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# Interpretation of gravity anomalies due to some two-dimensional structures: A Hilbert transform technique

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**Abstract.** An interpretation procedure is formulated using the Hilbert transform for analysing the gravity effect of (a) two-dimensional horizontal circular cylinder, (b) semi-infinite thin fault block, and (c) a geologic contact. In all three cases the abscissa of the point of intersection of the gravity anomaly curve or the horizontal derivative curve and its Hilbert transform yields directly the depth of the body. The proposed method is tested on theoretical models. Also, the method is applied to the field data of a geologic contact taken over the Himalayan foothills across the Brahmaputra valley.

**Keywords.** Hilbert transform; gravity; interpretation; two-dimensional structures.

## 1. Introduction

In recent years, the Hilbert transform techniques have acquired importance in the quantitative interpretation of gravity and magnetic anomalies (Nabighian 1972; Stanley and Green 1976; Stanley 1977; Mohan *et al* 1982; Sundararajan *et al* 1982). In the present paper an interpretation procedure is developed using Hilbert transforms for evaluating the body parameters of two dimensional horizontal circular cylinder, semi-infinite thin fault block and a geologic contact. An interesting feature observed is that the point of intersection of the curves of the gravity anomaly $g(x)$ of any body, and its Hilbert transform $H(x)$, gives the depth of the body. Further the amplitude of $g(x)$ and $H(x)$, is always maximum over the origin.

## 2. Hilbert transform

The Fourier transform of the gravity anomaly $g(x)$ of the body is given by

$$F(\omega) = \int_{-\infty}^{\infty} g(x) \exp(-\, i\omega x) \cdot \mathrm{d}x,$$

$$= \mathrm{Re}\, F(\omega) + \mathrm{Im}\, F(\omega), \tag{1}$$

where $\omega$ is the spatial frequency in radians per unit length and Re $F(\omega)$ and Im $F(\omega)$ are the real and imaginary components of the Fourier transform.

The Hilbert transform of $g(x)$ is defined as (Thomas 1969)

$$H(x) = \frac{1}{\pi}\int_{0}^{\infty} [\mathrm{Im}\, F(\omega) \cos \omega x - \mathrm{Re}\, F(\omega) \sin \omega x]\, \mathrm{d}\omega. \tag{2}$$

The Hilbert transform of the field data, which is always discrete is computed by making use of the discrete Fourier transform (DFT) and the discrete Hilbert transform (DHT).

The components of DFT are (Gold and Rader 1969)

$$\text{Re } F(n\omega_0) = \sum_{l=0}^{N-1} g(l\cdot\Delta x)\cos(n\omega_0 l\cdot\Delta x), \tag{3}$$

$$\text{Im } F(n\omega_0) = \sum_{l=0}^{N-1} g(l\cdot\Delta x)\sin(\omega_0 n\cdot l\Delta x). \tag{4}$$

Then the discrete Hilbert transform is defined as:

$$H(l\cdot\Delta x) = \frac{1}{\pi}\left[\sum_{n=0}^{\frac{N}{2}-1} \text{Im } F(n\omega_0)\cdot\cos(n\omega_0 l\cdot\Delta x) - \sum_{n=0}^{\frac{N}{2}-1} \text{Re } F(n\omega_0)\cdot\sin(n\omega_0 l\cdot\Delta x)\right] \tag{5}$$

where $\Delta x$ = sample interval; $N$ = total number of samples and

$$\omega_0 = 2\pi/(N\cdot\Delta x)$$

### 2.1 *Location of origin*

The abscissa where the amplitude curve

$$A(x) = \{[g(x)]^2 + [H(x)]^2\}^{1/2} \tag{6}$$

attains maximum gives the origin (Nabighian 1972).

## 3. Horizontal circular cylinder

The gravity effect due to a horizontal circular cylinder (figure 1a) is given by

$$g_1(x) = 2\pi^2 GR^2\sigma \frac{D}{D^2 + x^2} \tag{7}$$

where $G$ is the universal gravitational constant, $\sigma$ is the density contrast, $D$ is the depth of the centre of the cylinder and $R$ is the radius of the cylinder. The real and imaginary components of Fourier transform of $g_1(x)$ are given as:

