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Stability of a stationary Rossby wave embedded in barotropic monsoon zonal flow

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Abstract. Barotropic stability of a stationary Rossby wave of wavelength 30° longitude superposed on the uniform monsoon zonal flow has been examined. The wave is unstable to perturbations and the growth rate depends on the meridional scale. These perturbations grow by drawing on the kinetic energy of the stationary Rossby wave.

Keywords. Barotropic instability; baroclinic instability; beta-plane; zonal flow; meridional wind; Rossby wave.

1. Introduction

The barotropic instability of a zonal flow with superposed Rossby wave was investigated by Lorenz (1972). Dash and Keshavamurty (1982) have shown that the realistic winds in the monsoon atmosphere can sustain a finite amplitude baroclinic stationary Rossby wave of wavelength about 30° longitude and the amplitude of the Rossby wave is mainly confined to the lower troposphere. Such a stationary wave is observed over the Bay of Bengal and it may be induced by the topography of peninsular India (Gadgil 1977). It was found that such a stationary Rossby wave is unstable to perturbations (Dash and Keshavamurty 1982).

In the present study, we intend to perform the stability analysis by superposing a stationary Rossby wave of wavelength 30° longitude on the monsoon zonal flow having no vertical wind shear. The zonal flow corresponding to 18° N latitude is considered as the basic flow. The rate of conversion of kinetic energy is also calculated.

2. Barotropic zonal flow sustaining the stationary Rossby wave

First we calculate the barotropic zonal wind which can sustain a stationary Rossby wave of wavelength 30° longitude along the zonal direction. For that we consider the Rossby wave speed formula

$$c = U - \frac{\beta}{k_0^2}, \tag{1}$$

where k_0 is the zonal wavenumber of the Rossby wave.

Considering beta-plane centred at 18° latitude and substituting $k_0 = 1.84 \times 10^{-6}\ \mathrm{m}^{-1}$, which corresponds to the stationary wave of wavelength 30° longitude, in (1) we calculate,

$$U_s \equiv \frac{\beta}{k_0^2} = 6.5 \text{ m sec}^{-1}.$$

Thus a zonal wind $U_s = 6.5$ m sec^{-1}can sustain a stationary Rossby wave of wavelength 30° longitude.

3. The stability analysis

The vorticity equation is linearised by expressing the field variables as

$$X = [\bar{X}] + \bar{X}^* + X' \tag{2}$$

where the transient eddy $X' = X - \bar{X}$ and the stationary eddy $\bar{X}^* = \bar{X} - [\bar{X}]$. Here the bar and the square brackets represent time average and zonal average respectively. Thus the perturbation vorticity equation can be written as

$$\left(\frac{\partial}{\partial t} + U_s \frac{\partial}{\partial x} + \bar{v}^* \frac{\partial}{\partial y}\right) \zeta' + u' \frac{\partial \bar{\zeta}^*}{\partial x} + \beta v' = 0. \tag{3}$$

The basic flow is taken as

$$\Phi = -f_0 U_s Y + f_0 A \sin k_0 x, \tag{4}$$

where the meridional wind component is given by,

$$\bar{v}^* = k_0 A \cos k_0 x = \bar{v} \cos k_0 x. \tag{5}$$

Here $\bar{v}$ is substituted for $k_0 A$.

We propose to do normal mode analysis and following Lorenz (1972) assume solutions of the type

$$\phi' = \sum_{n=-\infty}^{\infty} \phi_n \exp\{i(nk_0 x + ly + \lambda t)\} \tag{6}$$

where k_0 and l are real and λ may be complex. Here l is the meridional wavenumber of the perturbation. Equation (6) possesses solutions, which amplify with time if the imaginary part of λ were negative. For side boundary conditions we impose cyclic continuity. The zonal wind U_s is taken to be independent of y. Substituting (4), (5) and (6) in (3) and collecting the coefficients of $\exp\{i(nk_0 x + ly + \lambda t)\}$ we get the following infinite system of linear, homogeneous algebraic equations while n is varied from $-\infty$ to $+\infty$.

$$\begin{aligned} &-1/2\, l\bar{v}(b_{n-1} - k_0^2)\phi_{n-1} + a_n(\beta - b_n U_s)\phi_n \\ &-1/2\, l\bar{v}(b_{n+1} - k_0^2)\phi_{n+1} - \lambda b_n \phi_n = 0, \end{aligned} \tag{7}$$

where

$$a_n \equiv nk_0 \quad \text{and} \quad b_n \equiv (n^2k_0^2 + l^2).$$

Generally, one is able to obtain a good approximation by truncating the series (6) at a finite number $n = N$. When N is large enough such that λ_N has converged, a good approximation to the 'true' solution is obtained. In our analysis, we have taken $N = 10$. The 21 homogeneous algebraic equations can be written in matrix form as

$$(\mathbf{P}\cdot\mathbf{Q}^{-1} - \lambda\mathbf{I})\,(\mathbf{Q}\cdot\boldsymbol{\phi}) = 0, \tag{8}$$

where **P** and **Q** are real square matrices and **I** is unit matrix. The eigenvalues λ and the eigenfunctions $(\mathbf{Q}\cdot\boldsymbol{\phi})$ are in general complex. The frequency of a disturbance and its doubling time are obtained from the real and imaginary parts of λ respectively. Similarly the amplitudes of different Fourier components ϕ_n are obtained from the eigenfunctions.

