Characteristic curves for interpreting gravity anomalies of vertical cylinders and horizontal circular discs

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Abstract. Characteristic curves for the complete interpretation of a wide range of parameters of a vertical cylinder and a horizontal circular disc are presented. It is established that the distance of the inflexion point on the gravity profile of a vertical cylinder from the point of maximum anomaly [i.e. the origin] is approximately equal to the radius of the vertical cylinder, thus enabling one to demarcate the boundary of the vertical cylinder directly from the contour map based on the cluster of contours. Two gravity profiles, one across an anomalous zone in central Alberta, Canada and the other over Borsad area, India are interpreted by the curves presented and the results are shown.

Keywords. Gravity interpretation; characteristic curves; vertical cylinder; horizontal circular disc.

1. Introduction

It has been well established that interpretation of gravity and magnetic anomalies is greatly facilitated by the use of characteristic curves which provide information of the body parameters of the causative bodies very rapidly and without using computers. The method of characteristic curves has been used in one form or the other for a very long time in gravity and magnetic interpretation, but it is only in 1966 that the method has been put in to a systematic format (Grant and Martin 1966). Characteristic curves have been constructed for interpreting magnetic anomalies of individual geometric models by several workers (Smellie 1956; Hood 1964; Moo 1965; Bruckshaw and Kunaratnam 1963; Bean 1966; Grant and West 1965; Am 1972). Rao *et al* (1974), Rao and Murthy (1978) and Nageswara Rao *et al* (1979) have constructed characteristic curves for all geometric models frequently used in gravity and magnetic interpretation which make use of readily measurable distances and anomalies on the field profile. It has been observed by experience that these curves yield reliable results in the majority of cases. Some extreme cases of a vertical cylinder involving large radii could not, however, be satisfactorily interpreted with their curves. This drawback may be overcome

if one uses the curves prepared with the characteristic distances involving the inflexion point and the points of one-third and two-thirds the maximum horizontal gradient. Such a set of characteristic curves for interpreting gravity anomalies of vertical circular cylinder are presented in this paper along with the relevant data to facilitate the interpreter to construct the charts to an elongated scale. Two extreme cases of vertical cylinder, viz., the circular disc and the bottomless circular cylinder are also considered. Some thumb rules helpful for rapid interpretation are also included.

2. Vertical circular cylinder

The vertical circular cylinder has three parameters, viz., Z_1 , Z_2 and R , explained in figure 1, to be interpreted from the shape of the profile, while the density contrast (σ) can be determined from the maximum gravity anomaly. The three length parameters can be made into two dimensionless ratios Z_2/Z_1 and R/Z_1 . The characteristic curves constructed for vertical cylinder are shown in figures 2, 3 and 4 and the related data are presented in tables 1, 2, 3 and 4. All these data have been obtained using IBM 1130 computer available at Andhra University.

Figure 1. Parameters of vertical circular cylinder and characteristic distances used in its interpretation.

Figure 2. Characteristic curves for interpreting gravity anomalies of vertical cylinders.

Figure 3. Complementary curves for interpreting gravity anomalies of vertical cylinders.

Figure 4. Complementary curves for determining the density contrast of the vertical cylinder.

Table 1. Data for the construction of characteristic curves in figure 2.

R/Z_1	The value of the ratio $X'_{1/3}/X'_{2/3}$ for Z_2/Z_1								
	1.2	2.0	5.0	7.5	10.0	15.0	20.0	50.0	100.0
1.0	0.476	0.470	0.469	0.469	0.468	0.468	0.469	0.469	0.469
2.0	0.562	0.517	0.494	0.494	0.493	0.493	0.493	0.493	0.493
3.0	0.676	0.598	0.525	0.518	0.517	0.516	0.516	0.516	0.516
4.0	0.757	0.679	0.562	0.544	0.539	0.536	0.536	0.536	0.536
5.0	0.810	0.742	0.604	0.571	0.561	0.555	0.553	0.552	0.552
7.5	0.879	0.834	0.703	0.645	0.617	0.595	0.589	0.584	0.584
10.0	0.913	0.880	0.774	0.713	0.673	0.635	0.621	0.609	0.608
20.0	0.958	0.944	0.892	0.854	0.821	0.768	0.732	0.674	0.667
50.0	0.984	0.978	0.959	0.945	0.932	0.907	0.884	0.786	0.751

These characteristic curves involve (i) $X'_{1/3}$ and $X''_{1/3}$, the distances between two sets of points of one-third the maximum horizontal gradient, one set near the origin—the point of maximum gravity anomaly, and the other far away from

Table 2. Data for the construction of characteristic curves in figure 2.

