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A second derivative method of resistivity sounding

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Abstract. The second vertical derivatives of gravity and magnetic potential are widely used in geophysical prospecting because of their better resolution. On the same basis an attempt has been made to obtain the expressions for the second vertical derivative of the electrical potential and to compute its nature for comparison. Derivative responses over a two-layered earth and also over an anticlinal structure have been computed and it is shown that the second vertical derivative sounding could be employed for greater accuracy in finding out the thickness of such beds or the inclination of the sides of the anticline and its depth when compared with normal resistivity sounding.

Keywords. Vertical derivative ; accuracy ; resolution ; layered earth.

1. Introduction

The use of higher derivatives of potential and in particular in the second derivatives is widely prevalent in geophysical prospecting in gravity and magnetic methods (Elkins 1951 ; Nettleton 1976). They are known to be capable of high resolution permitting not only a fine discrimination between different subsurface bodies, but also in the estimation of body parameters. Both horizontal and vertical derivatives are equally well used (Peters 1949 ; Roy 1969 ; Rao *et al* 1971, 1972) for various purposes. However, derivatives are not so popular in electrical resistivity methods, even though simple and direct means of measurement of derivatives exist in electrical methods in contrast to gravity or magnetic methods where they are often computed from field data. In the present paper an attempt has been made to compute and analyse the resolution of second vertical derivative sounding for the electrical methods where artificial source fields are employed in horizontal and inclined beds.

Only a few efforts (Sapuzhak 1967 ; Roy 1969) appear to have been made in this direction earlier. Certain electrode configurations to measure the derivatives were proposed by Rabinovich and Bukhimastor (1962), Rabinovich (1963) and Mosetti (1963), etc., which have remained of academic interest. However, Sapuzhak (1967) discussed some electrode configurations for measuring the higher derivatives which were employed in practical experiments. The use of horizontal derivatives was examined by him in some detail, mainly for profiling applications (Sapuzhak *et al* 1969). Murali *et al* (1980a, b) have computed the second horizontal derivatives $\partial^2 U/\partial X^2$ both in profiling and sounding and demonstrated their usefulness in both cases when dealing with plane boundaries between media. The second vertical derivative was employed (implicitly) by Rao *et al* (1981) in describing the vertical resolution characteristics of different electrode configurations in the form of $d\rho a/dh$ (the differential of apparent resistivity with respect to depth). These efforts notwithstanding, a direct approach to the measurement or use of second derivatives of potentials in the vertical direction is necessary particularly in the case of sounding since horizontal boundaries and horizontal discontinuities are to be expected in soundings.

The present paper deals with a simple method of obtaining the second vertical derivative and utilizes the results for computing the vertical electrical sounding curves over a layered earth as well as over an anticlinal structure.

2. Theoretical basis and computational approach

The vertical derivatives of electrical potential can be computed in different ways; one of them is that the potential, at a given point, may be differentiated twice with respect to the vertical. This procedure is sometimes difficult and not directly realised in practice. Hence, we resort here to the method of finite differences in two perpendicular horizontal directions (figure 1) employing a single point source located on the earth's surface and five-potential measuring electrodes. Then, if the earth were homogeneous with a resistivity ρ, U^M the potential at point M due to source A, etc., and if I is the amount of current introduced into the earth then

$$\triangle U_1 = U^M - U^o \quad \text{and} \quad \triangle U_3 = U^{M'} - U^o,$$

$$\triangle U_2 = U^o - U^N \quad \text{and} \quad \triangle U_4 = U^o - U^{N'}.$$

