Excitation of electromagnetic lower hybrid instability in the region of bouncing electron beams

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Abstract. The region of bouncing electron beams in the earth's magnetosphere can be unstable against a non-resonant electromagnetic lower hybrid instability. The instability is purely growing in the rest frame of the plasma, and can be excited either by the temperature anisotropy or the drift velocity of the bouncing electron beams. The growth rates of the instability decrease with the increase of cold electron density. Consequently the growth rate is maximum at the equator where the cold electron density is minimum. The intense turbulence generated by this instability could broaden the bouncing electron beams thereby explaining the observed wider cone width of the beams at the equator. The instability could generate magnetic pulsations in the frequency range of order $Pc_1 - Pc_2$ with typical wavelength $\sim (3-10)$ km in the magnetosphere during magnetic storms or substorms.

Keywords. Bouncing electron beams; electromagnetic lower hybrid instability; magnetosphere; magnetic pulsations.

1. Introduction

McIlwain (1975) has reported the presence of beams of bouncing electrons, within 10 minutes of the onset of magnetospheric substorms, from ATS-6 and ATS-5 satellite observations. The beams have source cone width of $\sim 20^\circ$ in pitch angles. Similar bouncing electron beams have been reported by Borg *et al* (1978) from GEOS 1 observations at $6.6 R_E$. Hasegawa (1979) has suggested that such bouncing beams of electrons can result from the injection of plasma sheet protons into the magnetosphere. The ionospheric electrons from both the hemispheres are sucked up to the equatorial region to neutralise these additional positive charges, thereby forming literally the beams of bouncing electrons. Recently Sharp *et al* (1980) have observed intense contrastreaming electron beams with cone width of about 1° pitch angle (energies of order keV range) at altitude of $1 R_E$ on the auroral field lines during magnetically disturbed periods. The cone width of auroral beams are considerably smaller than that would be expected for the equatorial beams if they move adiabatically to the auroral region down to $1 R_E$ altitude. The widening of the source cone width for the equatorial beams may

possibly be due to some plasma instabilities generated by the bouncing electron beams as suggested by Sharp *et al* (1980). Hasegawa (1979) has shown that these bouncing beams of electrons will excite Alfvén waves of short perpendicular wavelength and can account for the experimental observations of dusk side long period magnetic pulsations (Pc_2 to Pc_k).

In this paper we report our investigations on the excitation of electromagnetic lower hybrid instability by the bouncing electron beams. This instability is of non-resonant type and occurs on the fast magnetosonic (FMS) branch of hot plasma dispersion relation (Stix 1962; Mikhailovskii 1975; Akhiezer *et al* 1975; Lakhina 1979). Recently Lakhina (1980) has discussed this instability in the presence of field aligned currents and in the absence of cold plasma, to explain the observed low frequency magnetic noise (Gurnett and Frank 1977) on the auroral field lines. His analysis is valid for the wavelengths smaller than the scale lengths of plasma inhomogeneity produced by the field aligned current. For the case of field aligned bouncing electron beams, which is of interest to us, there is no net field aligned current and therefore the plasma can remain uniform. Further at altitudes of the order of a few R_E on the auroral field lines, the cold plasma density can be appreciable and can frequently exceed the hot particle density (Hudson *et al* 1978; Mozer *et al* 1979). Thus, it is essential to take account of the cold plasma in the analysis of electromagnetic lower hybrid instability in the region of bouncing electron beams.

We find that the instability can be driven by temperature anisotropy and bouncing electron drift velocity V_0 . In the presence of background electrons, the two electromagnetic modes, namely, a hot plasma mode and a cold electron mode, are generated whereas only a single electromagnetic mode is excited in the absence of cold electrons. The cold electron mode is excited only when there are sufficient number of cold particles, and its growth rate decreases with the increase of cold electron density. The variation of temperature anisotropy and $\beta_\perp$ (ratio of perpendicular electron pressure to magnetic field pressure) on the instability has been studied in detail.

2. Dispersion relation

We consider a homogeneous collisionless plasma in which two hot electron beams are contra-streaming with the velocity $\pm V_0$ relative to protons and cold electrons, along the uniform external magnetic field $\mathbf{B}_0 = B_0 \hat{e}_z$. The beam electrons are described by the drifted bi-Maxwellian equilibrium distribution functions, and the protons by the Maxwellian distribution function. The temperature of cold electrons is taken to be zero, so that they are described by a delta-type distribution function. The density of hot electrons in each beam is taken to be $N_H/2$, whereas the density of cold electrons and protons are taken to be N_c and N_p such that $N_p = N_c + N_H$. This ensures the charge and current neutrality in the equilibrium state. We shall make the following assumptions:

(i) $|\Omega_p^2| \ll |\omega|^2 \ll |\Omega_e^2|$, where Ω_p and Ω_e are the cyclotron frequencies of protons and electrons respectively and ω is the wave frequency with $k\rho_p > 1$ (ρ_p and k being the proton Larmor radius and wave number respectively). Under this assumption the protons can be treated as unmagnetized.