$$\text{Re } F(\omega) = 2\pi^2 GR^2\sigma \exp(-\omega D), \tag{8}$$

$$\text{Im } F(\omega) = 0. \tag{9}$$

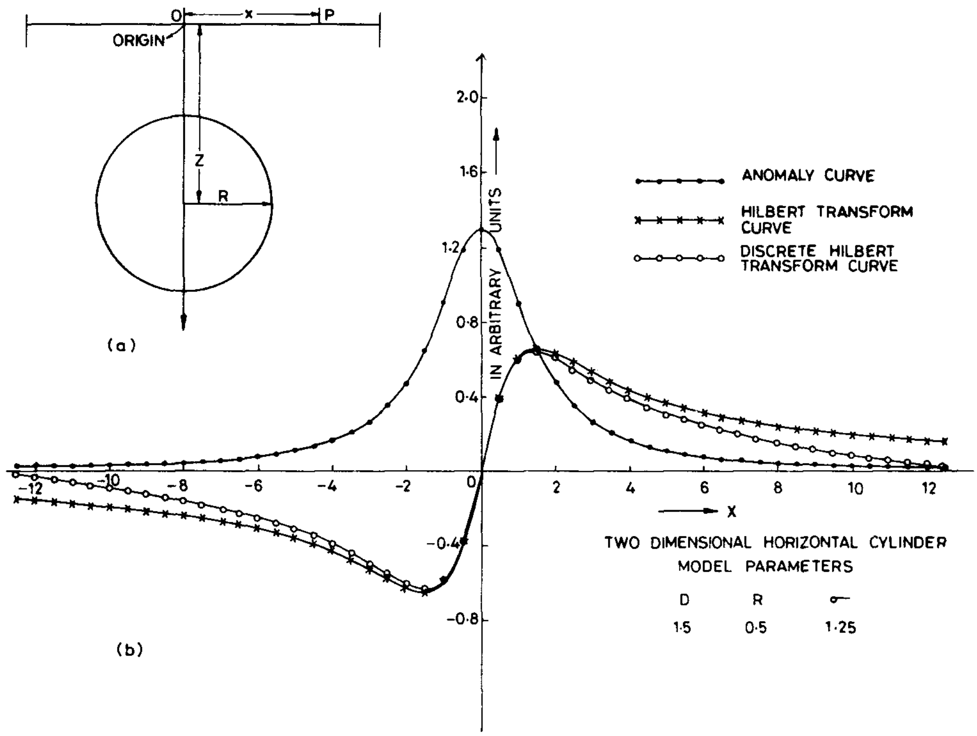


**Figure 1.** Computed gravity effect the Hilbert transform and the discrete Hilbert transform of the horizontal circular cylinder.

Using (8) and (9) in (2), it is obtained as

$$H(x) = 2\pi^2 G R^2 \, \sigma \int_0^{\infty} \exp(-\ \omega D) \sin \omega x \cdot d\omega.$$

On integrating, we get

$$H_1(x) = 2\pi G R^2 \sigma \, \frac{x}{D^2 + x^2} \tag{10}$$

The gravity effect due to a horizontal circular cylinder $g_1(x)$ and its Hilbert transform $H_1(x)$ are shown in figures 2 and 3. The two curves intersect at a point $X = X_1$, hence

$$g_1(x_1) = H_1(x_1) \quad \text{yields,} \qquad D = x_1. \tag{11}$$

Therefore, the depth to the centre of the cylinder is the abscissa of the point of intersection of $g_1(x)$ and $H_1(x)$. It is conspicuous that the magnitude of $D$ remains unaltered even if we consider the Hilbert transform with the minus sign.

Also, squaring and adding $g_1(x)$ and $H_1(x)$ at $x = 0$ we get,

$$R^2\sigma = \frac{D}{2\pi G} \{[g_1\ (0)]^2 + [H_1\ (0)\ ]^2\}^{1/2} \tag{12}$$