Then $$\frac{\partial U}{\partial X} = \frac{\triangle U_1}{MO} \quad \text{and} \quad \frac{\partial^2 U}{\partial X^2} = \frac{\triangle U_1 - \triangle U_2}{l^2},$$

where l is the distance MO. Similarly $\dfrac{\partial^2 U}{\partial Y^2} = \dfrac{\triangle U_3 - \triangle U_4}{l^2}$

and from the Laplacian

$$\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2} = 0$$

we have

$$\frac{\partial^2 U}{\partial Z^2} = -\left\{\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right\}, \tag{1}$$

$$= \frac{-1}{l^2}\{\triangle U_1 - \triangle U_2 + \triangle U_3 - \triangle U_4\},$$

$$= \frac{-1}{l^2}\{U^M - 4U^o + U^N + 2U^{M'}\}, \tag{2}$$

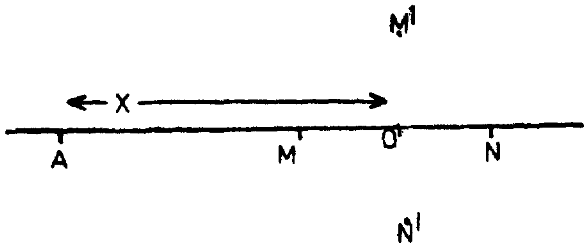

Figure 1. Placement of source and measuring electrodes for obtaining the second vertical derivative ($\partial^2 U/\partial Z^2$).

or
$$\frac{\partial^2 U}{\partial Z^2} = \frac{-I\rho}{2\pi l^2}\left\{\frac{1}{X-l} - \frac{4}{X} + \frac{1}{X+l} + \frac{2}{(X^2+l^2)^{1/2}}\right\}, \tag{3}$$

and the resistivity can be given by

$$\rho_a = \frac{2\pi l^2}{I}\left\{\frac{1}{\frac{4}{X} - \frac{1}{X-l} - \frac{1}{X+l} - \frac{2}{(X^2+l^2)^{1/2}}}\right\} \tag{4}$$

A practically realisable system for vertical second derivative measurement follows the same principle and is obtained as the sum of horizontal derivatives in two mutually perpendicular directions. The horizontal derivatives themselves are measured by using finite differences of potential over small elements of the earth. *MON* and *M'ON'* (see figure 1) are two sets of measuring electrodes placed on the earth in the perpendicular directions. A differential amplifier system can be used to measure directly $\partial^2 U/\partial Z^2$ to be substituted in equation (1).

3. Response over a layered earth

It is desirable to examine the response of the vertical derivative measuring system (sounding) over a layered earth. To start with we may consider as an example the case of a simple two-layered earth.

3.1. *Horizontally-layered earth*

We know that over a two-layered earth consisting of layers with resistivities ρ_1 and ρ_2 and thickness of the first layer as h the potential due to a point source is given by (Bhattacharyya and Patra 1968)

$$U^o = \frac{I\rho_1}{2\pi}\left\{\frac{1}{x} + \sum_{n=1}^{\infty} \frac{K_{12}^n}{[X^2+(2nh)^2]^{1/2}}\right\} \tag{5}$$

where $K_{12} = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$ $\quad x$ = distance *AO*.

U^{M}, U^{N}, $U^{M'}$ and $U^{N'}$ can be given in a similar fashion using appropriate values for distances between current and individual potential electrodes (see figure 1). Thus, after substituting for each we have

$$\frac{\partial^2 U}{\partial Z^2} = \frac{-I\rho_1}{2\pi l^2}\left[\frac{1}{x-l} - \frac{4}{x} + \frac{1}{x+l} + \frac{2}{\sqrt{x^2+l^2}}\right.$$

$$+ 2\sum_{n=1}^{\infty}\frac{K_{12}^n}{\{(x-l)^2+4n^2h^2\}^{1/2}} + 2\sum_{n=1}^{\infty}\frac{K_{12}^n}{\{x+l)^2+4n^2h^2\}^{1/2}}$$

$$\left.+ 4\sum_{n=1}^{\infty}\frac{K_{12}^n}{\{x^2+l^2+4n^2h^2\}^{1/2}} - 8\sum_{n=1}^{\infty}\frac{K_{12}^n}{\{x^2+4n^2h^2\}^{1/2}}\right]. \tag{6}$$

For computational purposes over two-layered horizontally stratified earth the second derivative can be written (after simplification and with the following assumptions) as

$$AO = x\ ;\quad OM = OM' = ON' = ON = l\ ;\ \text{and}\ x \gg l.$$

To facilitate numerical computation, we have assumed that $x = 10l$. In fact x can assume any value, and l can be kept at a negligible fraction of x. Under cfield onditions therefore a sounding can be very easily carried out with five stationary potential measuring electrodes spaced l apart and a moving single source electrod: at a distance x which gradually increases from about $10l$ to ∞.

