ackerson and Frank 1972; Mizera *et al* 1976; Lin and Hoffman 1979; Venkataranjan and McEwen 1979). The kinematics, trajectory and velocity distribution function of charged particles undergo an additional change in the presence of the electrostatic field. The dispersion relation contains a complex term which affects the wave-particle interaction in the presence of electric field along the geomagnetic field.

The results of this study clearly show that the charged particles influx changes with changing magnitude of electric field. Invariably the solar wind interaction and injection of energetic charged particles into the magnetosphere deform the dipolar shape of the geomagnetic field lines. Therefore, we have also considered the effect of distorted geomagnetic field. Choe and Beard (1974) have shown that at subsolar point, even during quiet times, solar wind induced distortions in the dipolar geomagnetic field are over 60%. These geomagnetic field distortions generally increase with increasing solar activity and extend to lower  $L$ -values. The combined effect of geomagnetic field distortions and parallel electric field on the electronic interaction with propagating whistlers have been studied. The theoretical formalism is similar to that given earlier (Prasad and Singh 1981). Using a definite form of distortion in the geomagnetic field, the existence of parallel electric field and charged particle parameters, we have first estimated the change in the pitch angle of resonating electrons. The precipitating electron influx for a velocity distribution function which is in conformity with measured energy spectra has been computed and its variations with other control parameters have been studied. It is shown that the existence of parallel electric field and geomagnetic field distortions play an important role in controlling the precipitating electron influx in the high latitude lower ionosphere. The precipitating electron influx simultaneously governed by various field changes may result in a complex auroral form which is neither discrete nor fully diffuse. A detailed study of fast varying auroral forms may establish the relevance of such a study.

## 2. Pitch angle diffusion of resonating charged particles

The effect of parallel electric field and distorted geomagnetic field model on the pitch angle diffusion of charged particles resonating with propagating whistlers has been reported in earlier papers (Prasad and Singh 1981; Singh and Prasad 1983). Under similar physical conditions and using same assumptions as made in the two earlier papers, we proceed to study the combined effect of parallel electric field and the distorted geomagnetic field as given by Mead model. The dispersion equation for propagating whistler mode wave is written as

$$\frac{c^2 k^2(z)}{\omega^2} = \frac{\omega_p^2(z)}{\omega[\omega_{cd}(z) - \omega]} \left\{ 1 + \frac{V_T^2 [k(z) + jK_1]^2}{2[\omega_{cd}(z) - \omega]^2} \right\}. \quad (1)$$

Here the parallel electric field-dependent parameter  $K_1 = eE_0/k_B T$  ( $k_B$  is Boltzmann constant) and plasma parameters have spatial dependence. In the presence of propagating whistler wave in the magnetosphere, the modified invariant equation holds good which combines the total charged particle energy and first adiabatic invariant of the resonant charged particles (Prasad and Singh 1981)

$$V^2 - \mu\omega - \frac{2e\phi}{m} = K_2^2, \quad (2)$$

where  $\phi$  is the electrostatic potential and  $K_2$  is a constant. The transformed kinetic equation for nearly resonant charged particles distributions satisfying conservation of (2), under the influence of whistler wave, distorted geomagnetic field and parallel electric field is now written as (Prasad and Singh 1981)

$$\frac{\partial f}{\partial t} + V_R(z) \frac{\partial f}{\partial z} + 2\eta \frac{\partial f}{\partial \xi} + \left( \frac{\cos \xi}{\tau_d^2} - 2\alpha_d \right) \frac{\partial f}{\partial \eta} = 0. \quad (3)$$

The transformation parameters used in obtaining this equation are similar to those used in Prasad and Singh (1981) and Singh and Prasad (1983). The parameters  $\tau_d$  and  $\alpha_d$  signify the combined effect of parallel electric field and distorted geomagnetic field.