(ii) $k_{\parallel}^2 \ll k_{\perp}^2$, where $k_{\parallel}$ and $k_{\perp}$ are parallel and perpendicular wave numbers with respect to the magnetic field.

$$(iii) \frac{\omega^2}{k_{\perp}^2 a_p^2} \ll 1, \quad \frac{(\omega \pm k_{\parallel} V_0)^2}{k_{\parallel}^2 a_{\parallel}^2} \ll 1, \quad \frac{\omega^2}{C^2 k^2} \ll 1 \quad \text{and} \quad \frac{k_{\perp}^2 a_{\parallel}^2}{\omega_{pH}^2} \ll 1;$$

where a_p and $a_{\parallel}$ are the protons and hot electron thermal velocities and V_0 is the drift speed of the bouncing electron beams. Under these assumptions, the general dispersion relation obtained from linearized Vlasov equation and full set of Maxwell equations reduces to the following simple form (Lakhina 1979, 1980):

$$P\omega^4 + R\omega^2 + T = 0, \quad (1)$$

where

$$P = \frac{A(1-\phi)\Omega_e^2}{\omega_{pH}^4 \psi^2} \left[1 + \frac{2\phi\omega_{pH}^2}{k_{\perp}^2 a_{\perp}^2} + \frac{2\omega_{pp}^2}{k_{\perp}^2 a_p^2} + \frac{\omega_{pH}^2}{\Omega_e^2} \frac{N_c}{N_H} \right] + 1 + \left(\frac{T_{\parallel}}{T_{\perp}} - 1 \right) \phi \\ + \frac{\omega_{pp}^2}{\omega_{pH}^2} \frac{a_{\parallel}^2}{a_p^2} + \frac{k_{\perp}^2 a_{\parallel}^2}{2\Omega_e^2} \frac{N_c}{N_H} + \frac{2(1-\phi)}{\psi} \frac{N_c}{N_H} + \frac{(1-\phi)}{\psi^2} \frac{N_c^2}{N_H^2}, \quad (2)$$

$$R = A \left\{ -\frac{C^2 k_{\parallel}^2 \Omega_e^2}{\omega_{pH}^4 \psi^2} \left[(1-\phi) + \phi \frac{T_{\parallel}}{T_{\perp}} + \frac{T_{\parallel}}{T_p} \frac{N_p}{N_H} + \frac{k_{\perp}^2 a_{\parallel}^2}{2\Omega_e^2} \frac{N_c}{N_H} \right] \right. \\ + \frac{k_{\parallel}^2 a_{\parallel}^2}{\omega_{pH}^2 \psi^2} \left(\phi^2 \left(\frac{T_{\parallel}}{T_{\perp}} - 1 \right) \frac{\Omega_e^2}{k_{\perp}^2 a_{\perp}^2} - \frac{\Omega_{pp}^2}{k_{\perp}^2 a_p^2} \frac{\Omega_e^2}{\omega_{pH}^2} \frac{N_c}{N_H} - \frac{\phi\Omega_e^2}{k_{\perp}^2 a_{\perp}^2} \frac{N_c}{N_H} \right. \\ \left. - \frac{1}{2} \frac{N_c^2}{N_H^2} \right) + \phi \left(\frac{2V_0^2}{a_{\perp}^2} + \frac{T_{\parallel}}{T_{\perp}} - 1 \right) \frac{\Omega_e^2}{\omega_{pH}^2 \psi^2} \frac{k_{\parallel}^2}{k_{\perp}^2} \left((1-\phi) + \frac{T_{\parallel}}{T_p} \frac{N_p}{N_H} \right. \\ \left. + \frac{k_{\perp}^2 a_{\parallel}^2}{2\Omega_e^2} \frac{N_c}{N_H} \right) \left. \right\} + k_{\parallel}^2 V_0^2 \left[(1-\phi) + \phi \frac{T_{\parallel}}{T_{\perp}} + \frac{N_p}{N_H} \frac{T_{\parallel}}{T_p} + \frac{k_{\perp}^2 a_{\parallel}^2}{2\Omega_e^2} \frac{N_c}{N_H} \right. \\ \left. - \frac{4\phi}{\psi} \frac{T_{\parallel}}{T_{\perp}} \frac{N_c}{N_H} \right] - \frac{k_{\parallel}^2 a_{\parallel}^2}{2} \frac{N_c}{N_H} \left[1 - \frac{C^2 k^2}{\omega_{pH}^2} \right] + \frac{N_c^2}{N_H^2} \frac{k_{\parallel}^2 a_{\parallel}^2}{\psi} \\ \times \left(-1 + \left(\frac{2V_0^2}{a_{\perp}^2} + \frac{T_{\parallel}}{T_{\perp}} - 1 \right) \frac{\phi}{2\psi} - \frac{C^2 k^2}{\omega_{pH}^2} \frac{1}{2\psi} - \frac{1}{2\psi} \frac{N_c}{N_H} \right), \quad (3)$$