Reverting to the case of the two-layered earth

$$\frac{\partial^2 U}{\partial Z^2} = \frac{-I\rho_1}{\pi x^2}\left[P + 100\sum_{n=1}^{\infty}\left\{\frac{K_{12}^n}{(1\cdot 01+4n^2h^2/x^2)^{1/2}}\right.\right.$$

$$- \frac{4K_{12}^n}{(1+4n^2h^2/x^2)^{1/2}} + \frac{K_{12}^n}{(0\cdot 81+4n^2h^2/x^2)^{1/2}}$$

$$\left.\left.+ \frac{2K_{12}^n}{(1\cdot 21+4n^2h^2/x^2)^{1/2}}\right\}\right]. \tag{7}$$

where $P \approx 0\cdot 5126$ or the apparent resistivity is given by

$$\rho_a = \rho_1\left[1 + Q\sum_{n=1}^{\infty}\left\{\frac{K_{12}^n}{(1\cdot 01+4n^2h^2/x^2)^{1/2}} - \frac{4K_{12}^n}{(1+4n^2h^2/x^2)^{1/2}}\right.\right.$$

$$\left.\left.+ \frac{K_{12}^n}{(0\cdot 81+4n^2h^2/x^2)^{1/2}} + \frac{2K_{12}^n}{(1\cdot 21+4n^2h^2/x^2)^{1/2}}\right\}\right], \tag{8}$$

where $Q \approx 195\cdot 01$.

We have computed the response of such a system measuring the vertical derivative over a two-layered horizontal earth having a variable resistivity contrast, in terms of the change in ρ_a with separation AO. The curves shown in figure 2

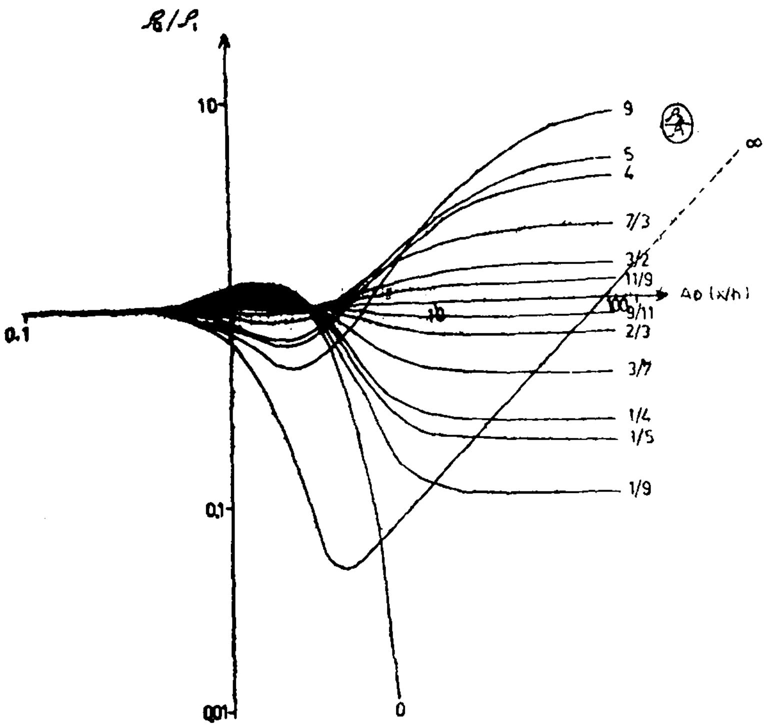

Figure 2. Apparent resistivity master curves obtained by sounding ($\partial^2U/\partial Z^2$) over a two-layered earth.

indicate the change in apparent resistivity ratio with separation (in terms of the layer thickness).