It is well known that the most significant wave-particle interaction occurs in the equatorial region of the magnetosphere. In the lower regions along the geomagnetic field the resonant velocity of charged particle is relatively high; therefore, the time of resonance is comparatively small. Considering axisymmetric Mead field approximation (Mead 1964) for distorted geomagnetic field in the near equatorial region at farther distances in the magnetosphere, the gyro-frequency distribution is written as

$$\omega_{cd}(z) = \omega_{ce} \left[ 1 + \frac{B_1 R^3 L^3}{B_0 b^3} + \frac{9}{2} \left( 1 - \frac{B_1}{4B_0} \frac{L^3 R^3}{b^3} \right) \left( \frac{z}{LR} \right)^2 \right], \quad (4)$$

where suffix  $e$  signifies the equatorial value,  $B_0 = 0.31$  gauss,  $B_1 = 0.25$  gauss,  $R$  is the earth's radius,  $L$  is the McIlwain parameter measured in earth radii and  $b$  is the equatorial 'stand-off' distance from the point dipole to the magnetopause in the noon meridian which for typical solar wind velocity and flux is  $b \approx 10R$  (Schulz and Lanzerotti 1974). Magnetopause is extremely responsive to solar and geomagnetic activity (Rudneva and Feldstein 1970; Choe and Beard 1974). Aubry *et al* (1971) have reported the magnetopause to move with a velocity of  $20 \text{ km sec}^{-1}$  in an oscillatory manner. The oscillatory motion of the magnetopause covers the spatial extent of  $0.2$  to  $2.2$  earth radii. Thus, we find that (4) is very useful in depicting the changes in the wave-particle interaction during magnetic storms. Substituting for  $d\omega_{cd}(z)/dz$  and  $(d/dz)[V_R^2(z)]$  for the Mead-distorted field, we obtain an expression for  $\alpha_d$  in the equatorial region of the magnetosphere

$$\alpha_d = \frac{3k_e |V_{Re}|}{2LR} (V^2 - V_{Re}^2)^{1/2} \left( \frac{\omega_{ce}}{\omega_{ce} - \omega} \right)^{1/2} \left( 1 - \frac{1}{4} \frac{B_1 L^3 R^3}{B_0 b^3} \right)^{1/2} \\ \times \left[ 1 + \frac{3(\omega_{ce} - \omega)^3}{2(\omega_{ce} - \omega)^3 - \omega \omega_{pe}^2 \frac{V_T^2}{c^2}} - \frac{2(\omega_{ce} - \omega)^2}{2(\omega_{ce} - \omega)^2 - K_1^2 V_T^2} \right]^{1/2}$$

$$\times \left\{ 1 + \frac{\theta_L^2}{2} \frac{\omega_{ce} - \omega}{\omega_{ce}} \left[ 1 + \frac{3(\omega_{ce} - \omega)^3}{2(\omega_{ce} - \omega)^3 - \omega \omega_{pe}^2 \frac{V_T^2}{c^2}} - \frac{2(\omega_{ce} - \omega)^2}{2(\omega_{ce} - \omega)^2 - K_1^2 V_T^2} \right]^{-1} \right\} - \frac{k_e e E_0}{2m}, \quad (5)$$

where  $\theta_L$  is the equatorial pitch angle defining the effective loss cone,  $k_e$  is the real equatorial value of the propagation constant.

The expression for  $\alpha_d$  derived in (5) is substituted into the equation of motion

$$\frac{\partial \eta}{\partial t} = \frac{\cos \xi}{\tau_d^2} - 2\alpha_d, \quad (6)$$

where

$$\tau_d = [h \text{Re} k(z) \omega_{ca}(z) V_{\perp}^{1/2}]^{-1/2}$$

This equation has been solved following the procedures used in our earlier paper (Prasad and Singh 1981). The solutions have been obtained under the following conditions governed by spatially varying inhomogeneity parameter  $\alpha_d$ .