$$T = A \frac{N_c}{N_H} \frac{k_{\parallel}^2 a_{\parallel}^2 \Omega_e^2}{2\omega_{pH}^2 \psi^2} \left[\frac{C^2 k_{\parallel}^2}{\omega_{pH}^2} - \left(\frac{2V_0^2}{a_{\perp}^2} + \frac{T_{\parallel}}{T_{\perp}} - 1 \right) \phi \frac{k_{\parallel}^2}{k_{\perp}^2} \right] \\ - k_{\parallel}^2 V_0^2 \frac{k_{\parallel}^2 a_{\parallel}^2}{2} \frac{N_c}{N_H}, \quad (4)$$

$$A = C^2 k^2 \left[1 + \beta_{\perp} \psi \left(1 - \frac{T_{\perp}}{T_{\parallel}} \right) \right]. \quad (5)$$

In writing equation (1) we have neglected the resonant terms and therefore the dispersion relation (1) can describe only the non-resonant instability. For $(N_c/N_H) = 0$ equation (1) allows only a purely electromagnetic hot plasma mode, given by

$$\omega^2 = -\frac{R}{P},$$

which has been discussed by Lakhina (1980). This mode describes the high frequency part of dispersion curve for the fast magnetosonic (FMS) waves (Akhiezer *et al* 1975).

In general equation (1) allows two modes namely the hot plasma mode modified by the cold plasma effects, and a new mode arising due to the presence of cold plasma. As we shall see below, the cold plasma mode is stable under the electrostatic limit ($C^2 k^2/\omega_{pH}^2 \rightarrow \infty$) but is unstable in the electromagnetic approximation ($C^2 k^2/\omega_{pH}^2 \ll 1$). We shall call the latter mode as cold electron mode.

3. Analysis of the dispersion relation

In the limit $C^2 k^2/\omega_{pH}^2 \rightarrow \infty$, the term $P\omega^4$ can be neglected as compared to the other two terms in equation (1), and consequently the dispersion relation simplifies to

$$\omega^2 = \frac{k_{\parallel} (N_C/N_H) V_s^2}{\left[1 + k^2 \lambda_{DP}^2 \frac{\omega_{pH}^2}{\Omega_e^2} \left(\frac{N_C}{N_H} + \frac{\phi}{\lambda_e} + \frac{T_{\perp}}{T_{\parallel}} \frac{(1-\phi)}{\lambda_e} \right) \right]}, \quad (6)$$

where $V_s = (k_B T_P/m_e)^{1/2}$, is electron acoustic velocity and $\lambda_{DP} = (a_P/\sqrt{2} \omega_{pP})$ is the Debye length for protons.

Equation (6) describes the new mode, i.e. the cold electron mode as it arises because of the presence of cold electrons. In the electrostatic approximation, we can see that this mode is stable as stated earlier. However the mode may be unstable against resonant instability.

Under the electromagnetic approximation $C^2 k^2/\omega_{pH}^2 = (2\lambda_e/\beta_{\perp}) \ll 1$, which for $\beta_{\perp} \sim 1$, demands $\lambda_e \ll 1$; the coefficients P , R and T in equation (1) are simplified as :

$$P = [1 + \beta_{\perp} (1 - T_{\perp}/T_{\parallel})] \frac{2N_P}{\beta_{\perp} N_H} + \frac{N_P T_{\parallel}}{N_H T_P} + \left(1 + \frac{N_C}{N_H} \right)^2, \quad (7)$$

$$R = -[1 + \beta_{\perp} (1 - T_{\perp}/T_{\parallel})] \left(\frac{C^4 k_{\perp}^2 k_{\parallel}^2 \Omega_e^2}{\omega_{pH}^4} \right) \left\{ \left[1 - \frac{\beta_{\perp}}{2} \left(\frac{2V_0^2}{\alpha_{\perp}^2} + \frac{T_{\parallel}}{T_{\perp}} - 1 \right) \right] \right. \\ \times \left(1 + \frac{N_P T_{\parallel}}{N_H T_P} \right) + \frac{T_{\parallel} N_P N_C \omega_{pH}^2}{T_P N_H^2 C^2 k^2} \left. \right\} + k_{\parallel}^2 V_0^2 \left(1 + \frac{N_P T_{\parallel}}{N_H T_P} \right) \\ - \frac{k_{\parallel}^2 \alpha_{\parallel}^2}{2} \frac{N_C}{N_H} (1 + N_C/N_H)^2, \quad (8)$$