3.2. *Response over an anticlinal structure*

The logical sequence to the above treatment would be to consider a case where the boundary between the two media is inclined. Such a case for the horizontal derivative had already been discussed earlier (Murali *et al* 1980b). However, there are certain computational restrictions on the angle of inclination in the method. Hence, a variation of the inclined layer case, *i.e.*, that of an anticlinal structure is considered below.

The response of a vertical derivative measuring system over a local inhomogeneity in the form of an anticlinal rise in the basement buried under an overburden was also estimated using the same technique as above.

The potential at any point M due to a source A (which is situated at the epicentre of the anticline) is given by (Kraev 1965)

$$U^M = \frac{I\rho_1}{2\pi}\left[\frac{1}{x} + \frac{\beta}{\sqrt{h^2 + x^2}}\right], \tag{9}$$

where

ρ_1 = resistivity of the medium surrounding the anticline,

$\rho_2 = \infty$, *i.e.*, the resistivity of the anticlinal material,

x = distance between points A and M,

h = depth of the epicentre of the anticline from the surface, and

$\beta = \dfrac{1-\cos\alpha}{\cos\alpha}$, where 2α is the solid angle which anticline makes at the epicentre.

Using (1) and (9) and the same assumptions for interelectrode distances as in (7) above the second vertical derivative of electrical potential over an anticlinal structure is given by

$$\frac{\partial^2 U}{\partial Z^2} = \frac{100\, I\rho_1}{2\pi x^2}\left[\frac{2}{99x} - \frac{4}{x} + \frac{2}{x\sqrt{1\cdot 01}} + \frac{\beta}{x}\left\{\frac{1}{\sqrt{0\cdot 81 + h^2/x^2}} - \frac{4}{\sqrt{1 + h^2/x^2}} + \frac{1}{\sqrt{1\cdot 21 + h^2/x^2}} + \frac{2}{\sqrt{1\cdot 01 + h^2/x^2}}\right\}\right], \tag{10}$$

or the apparent resistivity can be given by

$$\rho_a = \rho_1\left[1 + Q'\beta\left\{\frac{1}{\sqrt{0\cdot 81 + h^2/x^2}} - \frac{4}{\sqrt{1 + h^2/x^2}} + \frac{1}{\sqrt{1\cdot 21 + h^2/x^2}} + \frac{2}{\sqrt{1\cdot 01 + h^2/x^2}}\right\}\right]. \tag{11}$$

where $Q \approx 97\cdot 31$ and $\beta = \dfrac{1-\cos\alpha}{\cos\alpha}$.

Using (11) the apparent resistivity values were calculated over an anticlinal structure of varying width (*i.e.*, for different values of β or solid angle 2α) for expanding separation (AO), *i.e.*, x/h. The curves shown in figure 3 indicate change in apparent resistivity ratio with separation.

4. Results and comparison with other electrode set-ups

4.1. *Two-layered earth*

The curves obtained for ρ_a using $\partial^2 U/\partial Z^2$ over a two-layered earth with a horizontal boundary differ substantially from curves obtained for ρ_a using either $\partial^2 U/\partial X^2$ (figure 4) or the Schlumberger array. The Schlumberger array results have already been compared with those obtained by using the horizontal second derivative (Murali *et al* 1980b) and shown that the latter has some specific advantages. Thus the horizontal derivative results and the vertical derivative results are now being compared.