$$\eta/(4|\alpha_d|)^{1/2} \gg 1, \quad \eta_0/(4|\alpha_d|)^{1/2} \gg 1 \quad \text{and} \quad |\beta_d| < 1,$$

where  $\eta_0$  and  $\eta$  are respectively values of the parameter at the instant of beginning of resonant interaction and at the end of resonant interaction. The duration of resonant interaction is of the order of  $\beta_d \tau_d$  (Karpman and Shklyar 1977). Here  $\beta_d$  is a dimensionless parameter which governs the trapping and non-trapping of the resonant charged particles in the presence of the waves and external electric and magnetic fields and is defined in terms of  $\alpha_d$  and  $\tau_d$  as

$$\beta_d = \frac{1}{2\alpha_d \tau_d^2}. \quad (7)$$

It is obvious that  $\beta_d$  is controlled by parallel electric field and distortions in the geomagnetic field through  $\alpha_d$  and  $\tau_d$ .  $\beta_d$  plays an important role in estimating the effective range of resonant interaction of charged particles with whistler waves (Karpman and Shklyar 1977).

The solution of (6) gives an expression for effective change in pitch angles of the resonating electrons as induced by the geomagnetic field distortions in the presence of parallel electric field which is written as

$$(\Delta \theta_d)_{\text{eff}} = \left( \frac{\omega_{ce}}{\omega_{ce} - \omega} \right)^{3/2} \left( \frac{\pi h_e |\beta_d|}{\theta_L} \right)^{1/2} + \frac{2eE_0}{m} \frac{\omega_{ce}}{k_e V_{Re}^2} \beta_d \tau_d. \quad (8)$$

Substituting for  $\tau_d$  and  $\beta_d$  into (8), an estimate of effective pitch angle change of resonating charged particles as induced by distortions in axisymmetric Mead geomagnetic field and parallel electric field can be made.

### 3. Influx of precipitating electrons

Electrons with small pitch angles in the equatorial region of the magnetosphere are known to precipitate in the high latitude and low altitude ionosphere. First, we wish to find out

the stationary state velocity distribution function of the trapped electrons in the vicinity of loss cone. It is well known that in stationary state the velocity distribution function suffers significant diffusion caused by a variety of geophysical processes (Schulz and Lanzerotti 1974). The problem of resonant-charged particle diffusion is, in general, rather complex. However, Lyons (1974) has shown that for  $\omega/\omega_c < 0.828$  the pitch angle diffusion dominates over energy and mixed diffusion processes. Therefore, in the present analysis of wave-particle interaction for ducted whistlers, we have accounted for pitch angle diffusion only. The electrons scattered within the loss cone are lost in a time comparable with the escape time of the electrons,  $\tau_E(\theta) (\approx \tau_B(\theta)/4)$ . This loss within the loss cone is estimated by a sink term  $f(V, \theta)/\tau_E(\theta)$  whereas no such sink term is considered operative outside the loss cone of electrons. The radial diffusion process is operative and injects fresh electrons in the interaction region. However, the contribution of radially diffused electrons in the vicinity of the loss cone has been assumed to form an insignificant source term. Therefore, the velocity distribution function  $f(V, \theta)$  on the time scale of the bounce period is considered to be in steady state, i.e.  $\partial f(V, \theta)/\partial t = 0$ . Thus, we write the steady-state diffusion equation averaged over bounce period as (Sentman and Goertz 1978).

$$\frac{1}{\tau_B(\theta) \sin 2\theta} \frac{\partial}{\partial \theta} \left[ D(\theta) \tau_B(\theta) \sin 2\theta \frac{\partial f(V, \theta)}{\partial \theta} \right] - \frac{4g(\theta)f(V, \theta)}{\tau_B(\theta)} = 0, \quad (9)$$

where  $D(\theta)$  is pitch angle diffusion coefficient averaged over a bounce period,  $\tau_B$ . Both  $D$  and  $\tau_B$  depend, in general, on pitch angle. However, for small changes in pitch angle they are taken as  $\theta$  independent. In (9) the second term on the left side is the sink term with  $\tau_E$  replaced by  $\tau_B/4$ . Analysis of experimentally measured electron energy spectrum yields the forms of electron velocity distribution function and for specific case have been given by many workers (Schield and Frank 1970; Lyons and Williams 1975, 1980; Maeda *et al* 1976). In the present study of stationary state of wave-particle interaction and diffusion processes, we use the electron velocity distribution function which is derived from the result of Schield and Frank (1970)

$$f_0(V) = \frac{n_0(2\nu - 3)}{4\pi V_0^3} (V_0/V)^{2\nu} \quad \text{for } \nu > 3/2. \quad (10)$$

Here  $n_0$  is the number density of energetic electrons with velocity higher than the thermal velocity,  $V_0$ . The index  $\nu$  is obtained by analysing the experimental differential energy spectrum of energetic electrons. These parameters are highly dependent on the solar activity conditions.