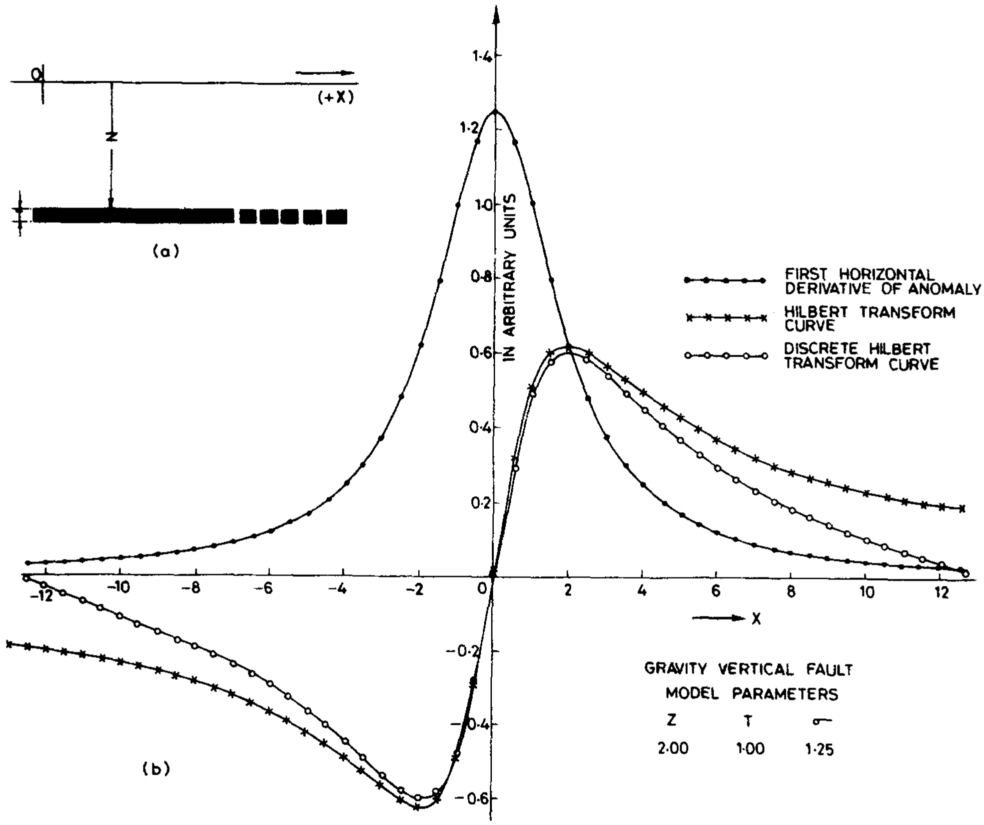


**Figure 2.** Computed first horizontal derivative of the Hilbert transform and the discrete Hilbert transform of the semi-infinite thin fault block.

**Table 1.** Theoretical models of horizontal circular cylinder, semi infinite thin fault block and geological contact.

| Model | Parameters | Assumed | Evaluated |
|---|---|---|---|
| Cylinder | D | 1.5 | 1.5 |
| | R | 0.5 | 0.5 |
| Semi infinite Thin fault block | Z | 2.00 | 2.05 |
| | T | 1.00 | 1.03 |
| Geologic contact | h | 0.5 | 0.5 |
| | $\theta$ | 110°.00 | 110° 30′ |
| | $\sigma$ | 1.00 | 0.97 |

*Note*: Except $\theta$ other parameters in arbitrary units.