$$T = \frac{N_C}{N_H} \frac{k_{\parallel}^4 \alpha_{\parallel}^4}{2\beta_{\parallel}} \frac{k_{\perp}^2 C^2}{\omega_{pH}^2} \left\{ [1 + \beta_{\perp} (1 - T_{\perp}/T_{\parallel})] \left[1 - \frac{\beta_{\perp}}{2} \left(\frac{2V_0^2}{\alpha_{\perp}^2} - 1 \right) - \frac{\beta_{\parallel}}{2} \right] \right. \\ \left. - \frac{\beta_{\perp} V_0^2}{\alpha_{\perp}^2} \frac{\omega_{pH}^2}{C^2 k_{\perp}^2} \right\}. \quad (9)$$

From equations (7), (8) and (9) we note that $R^2 \gg PT$ when either

$$N_C/N_H \ll C^2 k^2/\omega_{pH}^2 \text{ or } N_C/N_H \gg C^2 k^2/\omega_{pH}^2;$$

under such cases the roots of equation (1) are given by the following simple expressions :

$$\omega^2 = -R/P, \quad (10)$$

$$\text{and } \omega^2 = -T/R. \quad (11)$$

Equations (10) and (11) describe respectively the hot plasma mode and cold electron mode. We shall discuss these modes separately.

3.1. Hot plasma mode

From equation (10) we note that the instability arises whenever R and P have the same sign. From equations (7) and (8) we note that $R > 0$ and $P > 0$ under the following two cases.

3.1a. $T_\perp > T_\parallel$ anisotropy instability: In this case the instability arises when the parameter

$$\Gamma = \left(\frac{T_\perp}{T_\parallel} - 1 \right)$$

satisfies the following inequalities :

$$\Gamma_{\max} > \Gamma > \Gamma_{\min} \quad (12)$$

$$\text{where } \Gamma_{\max} = \frac{1}{\beta_\perp} + \frac{\beta_p}{2} \left[\frac{\beta_\parallel}{\beta_p} + \left(1 + \frac{N_C}{N_H} \right)^2 \frac{N_H}{N_P} \right] \quad (13)$$

$$\text{and } \Gamma_{\min} = \frac{1}{\beta_\perp} + \frac{\left[\frac{1}{2} \frac{N_C}{N_H} \left(1 + \frac{N_C}{N_H} \right) - \frac{V_0^2}{a_\parallel^2} \left(1 + \frac{T_\parallel}{T_P} \frac{N_P}{N_H} \right) \right]}{\left\{ \frac{N_C}{N_H} \frac{N_P}{N_H} \frac{T_\perp}{T_P} + \frac{2\lambda_0}{\beta_\parallel} \left(1 + \frac{T_\parallel}{T_P} \frac{N_P}{N_H} \right) \left[1 - \frac{\beta_\perp}{2} \left(\frac{2V_0^2}{a_\perp^2} - 1 \right) - \frac{\beta_\parallel}{2} \right] \right\}} \quad (14)$$

From equation (14), it is clear that $\Gamma_{\min}$ increases with the increase of N_C/N_H , and that for some value of N_C/N_H the inequality $\Gamma > \Gamma_{\min}$ may break down. Hence the increase of N_C/N_H will stabilize this mode. The increase of V_0 reduces $\Gamma_{\min}$ and therefore instability will occur at lower values of Γ . Further the parameter $\Gamma_{\min}$ is positive when $V_0 < V_{Cr}$, where V_{Cr} is given by

$$V_{Cr}^2 = \frac{a_p^2 m_P T_\parallel}{m_e T_P} \left\{ \frac{N_C}{N_H} \left(1 + \frac{T_\parallel}{T_P} \frac{N_P}{N_H} \right)^{-1} \left[\frac{1}{\beta_\perp} \frac{N_P T_\perp}{N_H T_P} + \frac{1}{2} \left(1 + \frac{N_C}{N_H} \right)^2 \right] + \frac{1}{\beta_\perp} \frac{C^2 k^2}{\omega_{pH}^2} \frac{T_\perp}{T_\parallel} \left(1 + \frac{\beta_\perp}{2} - \frac{\beta_\parallel}{2} \right) \right\}. \quad (15)$$

For $V_0 > V_{Cr}$, $\Gamma_{\min}$ becomes negative and the instability is always excited as long as $\Gamma_{\max} > \Gamma$, and particularly for $T_\perp/T_\parallel \approx 1$. Therefore the instability can be called a bouncing electron beam instability when $V_0 > V_{Cr}$. For $T_\perp \approx T_\parallel$, the growth rate of the instability as seen from equation (10) is given by:

$$\gamma \approx k_\parallel (V_0^2 - V_{Cr}^2)^{1/2} \left\{ \frac{\left(1 + \frac{T_\parallel N_P}{T_P N_H} \right)}{\left[\left(1 + \frac{N_C}{N_H} \right)^2 + \frac{T_\parallel N_P}{T_P N_H} \left(1 + \frac{2}{\beta_\parallel} \right) \right]} \right\}^{1/2}. \quad (16)$$

Since V_{cr} increases with the increase in N_c/N_H and a decrease in $\beta_\perp$, the growth rate will increase with the decrease of N_c/N_H and increase of $\beta_\perp$.