To solve (9), we further assume that the effective velocity distribution function is pitch angle dependent and is expressible as a product term.

$$f(V, \theta) = f_0(V)f(\theta). \quad (11)$$

Different solutions for  $f(\theta)$  inside and outside the loss cone have been obtained following earlier procedure (Prasad and Singh 1981) in the angular range of  $\theta < \theta_L$  or  $\theta > \theta_L$  respectively. The width of the loss cone determines the number of precipitating charged particles in the high latitude and low altitude dense atmosphere. The resonant interaction of energetic charged particles with propagating whistler wave changes pitch angle and velocity distribution function of particles. These changes depend on various plasma and field parameters. Considering the loss cone as empty, the influx of

precipitating electron is written as (Karpman and Shklyar 1977)

$$J_{\parallel}(z_p) = \frac{2\pi}{\theta_L^2} \int_{|V_{Re}|}^{\infty} f_0(V) V^3 dV \int_{\theta_L}^{\theta_L + (\Delta\theta_d)_{\text{eff}}} \theta f(\theta) d\theta, \quad (12)$$

where  $z_p$  is the height of precipitation point (Prasad and Singh 1981). Substituting for  $f(\theta)$  and performing the  $\theta$  integration (12) is rewritten as

$$J_{\parallel}(z_p) = \frac{2\pi}{\theta_L^2} \int_{|V_{Re}|}^{\infty} V^3 f_0(v) \left[ \frac{1}{2} \left\{ \theta_L + (\Delta\theta_d)_{\text{eff}} \right\}^2 \ln \left\{ 1 + \frac{(\Delta\theta_d)_{\text{eff}}}{\theta_L} \right\} - \frac{1}{4} \left\{ (\Delta\theta_d)_{\text{eff}}^2 + 2\theta_L (\Delta\theta_d)_{\text{eff}} \right\} + \delta\theta \left\{ (\Delta\theta_d)_{\text{eff}} + \frac{(\Delta\theta_d)_{\text{eff}}^2}{2\theta_L} \right\} \right] dV \quad (13)$$

where  $\delta\theta = (D\tau_E)^{1/2}$ . The influx of precipitating electrons as expressed by (13) depends on parallel electric field and geomagnetic field distortions through the effective change in the pitch angle of resonantly interacting charged particles,  $(\Delta\theta_d)_{\text{eff}}$ . The interacting particles are considered to be phase-trapped in the potential troughs which appear for  $|\beta_d| < 1$ . Substituting  $f_0(V)$  and  $(\Delta\theta_d)_{\text{eff}}$  from (10) and (8) into (13) we perform the numerical integration of the resulting equation over velocity. The integration limit is divided into smaller steps and for each step a 40 point gauss integration is performed. The upper limit of the velocity integration is gradually increased until required convergence of the integral is achieved. Effects of wave, parallel electric field and geomagnetic field distortions have been studied using this numerical integration process.