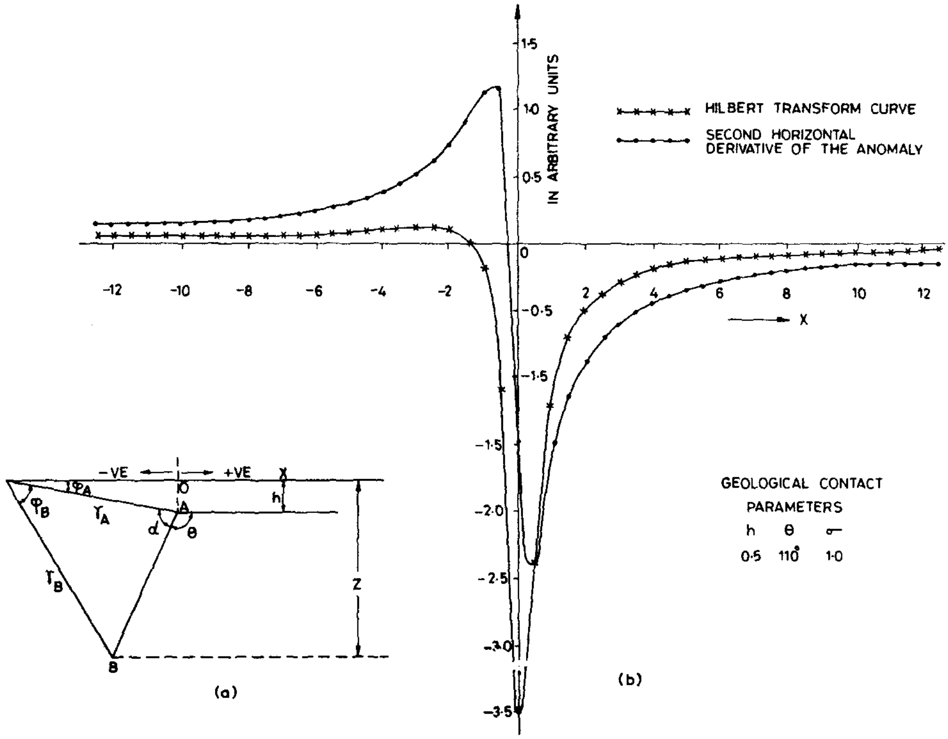


**Figure 3.** Computed second horizontal derivative of the geologic contact and its Hilbert transform.

From (12), either $R$ or $\sigma$ is calculated by assuming the other.

Figure 1(b) shows an example (table 1) of the gravity effect $g_1(x)$ of the cylinder and its Hilbert transform $H_1(x)$. It is observed that $g_1(x)$ and $H_1(x)$ intersect at a point, which yields directly the depth to the centre of the cylinder.

In practice, we get the field data in discrete form, and hence the discrete Hilbert transform of $g_1(x)$ is computed by using (5) as shown in figure 1(b). It is observed that the discrete Hilbert transform curve intersects the anomaly curve nearly at the same point as that of the theoretical Hilbert transform and the anomaly curve. Based on the analysis given earlier, the parameters are calculated as shown in table 1.

## 4. A semi-infinite thin fault block

The gravity effect due to semi-infinite thin fault block (figure 2a) is expressed as

$$g_2(x) = 2G\sigma T(\pi/2 + \tan^{-1}x/Z), \tag{13}$$

where $G$ and $\sigma$ stand as in $g_1(x)$, $Z$ the depth to the thin plate and $T$ the thickness of the plate.

For the purpose of interpretation it is however convenient to use the first horizontal derivatives.

Differentiating (13) with respect to $x$ we get

$$g_2'(x) = 2G\sigma T \frac{Z}{Z^2 + x^2} \tag{14}$$

The real and imaginary components of the Fourier transform of $g'_2(x)$ are given as:

$$\operatorname{Re} F(\omega) = 2G\sigma T\cdot \pi \exp(-\omega Z), \tag{15}$$

$$\operatorname{Im} F(\omega) = 0. \tag{16}$$

Substituting (15) and (16) in (2) and then integrating the Hilbert transform is obtained as:

$$H_2(x) = 2G\sigma T \frac{x}{Z^2 + x^2} \tag{17}$$

The first horizontal derivative of the gravity effect due to a semi-infinite thin fault block $g_2^1(x)$ and its Hilbert transform $H_2(x)$ are shown in figures 4 and 5. It is observed that these two curves intersect at $x = x_1$, which is the depth to the fault block. Thus