R/Z_1	The values of the ratio $(X''_{2/3} - X'_{2/3}) / (X''_{1/3} - X'_{1/3})$ for Z_2/Z_1								
	1.2	2.0	5.0	7.5	10.0	15.0	20.0	50.0	100.0
1.0	0.586	0.576	0.541	0.529	0.523	0.520	0.519	0.518	0.518
2.0	0.552	0.560	0.539	0.525	0.517	0.511	0.509	0.508	0.508
3.0	0.524	0.531	0.522	0.510	0.502	0.494	0.491	0.488	0.488
4.0	0.513	0.512	0.503	0.493	0.485	0.476	0.472	0.468	0.467
5.0	0.508	0.503	0.485	0.477	0.470	0.460	0.455	0.449	0.449
7.5	0.503	0.494	0.460	0.446	0.438	0.428	0.422	0.414	0.413
10.0	0.501	0.491	0.449	0.429	0.418	0.406	0.399	0.387	0.385
20.0	0.500	0.488	0.439	0.412	0.392	0.367	0.352	0.327	0.322
50.0	0.499	0.487	0.437	0.407	0.384	0.352	0.330	0.273	0.252

Table 3. Data for the construction of complementary curves in figure 3.

R/Z_1	The values of the ratio $X'_{1/3}/Z_1$ for Z_2/Z_1								
	1.2	2.0	5.0	7.5	10.0	15.0	20.0	50.0	100.0
1.0	0.454	0.458	0.482	0.485	0.486	0.487	0.487	0.487	0.487
2.0	1.361	1.137	1.045	1.047	1.049	1.050	1.051	1.051	1.051
3.0	2.920	2.356	1.865	1.827	1.821	1.819	1.819	1.819	1.819
4.0	4.780	3.988	2.938	2.782	2.740	2.722	2.718	2.718	2.718
5.0	6.748	5.834	4.256	3.908	3.792	3.730	3.716	3.708	3.708
7.5	11.750	10.746	8.358	7.418	6.966	6.628	6.526	6.456	6.452
10.0	16.770	15.754	13.050	11.668	10.822	10.020	9.720	9.468	9.452
20.0	36.820	35.820	32.920	31.040	29.460	27.120	25.620	23.140	22.840
50.0	96.860	95.880	93.060	91.120	89.400	86.400	83.800	74.160	69.480

Table 4. Data for the construction of complementary curves in figure 4.

R/Z_1	The values of the function $g_{\max}/2\pi\gamma\sigma Z_1$ for Z_2/Z_1								
	1.2	2.0	5.0	7.5	10.0	15.0	20.0	50.0	100.0
1.0	0.052	0.178	0.315	0.348	0.364	0.381	0.389	0.404	0.409
2.0	0.103	0.407	0.851	0.974	1.038	1.103	1.136	1.196	1.216
3.0	0.113	0.557	1.331	1.584	1.722	1.865	1.938	2.072	2.117
4.0	0.147	0.651	1.719	2.123	2.353	2.598	2.727	2.963	3.043
5.0	0.157	0.714	2.028	2.585	2.918	3.287	3.483	3.849	3.974
7.5	0.171	0.804	2.552	3.459	4.066	4.796	5.206	6.007	6.288
10.0	0.178	0.851	2.869	4.049	4.908	6.022	6.689	8.059	8.551
20.0	0.189	0.925	3.409	5.165	6.664	9.025	10.741	15.173	17.044
50.0	0.195	0.970	3.761	5.950	8.019	11.808	15.158	28.299	37.206

it, and (ii) $X'_{2/3}$ and $X''_{2/3}$, the distances between two sets of points of two-third the maximum horizontal gradient, one set near the origin and the other far away from it. These distances are explained in figure 1. Figure 2 gives the ratios Z_2/Z_1 and R/Z_1 , while figures 3 and 4 provide information on Z_1 and the density contrast respectively.