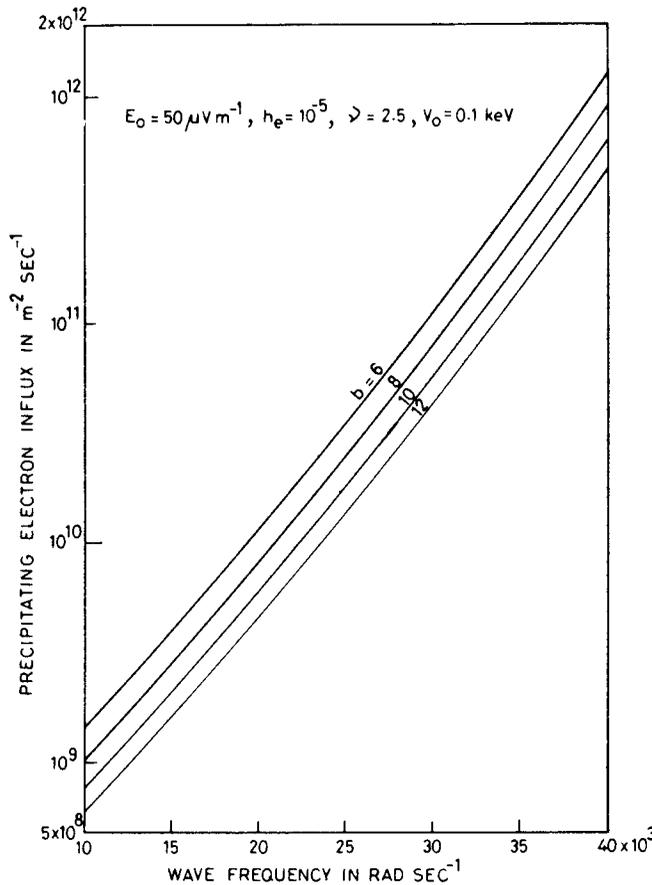
#### 4. Results and discussion

The quiet condition of the magnetosphere is governed by solar activity index. The field parameters in the magnetospheric region do not change violently and the precipitated influx may be small depicting spatially localized and temporally stable. However, on the other hand, the changes in field parameters under high solar activity periods may produce a corresponding change in wave field, plasma and solar wind parameters which may result in dynamic variations in position and fast temporal variations in precipitated influx of electrons. This situation is characteristically depicted by the measured influx of precipitated electrons (Gurnett and Frank 1973; Reasoner and Chappell 1973; Lyons *et al* 1979), optical emissions produced by precipitated electrons (Feldman and Doering 1975; Meng 1976; Parks *et al* 1977; Feldman 1978; Jones 1979), generation of bremsstrahlung flux (Prasad and Singh 1972; Luhmann 1977) and many other simultaneous geomagnetic phenomena. In the present study, we take  $\omega_{pe} = 565.5 \times 10^3 \text{ rad sec}^{-1}$  and  $T = 83.6 \times 10^3 \text{ }^\circ\text{K}$  for thermalized background plasma in the equatorial plane corresponding to  $L = 4$  where  $\omega_{ce} = 84 \times 10^3 \text{ rad sec}^{-1}$  and briefly outline some of the main features of precipitating electron influx as affected by the wave and field parameters.

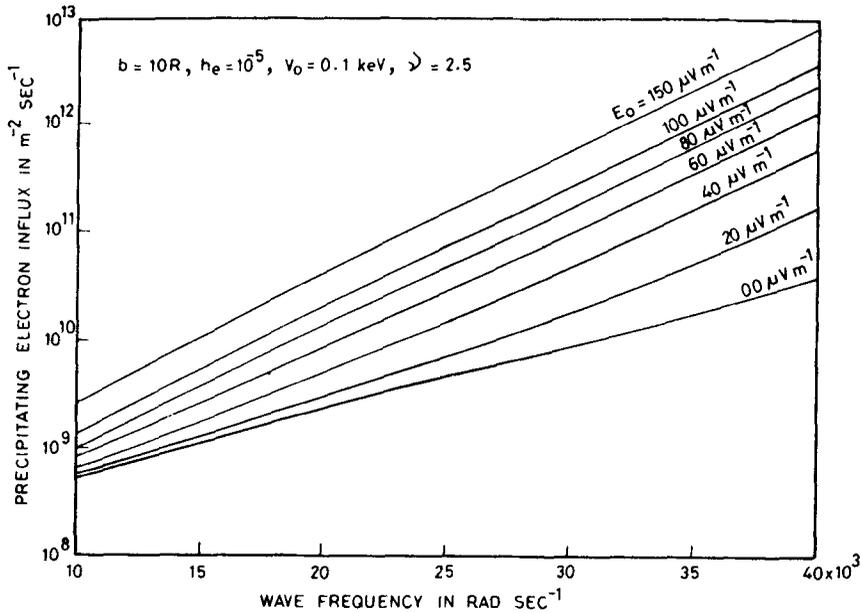
The enhanced solar wind impact on geomagnetic field results in a significant modification in the configuration of dipolar geomagnetic field as depicted by (4). The 'stand-off' distance  $b$  is a measure of distortion in the geomagnetic field (Choe and Beard 1974). Therefore, we have computed the variations in precipitating electron influx as a result of interacting ducted whistler wave at  $L = 4$  as governed by  $\nu = 2.5$ ,