$$g'_2(x_1) = H_2(x_1) \quad \text{yields} \quad Z = x_1 \tag{18}$$

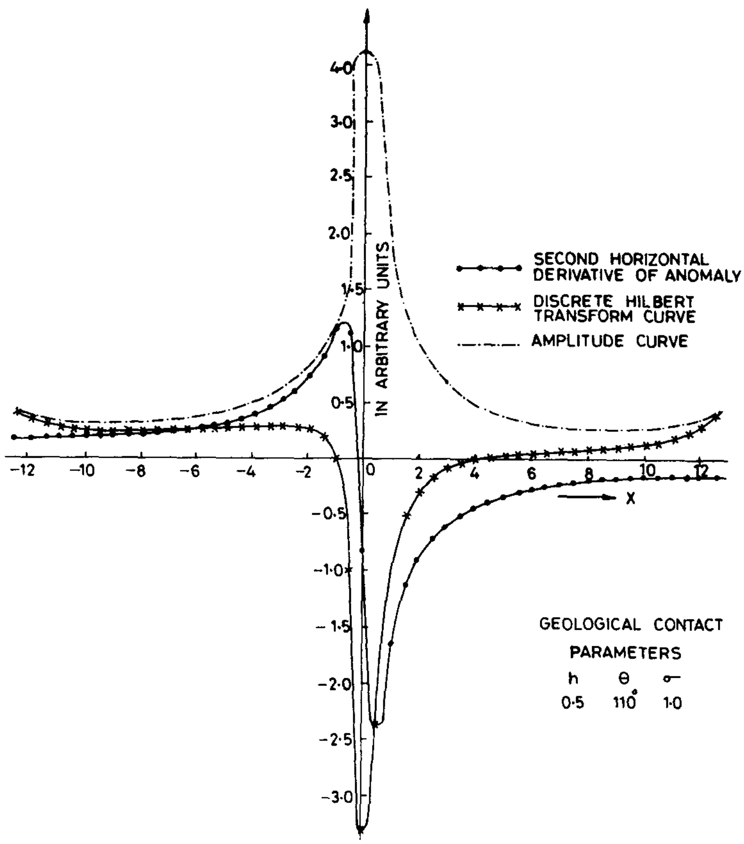


**Figure 4.** Computed second horizontal derivative of the geoloigc contact, its discrete Hilbert transform and the amplitude of the second horizontal derivative and its DHT.

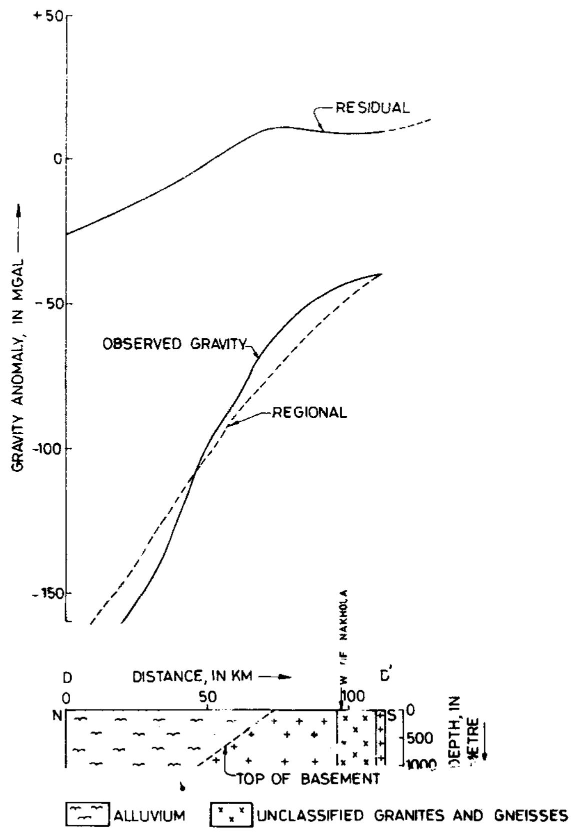


**Figure 5.** The gravity anomaly due to a geologic contact taken over the Himalayan foothills across the Brahmaputra valley.

Also, squaring and adding $g_2'(x)$ and $H_2(x)$ at $x = 0$, we get

$$T\sigma = Z\{[g_2'(0)]^2 + [H_2(0)]^2\}^{1/2}/2\pi G. \tag{19}$$

From (19) either $\sigma$ or $T$ is calculated by assuming the other.

The method is studied by considering a theoretical model (table 1). Using (14) and (17) the horizontal derivative of the gravity effect $g_2'(x)$ of the fault block and its Hilbert transform $H_2(x)$ is computed and shown in figure 2b. Also, $g_2'(x)$ is digitized for generating the discrete data. Using (5), the discrete Hilbert transform of the digitized data is computed and shown in figure 2b. It is observed that the Hilbert transform and the discrete Hilbert transform curves intersect the anomaly curve at the same point which directly gives the depth to the body. The thickness of the body is calculated using (19). The results are presented in table 1.

## 5. Geologic contact

The geometry of the geologic contact is shown in figure 3a. The gravity effect over the geologic contact is expressed as (Jung 1961)

$$g_3(x) = 2G\sigma\{(x + h/\tan\theta)\sin\theta[\sin\theta\ln(r_B/r_A) - \\ - \cos\theta(\phi_B - \phi_A)] + (Z\phi_B - h\phi_A)\}, \tag{20}$$

where the various parameters are explained in figure 3a.