3.1b. $T_\perp > T_\parallel$ anisotropy : In this case $P > 0$ always, and R becomes positive when the following conditions are satisfied:

$$\left\{ \frac{N_P}{N_H} \frac{N_c}{N_H} \frac{T_\parallel}{T_P} \frac{\omega_{pH}^2}{C^2 k^2} \left(1 + \frac{N_P T_\parallel}{N_H T_P} \right)^{-1} + 1 + \frac{\beta_\perp}{2} - \frac{\beta_\parallel}{2} \left(\frac{2V_0^2}{a_\parallel^2} + 1 \right) \right\} < 0 \quad (17a)$$

$$V_0^2 > \frac{a_\parallel^2}{2} \frac{N_c}{N_H} \frac{\left(1 + \frac{N_c}{N_H} \right)^{\frac{1}{2}}}{\left(1 + \frac{T_\perp}{T_P} \frac{N_P}{N_H} \right)}. \quad (17b)$$

It is clear that inequalities 17(a) and 17(b) can be satisfied only when $(N_c/N_H) \ll 1$, otherwise large anisotropies as well as large streaming speeds are required. Hence even a small fraction of cold electrons can stabilize the instability. For $N_c = 0$, however the instability is easily excited provided inequality 17(a) is fulfilled (Lakhina 1980).

3.2. Cold electron mode

From equation (11) it is clear that this mode becomes unstable whenever T and R have the same sign. From equations (8) and (9) we notice that T and R cannot be simultaneously positive. However both can be negative simultaneously under the following cases.

3.2a. $T_\perp > T_\parallel$ anisotropy : In this case the instability is possible provided the inequalities

$$\Gamma_{min} > \Gamma > \left\{ \frac{1}{\beta_\perp} - \frac{V_0^2}{a_\perp^2} \frac{\omega_{pH}^2}{C^2 k_\perp^2} \frac{1}{\left[1 + \frac{\beta_\perp}{2} - \frac{\beta_\parallel}{2} \left(1 + \frac{2V_0^2}{a_\parallel^2} \right) \right]} \right\}, \quad (18)$$

are satisfied. The growth rate is given by

$$\gamma \approx \frac{\left(\frac{T_P}{2T_\parallel} \frac{N_H}{N_P} \right)^{1/2} k_\parallel a_\parallel \left[\frac{V_0^2}{a_\perp^2} + \frac{C^2 k_\perp^2}{\omega_{pH}^2} \left(\frac{T_\perp}{T_\parallel} - 1 - \frac{1}{\beta_\perp} \right) \right]^{1/2}}{(\Gamma_{min} - \Gamma)^{1/2}}. \quad (19)$$

Comparing the inequalities (12) and (18) we note that the instability is excited for lower values of $T_\perp/T_\parallel$ than that for the case of hot plasma mode. It is interesting to note that increase of $V_0^2/a_\perp^2$ has a destabilizing effect in the sense that smaller values of $T_\perp/T_\parallel$ are required to satisfy the right-hand side inequality (18). This is true only when $V_0^2 < V_{cr}^2$; since for $V_0 \geq V_{cr}$, Γ_{min} becomes negative and the inequality $\Gamma_{min} > \Gamma$ cannot be satisfied. This results in a stable case. Since Γ_{min} increases with N_c/N_H , the increase of N_c/N_H will reduce the growth rate.

3.2b. $T_{\parallel} > T_{\perp}$ anisotropy : For $V_0 = 0$, the instability is excited when the inequalities

$$1 + \frac{\beta_{\perp}}{2} < \frac{\beta_{\parallel}}{2} < 1 + \frac{\beta_{\perp}}{2} + \frac{N_P}{N_H} \frac{N_C}{N_H} \frac{T_{\parallel}}{T_P} \frac{\omega_{pH}^2}{C^2 k_{\perp}^2} \left(1 + \frac{N_P}{N_H} \frac{T_{\parallel}}{T_P}\right)^{-1}, \quad (20)$$

are satisfied. The growth rate is then given approximately as

$$\gamma \approx \frac{k_{\parallel} a_{\perp} k_{\perp} C}{\beta_{\perp}^{1/2} \omega_p} \frac{\left(\frac{\beta_{\parallel}}{2} - \frac{\beta_{\perp}}{2} - 1\right)^{1/2}}{\left[\left(1 + \frac{N_C}{N_H}\right)^2 + 2 \frac{N_P}{N_H} \frac{1}{\beta_P}\right]^{1/2}}. \quad (21)$$