When the second horizontal derivative is used the sounding curves show only a smooth variation from ρ_1 to the asymptotic value (equivalent to ρ_2), whereas for the second vertical derivative ρ_a becomes less than ρ_1 for small values of x/h and then it increases to reach to the asymptotic value where $\rho_a = \rho_2$, for

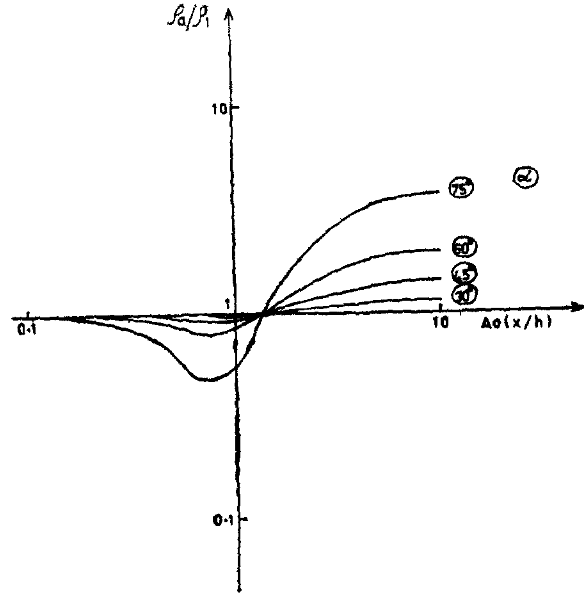

Figure 3. Apparent resistivity curves t obtained by sounding ($\partial^2 U/\partial Z^2$) over an anticlinal structure of infinite resistivity showing the variation in ρ_a/ρ_1 with the inclination of the sides (α).

positive values of K (*i.e.*, $\rho_2 > \rho_1$). The converse happens when the value of K is negative (*i.e.*, the case when $\rho_2 < \rho_1$).

With the vertical derivative curves it will be seen that the ρ_a curves are steeper and the final value reached (asymptote) is always nearer the true value of ρ_2. By virtue of the larger change in ρ_a with any variation in h it will be possible to match the field curves with the master curves more accurately resulting in a more precise determination of h or ρ_2. It will be noted that the ρ_a curves for vertical derivative sounding are distinguished from the corresponding Schlumberger curves by an intermediate peak or trough in the curve. This peak occurs (for ρ_2 values less than ρ_1, *i.e.*, K negative) at $x/h = 1 \cdot 3$ and ρ_a values cross over $\rho_a = \rho_1$ line at $x/h = 2 \cdot 5$; thus facilitating the direct determination of h. However for increasing positive values of K (*i.e.*, ρ_2 values higher than ρ_1) the trough occurs at correspondingly larger values of x/h and the cross-over point also shifts along the x/h axis.

4.2. *Anticlinal structure*

Similar to the two-layered case described above, in the case of anticlinal structure also ($\rho_2 = \infty$) the angle of dip of sides may be easily estimated from the position of the minimum in the apparent resistivity curve, the corresponding spacing for which may be denoted as AO_{min}. The depth to the hypocentre of the anticline from the surface may also then be computed from $h_0 = (1 \cdot 224) \cdot AO_{min}$. We may compare the above results of sounding obtained with a second horizontal derivative measuring three electrode set-up (AMON) (figure 5).