In this case it is convenient to use the second derivatives in the analysis. The second derivative of $g_3(x)$ is obtained as are obtained by differentiating (20) with respect to $x$ twice,

$$g_3''(x) = 2G\sigma\sin\theta\left(\frac{h\cos\theta - x\sin\theta}{r_A^2} - \frac{(2Z - h)\cos\theta - x\sin\theta}{r_B^2}\right) \tag{21}$$

For the case when $Z \gg h$ and $r_B \gg r_A$ the above equation reduces to,

$$g_3''(x) = \left(\frac{2G\sigma\sin\theta(h\cos\theta - x\sin\theta)}{r_A^2}\right) \tag{22}$$

The Hilbert transform of $g_3''(x)$ is obtained as:

$$h_3(x) = 2G\sigma\sin\theta\left(\frac{x\cos\theta - h\sin\theta}{x^2 + h^2}\right) \tag{23}$$

$g_3''(x)$ and $H_3(x)$ intersect at $x_1$ given by

$$g_3''(x) = H_3(x) \quad \text{yields} \quad h = x_1. \tag{24}$$

The dip of the geologic contact is obtained as follows:

$$\theta = \tan^{-1}\left(\frac{xg_3''(x) - hH_3(x)}{hg_3''(x) - xH_3(x)}\right) \tag{25}$$

Putting $x = 0$ in (22) and (23) and solving we obtain an expression for $\sigma$ given by

$$\sigma = \frac{h}{2G\sin\theta}\{[g_3''(0)]^2 + [H_3(0)]^2\}^{1/2} \tag{26}$$

The theory is demonstrated by considering an example (table 1). Using (22) and (23) $g_3''(x)$, $H_3(x)$ are computed and shown in figure 3b.

Using (5) and (6) the discrete Hilbert transform and amplitude are computed and shown in figure 4 and the parameters are evaluated and given in table 1.

It may be recalled that the curve of the discrete variable could never totally coincide with that of the continuous variable of the same function. This inherent theoretical concept is responsible for the departure of DHT curves from the theoretical curves in the above examples.

## 6. Field example

The Hilbert transform technique developed here is demonstrated in field data over a geologic contact (figure 5). This profile runs from the northern edge of the Shillong Plateau to the Himalayan foothills across the Brahmaputra valley (Verma and Mukhopadhyay 1976). The length of the profile considered is 30 km and it is digitized at an interval of 1.25 km. The second horizontal derivative of the anomaly, the discrete Hilbert transform and the amplitude are shown in figure 6. The abscissa of the point of intersection of the second horizontal derivative of the gravity anomaly and its discrete Hilbert transform gives the depth to the upper surface of the contact. The dip and the density contrast are calculated using (25) and (26). The results are given in table 2.

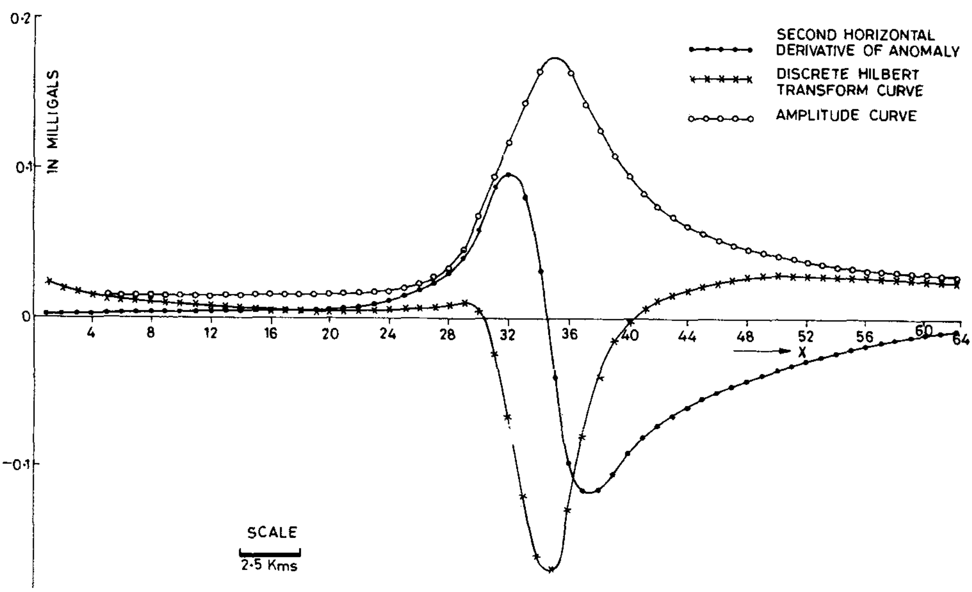


**Figure 6.** The second horizontal derivative of the anomaly, the discrete Hilbert transform and their amplitude.

**Table 2.** Evaluated parameters from the gravity data across the Brahmaputra valley.

| Parameters | $Z$ (km) | $\theta$ (deg.) | $\sigma$ (g/cc) |
|---|---|---|---|
| Evaluated values | 1.6875 | 102.17 | 0.008 |

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