The growth rate decreases with the increase of N_C/N_H and the decrease of $\beta_{\parallel}$. For the case $V_0 \neq 0$, the instability is excited, provided the inequalities

$$\begin{aligned} & \left\{ \left[1 + \beta_{\perp} \left(1 - \frac{T_{\perp}}{T_{\parallel}}\right)\right] \left(1 + \frac{\beta_{\perp}}{2} - \frac{\beta_{\parallel}}{2}\right) \frac{C^2 k_{\perp}^2}{\beta_{\parallel} \omega_{pH}^2} \right\} < \frac{V_0^2}{a_{\parallel}^2} < \frac{1}{2} \frac{N_C}{N_H} \\ & \times \left(1 + \frac{N_P}{N_H} \frac{T_{\parallel}}{T_P}\right)^{-1} \left\{ 2 \frac{N_P}{N_H} \frac{T_{\parallel}}{T_P} \left[1 + \beta_{\perp} \left(1 - \frac{T_{\perp}}{T_{\parallel}}\right)\right] + \left(1 + \frac{N_C}{N_H}\right)^2 \right\} \end{aligned} \quad (22)$$

are satisfied and the corresponding growth rate is approximately given as

$$\gamma \approx \frac{k_{\parallel} V_0}{\left[\left(1 + \frac{N_C}{N_H}\right)^2 + 2 \frac{N_P}{N_H} \frac{1}{\beta_P}\right]^{1/2}}. \quad (23)$$

The inequalities (22) show that the beam-driven instability will exist in a bounded ranges of $V_0/a_{\perp}$, the growth rate will however increase with the increase of $V_0/a_{\perp}$ and decrease of N_C/N_H .

We may point out that for $\beta_{\perp} \sim 1$ and $\cos \theta = (k_{\parallel}/k_{\perp}) \geq (m_e/m_p)^{1/2}$ the analysis presented in §§ 3.1 and 3.2 is consistent with the assumptions (1)–(3) made in § 2, provided the wave number lies in the range

$$\frac{\omega_{pe}^2}{C^2 \cos \theta} > k^2 > \frac{\omega_{pP}^2}{C^2 \cos \theta} \quad (24a)$$

$$\text{and} \quad \frac{V_0}{a_{\perp}} < \left(1 + \frac{N_C}{N_H}\right). \quad (24b)$$

where ω_{pe} is the plasma frequency of electrons with density equal to $(N_C + N_H)$. Further the temperature of cold electrons T_C , need not be exactly zero. From equations (16), (19), (21), (23), we note that the cold electron approximation will be valid as long as

$$T_C < (k_{\perp} C/\omega_{pe}) T_{\parallel} \text{ or } a_C < V_0/(1 + N_C/N_H),$$

where a_C is the thermal speed of the cold electrons.

3.3. Numerical results

The analytical expressions derived in §§ 3.1 and 3.2 are valid for $\lambda_e \ll 1$ and for some restricted values of N_C/N_H ; namely $(N_C/N_H) \rightarrow 0$ or for $(N_C/N_H) \gg C^2 k^2/\omega_{pH}^2$.

For arbitrary values of N_C/N_H and λ_e the roots of equation (1) should be evaluated numerically. Since the hot plasma mode is unstable for $R > 0$ and the cold electron mode is unstable for $R < 0$, the sign of R can directly give an idea of the range of unstable parameters corresponding to these modes. In figure 1 we have shown the variation of R , as computed from equation (3) against λ_e for different values of N_C/N_H .

It is clear that for $N_C = 0$, R is positive for all values of λ_e (cf. curve 1). For finite values of N_C/N_H close to zero, R is still positive, but for values of N_C/N_H shown by the curves 2, 3, 4 and 5, R is negative at smaller values of λ_e and is positive for larger values of λ_e . Hence for $(N_C/N_H) \neq 0$, the cold electron mode can be unstable at smaller values of λ_e , the hot plasma mode may be unstable at higher values of λ_e . In figures 2 to 4 we have shown some results for growth rates as calculated from equation (1).

From figure 2 it is clear that the growth rates are maximum at some particular values of λ_e . For $(N_C/N_H) = 0$ (cf. solid curves 1, 5, 6) the unstable mode is the hot plasma mode whereas for finite values of N_C/N_H shown here, the cold electron mode becomes unstable (cf. curves 2, 3, 4, 7, 8 and 9). Further we note that the growth rate decreases with the increase of N_C/N_H irrespective of whether the instability is driven by temperature anisotropy or the drift speed of electrons. The growth rate increases with the decrease of $(T_P/T_\perp)$ (cf. curves 5 and 6).