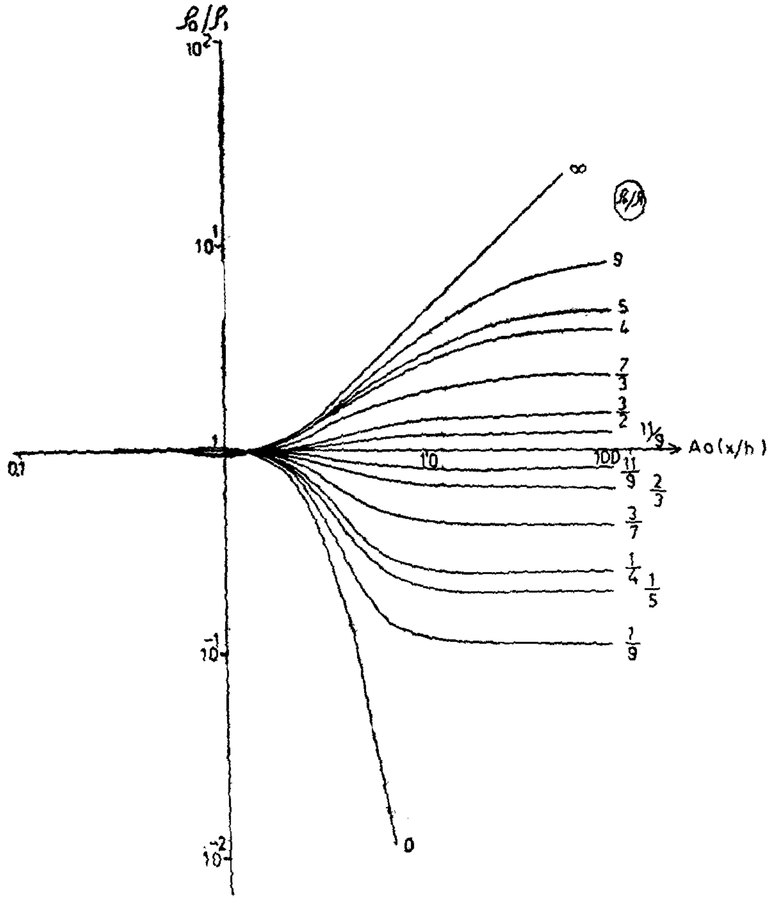

Figure 4. Apparent resistivity master curves for sounding over a two-layered earth obtained with a second horizontal derivative ($\partial^2 U/\partial x^2$) measuring set-up.

It will be noticed that the three electrode ρ_a curves are not much influenced by the dip of the formation, while on the vertical derivative, the ρ_a curves vary quite substantially with the dip and hence may be used in a more reliable fashion for estimating the dip.

5. Conclusions

The second vertical derivative of electrical potential due to a single point source may be estimated over an inhomogeneous earth by employing five measuring electrodes. The field measurement of $\partial^2 U/\partial Z^2$ can be carried out without involving much effort by moving only the current electrode and keeping the potential measuring electrodes constant in the same place. It will be relevant here to note that much more effort and time are required for the normal Schlumberger soundings.

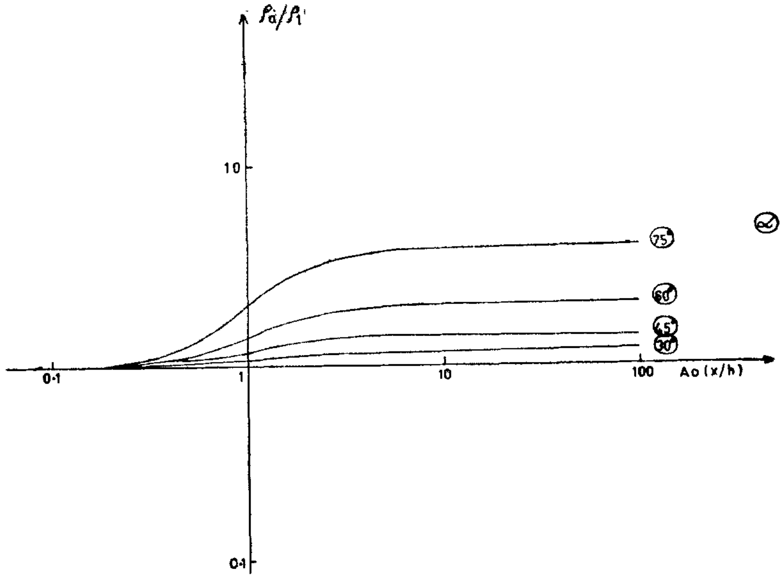

Figure 5. The relation between ρ_a/ρ_1 and angle of inclination of the sides of an anticline obtained using a simple three-electrode (AMN) system.

Resistivity sounding employing the second vertical derivative may be employed with additional advantages over horizontally-layered earth, as well as anticlinal structure in comparison to conventional resistivity sounding. The ρ_a curves have a distinct form and it is also possible to use a simple thumb rule for finding the thickness of the first layer.

It may also be noted that the above measurements of the derivatives (employing potential electrodes only) are valid also for natural electrical fields as well as for IP, etc.

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