$E_0 = 50 \mu\text{V m}^{-1}$  and  $\theta_L = 0.1$  rad for  $b = 6, 8, 10$  and  $12R$ . For  $b = 10R$ , we have taken  $\delta\theta = 0.001$  rad,  $V_0 = 0.1$  keV and  $n_0 = 10^7 \text{ m}^{-3}$ . Realistic energy spectrum of energetic charged particles of solar wind origin entering at different  $L$ -values during different phases of solar activity is not well documented. Therefore, in the absence of any concrete value, as a crude approximation we have accounted for 20% and 10% enhancements in  $V_0$  and  $n_0$  respectively for  $b = 6R$  and  $b = 8R$  whereas 10% reduction for  $b = 12R$ . We have also increased and reduced  $\delta\theta$  by the same amount as for  $V_0$  and  $n_0$ . It is assumed that due to distortion the shape of the field line is changed while its length remains the same. As a given whistler frequency requires a fixed resonance velocity, it is unlikely that  $\tau_E$  changes appreciably with geomagnetic field distortion. Therefore, change in pitch angle diffusion coefficient would produce a corresponding change in  $\delta\theta$ . With these parameters we have computed the precipitating electron influx and have shown its variation with whistler wave frequencies in figure 1. The changes in precipitating electron influx for  $b = 6, 8, 10$  and  $12R$  are appreciable.

Figure 2 shows the effect of presence of parallel electric fields for  $b = 10R$  and  $\nu = 2.5$  on precipitating electron influx. This shows that the parallel electric field results in



**Figure 1.** Variation of precipitating electron influx with whistler wave frequency for different 'stand-off' distances.



**Figure 2.** Variation of precipitating electron influx with whistler wave frequency for different values of parallel electric field.

enhanced precipitation in the lower dense atmosphere. The effect of parallel electric field becomes more pronounced as the frequency of ducted whistler wave increases. It is seen from the figure that the precipitating electron influx at wave frequency of  $10 \times 10^3 \text{ rad sec}^{-1}$  increases from  $5.4 \times 10^8 \text{ m}^{-2} \text{sec}^{-1}$  to  $2.4 \times 10^9 \text{ m}^{-2} \text{sec}^{-1}$  as the parallel electric field increases from  $0 \mu\text{V m}^{-1}$  to  $150 \mu\text{V m}^{-1}$  whereas this increase at  $40 \times 10^3 \text{ rad sec}^{-1}$  wave frequency is from  $3.7 \times 10^{10} \text{ m}^{-2} \text{sec}^{-1}$  to  $8 \times 10^{12} \text{ m}^{-2} \text{sec}^{-1}$ . The choice of different parallel electric fields is based on varying values of anomalous resistivity and field aligned currents. A set of two values of parallel electric field strengths chosen for this calculation may depict two solar activity conditions of enhanced solar wind impact.

The present analysis accounts only for modelled parameters which permit the analytical solution of the problem. For example, higher values of whistler wave amplitude gives rise to nonlinear effects and modulation of whistler amplitude (Gupta *et al* 1977; Prasad *et al* 1979). The presence of any modulation in the precipitated influx may be interpreted as indicator of higher whistler wave amplitude ratio apart from short term changes in plasma density contributing to modulational effect. The consequence of precipitated electron influx is beyond the scope of this paper. However, observed spatially and temporally varying features of auroral display and related optical radiations depict systematic control of various parameters discussed in this paper.

Thus, we conclude that the present formulation and computation predict the changes in precipitation of electrons as affected by ducted whistler waves, parallel electric fields and distortions in geomagnetic field of the earth.

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