The gravity effect (g) at a distance x from the origin of a vertical cylinder, extending from depth Z_1 to depth Z_2 , is obtained by subtracting the gravity effect of the vertical cylinder whose depth to the top is Z_2 from the gravity effect of the vertical cylinder whose depth to the top is Z_1 (Nabhigian 1962). The relevant equation is given by

$$g(x) = 2\gamma\sigma \left[\frac{R^2 - x^2}{\sqrt{(x+R)^2 + Z_1^2}} K(k) + \sqrt{(x+R)^2 + Z_1^2} E(k) \right. \\ \left. - \frac{R^2 - x^2}{\sqrt{(x+R)^2 + Z_2^2}} K(k) - \sqrt{(x+R)^2 + Z_2^2} E(k) \right. \\ \left. + \frac{\pi(Z_1 - Z_2)}{2} \Lambda(\phi, k) - \pi(Z_1 - Z_2) \right]$$

where $K(k)$ is the complete elliptic integral of the first kind, $E(k)$ is the complete elliptic integral of the second kind, $\Lambda(\phi, k)$ is the Neumann's Lambda function,

$$k = \sqrt{\frac{4xR}{(x+R)^2 + Z^2}} = \text{the modulus of the elliptic integrals,}$$

σ is the density contrast, and γ is the gravitational constant. The maximum gravity anomaly is given by

$$g_{\max} = g(0) = 2\pi\gamma\sigma \left[Z_2 - Z_1 + \sqrt{Z_1^2 + R^2} - \sqrt{Z_2^2 + R^2} \right]. \quad (1)$$

The first and second horizontal derivatives of gravity anomaly are obtained as

$$\frac{\partial g}{\partial x} = 2\gamma\sigma R \left[-\frac{R^2 + x^2 + Z_1^2}{x\sqrt{(R+x)^2 + Z_1^2}} K(k) + \frac{\sqrt{(R+x)^2 + Z_1^2}}{x} E(k) \right. \\ \left. + \frac{R^2 + x^2 + Z_2^2}{x\sqrt{(R+x)^2 + Z_2^2}} K(k) - \frac{\sqrt{(R+x)^2 + Z_2^2}}{x} E(k) \right], \quad (2)$$

$$\text{and } \frac{\partial^2 g}{\partial x^2} = \frac{2\gamma\sigma R^2}{x^2 [(R-x)^2 + Z_1^2] \sqrt{(R+x)^2 + Z_1^2}} \\ \times [(R^2 + Z_1^2) \{ (R-x)^2 + Z_2^2 \} K(k) \\ - \{ (Z_2^2 + R^2)^2 + x^2 (Z_2^2 - R^2) \} E(k)]$$

$$\begin{aligned}
& - \frac{2\gamma\sigma R^2}{x^2 [(R-x)^2 + Z_2^2] \sqrt{(R+x)^2 + Z_2^2}} \\
& \times [(R^2 + Z_2^2) \{ (R-x)^2 + Z_2^2 \} K(k) \\
& - \{ (Z_2^2 + R^2)^2 + x^2 (Z_2^2 - R^2) \} E(k)] \quad (3)
\end{aligned}$$

Equation 3, however, does not allow calculation of points of inflexion easily. Hence, the inflexion point is determined and other distances are calculated by numerical methods using (2) and (3) as follows :

Values of Z_2/Z_1 and R/Z_1 are first assumed. The first and second derivatives for different values of x are then calculated at reasonable intervals of x using (2) and (3). The values of x where the second derivative is zero is the inflexion point. Hence two values of x (say x_1 and x_2) are determined from the calculated second derivatives, between which x_{infl} lies and the second derivative becomes zero. The exact value of x_{infl} is determined by linear interpolation. If D_1 and D_2 are the second derivatives at x_1 and x_2 respectively, then a new point x_0 is located by the interpolation formula

$$x_0 = x_1 + \frac{(x_2 - x_1)}{(D_1 - D_2)} D_1.$$

The second derivative value D_0 is then calculated at x_0 by (3). It may be noted that D_1 is necessarily negative, while D_2 is positive. If D_0 is negative, further interpolation is performed between x_0 (treating it as x_1) and x_2 . If D_0 is positive, interpolation is performed between x_1 and x_0 (treating it as x_2). This procedure is repeated several times.

The maximum horizontal gradient is then calculated at x_{infl} using (2). Two values of x at which the gradient is nearer to one-third the maximum gradient and two more values of x at which the gradient is nearer to two-thirds the maximum gradient are then located and improved by Newton-Raphson's method. The ratio $g_{\text{max}}/2\pi\gamma\sigma Z_1$ for figure 4 is obtained from (1).

The characteristic curves presented in figures 2, 3 and 4 are used in the following way :

From the field profile, we find out the distances $X'_{1/3}$, $X'_{2/3}$, $X''_{2/3}$, $X''_{1/3}$, and then calculate the estimators $(X''_{2/3} - X'_{2/3}) / (X''_{1/3} - X'_{1/3})$ and $(X'_{1/3} / X'_{2/3})$ to determine Z_2/Z_1 and R/Z_1 from figure 2. Figure 3 gives the values of $X'_{1/3} / Z_1$ for the known values of Z_2/Z_1 and R/Z_1 . Since $X'_{1/3}$ is known from the field profile, Z_1 and hence Z_2 and R are calculated. Figure 4 provides the value of the function $g_{\text{max}} / 2\pi\gamma\sigma Z_1$ for the known values of Z_2 / Z_1 and R/Z_1 . g_{max} is measured from the field profile and is compared with this function to determine density contrast.