Figure 1. Variation of R with λ_e , for $N_C/N_H = 0.0, 0.1, 0.2, 0.3$ and 0.5 for the curves 1, 2, 3, 4 and 5 respectively. For all curves $T_{||}/T_{\perp} = 0.5$, $T_P/T_{\perp} = 2.0$, $\cos \theta = 0.1$, $\beta_{\perp e} = 1.5$, $V_0/a_{Le} = 0.0$.

Figure 2. Variation of normalized growth rate γ/Ω_p versus λ_e for the parameters $\beta_{\perp e} = 1.5$, $\cos \theta = 0.1$ and $T_p/T_{\perp} = 2.0$. For the curves 1, 2, 3 and 4 $T_{\perp}/T_{\parallel} = 1.0$, $V_0/a_{\perp e} = 0.3$ and $N_C/N_H = 0.0, 0.1, 0.5$ and 1.0 respectively. For the curves 6, 7, 8 and 9, $V_0/a_{\perp e} = 0.0$, $T_{\perp}/T_{\parallel} = 0.5$ and $N_C/N_H = 0.0, 0.5, 1.0$ and 10.0 respectively. For curve 5, $V_0/a_{\perp e} = 0.0$, $T_{\perp}/T_{\parallel} = 0.5$, $T_p/T_{\perp} = 0.5$ and $N_C/N_H = 0.0$. In this figure as well as in figures 3 and 4 the solid portion of curves corresponds hot plasma mode whereas the dashed portion denotes the cold electron mode. For the curves 1, 2, 3 and 4, the ordinate scale has been multiplied by a factor of 10 to avoid the crowding of the curves.

Figure 3. Variation of maximum growth rate γ_m/Ω_p (maximised with respect to λ_e) versus electron temperature anisotropy. For the parameters $V_0/a_{\perp e} = 0.0$, $T_p/T_{\perp} = 2.0$ and $\cos \theta = 0.1$, for curves 1, 2 and 3, $N_C/N_H = 0.0, 0.1$ and 1.5 respectively. For curves 4, 5 and 6, $\beta_{\perp e} = 1.5$ and $N_C/N_H = 0.1, 0.5$ and 10.0 respectively. The curves in this and the following figure, have been truncated when the parameter $\omega^2/k_{\perp}^2 a_{\perp e}^2$ exceeded the value 0.1.

Figure 4. Variation of γ_m / Ω_{ce} with the bouncing electron drift velocity for the parameters $T_{\parallel}/T_{\perp}=1.0$, $T_p/T_{\perp}=2.0$. For the curves 1, 2 and 3 $N_c/N_H=0.0$, $\beta_{\perp e}=0.2$ and $\cos \theta=0.05, 0.1$ and 0.2 respectively. For curve 4, $\beta_{\perp e}=1.0$ and $\cos \theta=0.1$. For curves 5 and 6, $\beta_{\perp e}=1.0$, $\cos \theta=0.1$ and $N_c/N_H=0.5$ and 1.0 .

In figures 3 and 4 we have shown the variation of growth rates maximised with respect to λ_e against the temperature anisotropy and drift speed of the electron beams. In both the figures the maximum growth rate γ_m occurs for $\lambda_e \sim 0.1$, and the curves are truncated whenever the parameter $|\omega| k_{\parallel} a_{\parallel}|$ exceeds the value 0.1. From figure 3 it is clear that γ_m decreases as the temperature anisotropy decreases [i.e., as $(T_{\parallel}/T_{\perp}) \rightarrow 1$]. The increase of $\beta_{\perp}$ results in an increase in γ_m (cf. curves 1, 2 and 3). From figure 4 it is clear that γ_m increases with the increase of V_0 and $\cos \theta$ (cf. curves 1, 2 and 3). The increase of N_c/N_H excites the unstable cold electron mode (dashed curves 4, 5 of figure 3 and 5 and 6 of figure 4) and the growth rate decreases with N_c/N_H .