In another study (Dash and Keshavamurty 1982), it is found that for Rossby wave amplitude $\bar{v} = 10\,\mathrm{m\,sec^{-1}}$ at the lower troposphere the growth rate of disturbances reasonably agree with those of monsoon disturbances. Hence, we have conducted the present stability analysis for $\bar{v} = 10\,\mathrm{m\,sec^{-1}}$ only. The meridional wavenumber l is varied from $k_0/12$ to k_0, satisfying the cyclic boundary conditions. This is done by putting $l = Jk_0/12$ and varying J from 1 to 12 by steps of 1.

The stability analysis yields that the stationary Rossby wave is unstable to perturbations. The growth rate depends on the meridional scale of the disturbance and the fastest growing mode has a doubling time of about 1.5 days (figure 1). Also, the growth rate is maximum for $l \leq k_0/2$ which agrees with the results of barotropic stability analysis of Lorenz (1972). The growing disturbances are found to be stationary. The Fourier coefficients of geopotential perturbation corresponding to the fastest growing mode are shown in figure 2. It is seen that the amplitudes are mainly confined to the first few low order harmonics. This justifies the truncation of the series (6) for $N = 10$.

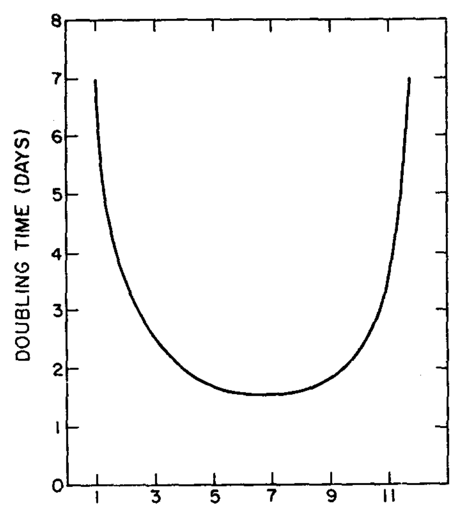

Figure 1. Meridional wavenumber ($l = Jk_0/12$) dependence of the growing modes.

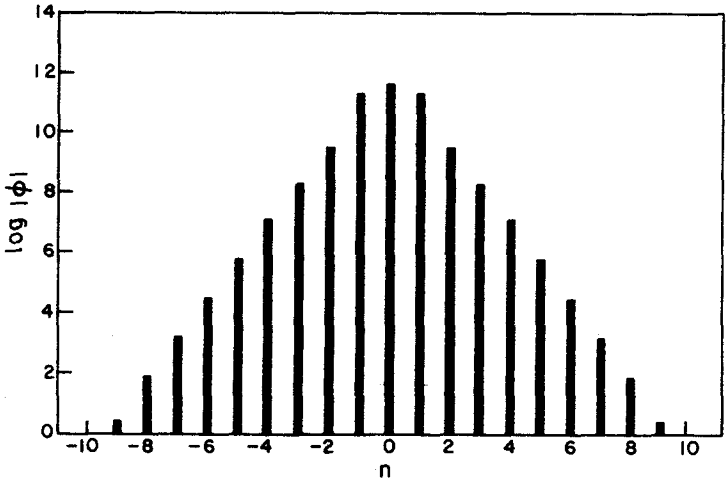

Figure 2. Fourier components of geopotential perturbation of the fastest growing mode in figure 1.

4. Energy conversion

As there is no vertical wind shear in the zonal flow and again the latter is independent of Y, the only source of energy for a perturbation to grow is the kinetic energy of the basic Rossby wave. The rate of conversion from kinetic energy of the basic wave to perturbation kinetic energy, $C(K_W, K')$ is calculated from

$$C(K_W, K') = -\frac{1}{g}\iiint u'v'\frac{\partial \bar{v}^*}{\partial x}\,\mathrm{d}x\mathrm{d}y\mathrm{d}p, \tag{9}$$

where the integrations are over one wavelength in both x and y directions and from $p = 0$ to $p = p_0$. In terms of eigenfunctions, (9) can be expressed as

$$C(K_W, K') = i\,\frac{2\pi^2 k_0 p_0}{g f_0^2}\exp(-2\lambda_i t)\sum_{n=-\infty}^{\infty}[\bar{v}\{(n+1)\times$$

$$(\phi_n\tilde{\phi}_{n+1} - \tilde{\phi}_n\phi_{n-1}) - (n - \;)\quad(\phi_n\tilde{\phi}_{n-1} - \tilde{\phi}_n\phi_{n-1})\}] \tag{10}$$

where ϕ is complex, $\tilde{\phi}$ is its complex conjugate and λ_i is the imaginary part of λ. In practice, the summation is carried out from $n = -N$ to N. Calculations show that the perturbation grows by drawing on the kinetic energy of the stationary Rossby wave.

5. Conclusions

In this paper we have carried out the barotropic stability analysis of a stationary Rossby wave superposed on the monsoon zonal flow having no vertical wind shear. It is found that the stationary Rossby wave of wavelength 30° longitude is unstable to perturbations. The growth rate depends on the meridional scale. The fastest growing mode has a doubling time of 1.5 days for Rossby wave amplitude $\bar{v} = 10\,\mathrm{m\,sec^{-1}}$. These perturbations grow by drawing on kinetic energy of the stationary Rossby wave.

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