2.1 Some thumb-rules

It is of interest to note the dependence of X_{infl}/Z_1 on R/Z_1 and Z_2/Z_1 which is shown in table 5. It may be observed that if $R > Z_1$, X_{infl} is less than R .

Table 5. Dependence of the ratio X_{infl}/Z_1 on R/Z_1 and Z_2/Z_1 of a vertical cylinder.

R/Z_1	The values of X_{infl}/Z_1 for Z_2/Z_1								
	1.2	2.0	5.0	7.5	10.0	15.0	20.0	50.0	100.0
1.0	0.983	1.040	1.115	1.125	1.127	1.129	1.129	1.129	1.129
2.0	1.916	1.911	1.954	1.969	1.976	1.979	1.981	1.982	1.982
3.0	2.925	2.902	2.907	1.291	2.929	2.935	2.937	2.938	2.938
4.0	3.937	3.913	3.896	3.904	3.912	3.919	3.921	3.924	3.924
5.0	4.947	4.924	4.898	4.901	4.906	4.913	4.916	4.919	4.920
7.5	7.462	7.444	7.414	7.409	7.409	7.413	7.416	7.421	7.422
10.0	9.971	9.956	9.929	9.921	9.918	9.919	9.921	9.927	9.928
20.0	19.985	19.977	19.960	19.953	19.948	19.944	19.943	19.945	19.947
50.0	49.994	49.991	49.983	49.979	49.977	49.974	49.972	49.968	49.969

Figure 5. Characteristic and complementary curves for interpreting gravity anomalies of horizontal circular discs,

Further, X_{infl} decreases first and then increases with increase in Z_2 . The initial decrease of X_{infl} in case of small ratios of Z_2/Z_1 is not brought out by the contents of table 5, because such a decrease might occur at values of $Z_2 < 1.2 Z_1$. For all practical purposes, however, X_{infl} is approximately equal to R and hence determines the radius of the cylinder. The value of R , thus obtained, is always an underestimate, the maximum error being not more than 3%. For shallow cylindrical bodies, the inflexion point can be easily determined, because at these points the anomaly contours are crowded. Thus, the boundary of the vertical cylinder can be easily determined straight from the contour map based on the cluster of contours.

3. Horizontal circular disc

The horizontal circular disc may be viewed as a vertical cylinder whose Z_2/Z_1 ratio has a very small value. The exact expression for the gravity anomaly of this model is given by (Rao and Murthy 1966, 1978)

$$g(x) = 2\gamma\sigma dZ \left[\pi - \frac{Z}{(x+R)^2 + Z^2} K(k) - \frac{\pi}{2} \wedge(\phi, k) \right],$$

where Z is the depth to the centre of the disc and other symbols carry the meaning as for the vertical cylinder.

The characteristic distances used in this case are $X'_{1/2}$, the distance between two points, nearer to the origin of half maximum gradient and X_{infl} , the distance between two points of inflexion gradient. These are calculated and their ratios determined for different values of R/Z following the same procedure outlined in the case of vertical cylinders. Figure 5 shows the characteristic and complementary curves for determining R and Z from the gravity anomalies of horizontal circular discs. The data for the construction of figure 5 are given in table 6. Figure 5 is used in the following way :

Table 6. Data for the construction of characteristic curves for circular disc.

R/Z	$X'_{1/2}/X_{\text{infl}}$	$X'_{1/2}/Z$
1.0	0.3616	0.7016
2.0	0.5267	2.0240
3.0	0.6602	3.8720
4.0	0.7414	5.8500
5.0	0.7927	7.8540
7.5	0.8623	12.8780
10.0	0.8971	17.8980
20.0	0.9489	37.9200
50.0	0.9797	97.9600

The characteristic distances $X'_{1/2}$ and X_{infl} are measured from the field profile. The ratio $X'_{1/2}/X_{\text{infl}}$ is calculated. The values of R/Z and $X'_{1/2}/Z$ are noted from figure 5. Since $X'_{1/2}$ is known from the field profile, Z and R are calculated.