4. Discussion

From the simplified analytical solutions of equation (1) given in §§ 3.1 and 3.2 and from the numerical results presented in § 3.3, it is clear that the non-resonant electromagnetic lower hybrid instability can exist in a system containing cold electrons, hot isotropic protons and the hot anisotropic bouncing electron beams. In the absence of cold electrons, the only hot plasma mode, which is purely electromagnetic with no electrostatic counterpart, is excited (Lakhina 1980). The addition of cold electrons stabilises the hot plasma mode by raising the threshold values of temperature anisotropy or the bouncing electron drift speed, required for the excitation of instability. However, the mere addition of cold electrons generates a new mode, namely, the cold electron mode, in the system. This mode is found to be stabilized in the electrostatic limit $C^2 k^2 / \omega_{pH}^2 \rightarrow \infty$ but becomes unstable under the electromagnetic limits $C^2 k^2 / \omega_{pH}^2 \ll 1$, in the parametric range where the hot plasma mode becomes stable. The hot plasma mode and the cold electron mode are never unstable simultaneously. Further the increase of N_c/N_H reduces the growth rate of the cold electron mode. Since the cold electrons do not possess any free energy, it is obvious that they cannot increase the growth rate. The free energy for the instability comes from the anisotropy

or drift of the hot electron distributions. However, it is not possible to give a simple physical mechanism for the decrease of growth rate with an increase of N_c/N_H for the cold electron mode because the cold electrons and hot electrons terms are coupled in the dispersion relation (11). The reduction in the growth rate may possibly be due to neutralisation of the wave electric field by the addition of cold electrons. For large values of cold electron density $N_c/N_H \approx 10^2$, the growth rate becomes smaller than Ω_p and the assumption of unmagnetized protons breaks down. This can happen at altitudes $< 1 R_E$ on the auroral field lines where $N_c/N_H > 10^2$. However, the cold electrons density decreases as the altitude increases (L being fixed) and at geostationary altitudes $\sim 6.6 R_E$, the cold electron density is usually much smaller than the hot electron density. Hence the electromagnetic lower hybrid instability, according to our theory, can occur on the auroral field lines at altitude $\geq 1 R_E$ and the growth rate would increase with the altitude.

In the rest frame of the plasma, the instability studied here has a zero real frequency, i.e. it is purely growing in contrast to the field aligned current case where the instability has a real frequency $\omega_r \simeq k_{\parallel} V_0$ (Lakhina 1980). Hence the instability will generate a zero frequency turbulence in the plasma rest frame, in the region of bouncing electron beams in the magnetosphere. The turbulence would be most intense in the equatorial region where the growth rates are largest. The scattering of beam electrons due to such turbulence would lead to wider cone width at the equatorial region than expected on the basis of adiabatic motion of the electron beams from auroral region ($\sim 1 R_E$ altitude) to the geostationary altitudes. Since the instability is purely growing, it can affect the proton dynamics either by direct heating or by pitch-angle scattering leading to their precipitation.

It is important to note that during the magnetically disturbed periods, the convection electric field increases and the whole of the plasma experience an inward drift $\mathbf{V}_p = \vec{E} \times \vec{B}/B^2$. In such a case, in an inertial frame of reference, the instability will have a frequency $f = \mathbf{K} \cdot \mathbf{V}_p/2\pi$ Hz. Since the maximum growth rates occur typically for $\lambda_e \sim 10^{-2} - 10^{-1}$; we can take $k_{\perp} \sim (0.1 - 0.3) \rho_e^{-1}$. Then for $\mathbf{V}_p \simeq (5 - 10)$ km sec $^{-1}$, $\rho_e \sim (1 - 2)$ km, the frequencies generated would be $f \approx (0.05 - 0.5)$ Hz, which lies in the range of Pc1 to Pc3 magnetic pulsations. The wavelength associated with these pulsations is $\lambda_{\perp} \sim (3 - 10) \rho_e \approx (3 - 20)$ km which is much smaller than the ionospheric height; therefore these waves would be completely shielded by the ionosphere (Hughes and Southwood 1976; Hasegawa 1979). It would, however, be possible to observe such waves by the satellites above $1 R_E$ during magnetic storms or substorms. We may point out that during disturbed magnetospheric conditions, the electric field does not remain stationary thereby causing a polarization drift in addition to $\mathbf{E} \times \mathbf{B}$ drift discussed above. The polarization drift is usually one order of magnitude smaller than $\mathbf{V}_p$ and therefore it may not change the frequencies of the pulsations to any appreciable extent.

Before concluding, we may point out that our choice of bi-Maxwellian equilibrium distribution function for hot electron beams seems to violate the H -theorem which demands that the equilibrium distribution function should be such that H has a minimum and the entropy has a maximum value. In other words, the

equilibrium distribution function ought to be a Maxwellian rather than a bi-Maxwellian type. This apparent violation of the H -theorem is justified when the relaxation time for the distribution is much larger than the instability growth time, γ^{-1} . For the hot electrons with $N_H \simeq (1 - 10) \text{ cm}^{-3}$ and energies $\simeq 1 \text{ keV}$, the relaxation time is typically $\tau_{ee} \simeq (10^8 - 10^7) \text{ sec}$ (τ_{ee} being the electron-electron collision time), whereas $\gamma^{-1} \simeq (10 \Omega_p)^{-1} < 1 \text{ sec}$ for the magnetosphere. Hence the choice of bi-Maxwellian distribution for the hot electrons is valid.

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