When R and Z are known, the surface density, σdZ of the circular disc can be calculated from the maximum gravity anomaly as follows :

$$\sigma dZ = \frac{g_{\text{max}}}{2\pi\gamma \left(1 - \frac{Z}{\sqrt{R^2 + Z^2}}\right)}$$

where dZ is the thickness of the circular disc. σ and dZ cannot be separated. One of them can be calculated if the other is known.

4. Bottomless vertical cylinder

When the depth Z_2 of the vertical cylinder has a very high value, the gravity anomaly is given by the equation (Nabhigian 1962)

Figure 6. Characteristic and complementary curves for interpreting gravity anomalies of bottomless vertical cylinder.

$$g(x) = 2\gamma\sigma \left[\frac{R^2 - x^2}{\sqrt{(x+R)^2 + Z^2}} K(k) + \sqrt{(x+R)^2 + Z^2} E(k) + \frac{\pi Z}{2} \wedge(\phi, k) - \pi Z \right],$$

where Z is the depth to the top of the bottomless vertical cylinder. The characteristic and complementary curves for this model are presented in figure 6 and the relevant data are given in table 7.

5. Examples of interpretation

The characteristic curves presented in this paper have been used to interpret some field profiles. In general, the accuracy of interpretation depends on how accurately the inflexion and other points are located.

Example 1 : Figure 7 shows the gravity profile across an anomalous zone in central Alberta (Skeels 1963) and its interpretation making use of characteristic curves of a vertical cylinder. The distances measured are given below.

$$X'_{1/3} = 1.92 \text{ km}, \quad X''_{1/3} = 16.80 \text{ km}$$

$$X'_{2/3} = 3.50 \text{ km and } X''_{2/3} = 11.68 \text{ km.}$$

The characteristic estimators are calculated as

$$X'_{1/3}/X'_{2/3} = 0.545 \text{ and } (X''_{2/3} - X'_{2/3})/(X''_{1/3} - X'_{1/3}) = 0.548$$

These values when located in figure 2 define the values of $Z_2/Z_1 = 2.1$ and $R/Z_1 = 2.25$. From figure 3, the values of $X'_{1/3}/Z_1$ is found to be 1.3 corresponding to these values of Z_2/Z_1 and R/Z_1 . Since $X'_{1/3} = 1.92 \text{ km}$, Z_1 is calculated as 1.48 km. From the known values of Z_1 , Z_2/Z_1 and R/Z_1

Table 7. Data for the construction of characteristic curves for bottomless cylinder.

R/Z	$X'_{1/2}/X_{\text{inf}}$	$X'_{1/2}/Z$
1.0	0.3637	0.8308
2.0	0.4075	1.6178
3.0	0.4609	2.7100
4.0	0.5052	3.9660
5.0	0.5414	5.3280
7.5	0.6073	9.0160
10.0	0.6520	12.9440
20.0	0.7467	29.7800
50.0	0.8390	83.8400

Figure 7. Interpretation of gravity anomaly profile in central Alberta [Skeels 1963].

Figure 8. Interpretation of residual gravity anomaly profile over Borsad area, India [Rao and Murthy 1978].

the values of Z_2 and R are calculated as 3.11 km and 3.33 km respectively. From figure 4, it is obtained that $g_{\max}/2\pi\gamma\sigma Z_1 = 0.46$. Since the value of $g_{\max} = -7.625$ m gals, the density contrast (σ) calculated is -0.267 gm/cc. The parameters obtained in this interpretation compare reasonably well with those obtained by Skeels ($Z_1 = 2.24$ km, $Z_2 = 3.74$ km and $R = 3.74$ km) with his master curves assuming the density contrast to be -0.3 gm/cc. The difference in Z_1 is due to the fact that the density contrast is assumed to be -0.30 gm/cc by Skeels, while in the present method it is solved from the field profile.

Example 2 : Figure 8 depicts the residual gravity anomaly profile over Borsad area, India, believed to be due to structure in the basement (Rao and Murthy,

1978). This anomaly is interpreted using characteristic curves of horizontal circular disc. As the profile is not perfectly symmetric, the characteristic distances on either flank of the profile are measured. The distances thus measured are $X'_{1/2} = 6.96$ km and $x_{\text{infl}} = 12.85$ km. The characteristic estimator is $X'_{1/2}/x_{\text{infl}} = 0.5437$ which when located in figure 5 gives that $X'_{1/2}/Z = 2.4$ and $R/Z = 2.1$. Since, the value of $X'_{1/2} = 6.96$ km, the depth to the top (Z) of the circular disc is calculated as 2.9 km and hence $R = 6.1$ km. The calculated value of Z roughly corresponds to the maximum thickness of sediments in this area.

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