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An application of complex demodulation technique to geomagnetic data and conductivity anomaly studies

A K AGARWAL, B P SINGH and N NITYANANDA
Indian Institute of Geomagnetism, Bombay 400 005

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Abstract. The method of complex demodulation has been used here to compute the amplitude and phase of a signal present in a geomagnetic series using, Banks' method. It is found that the results are quite accurate when four or more continuous cycles of the signal of interest are present. The same limitations have also been observed when band-pass filters are used to isolate a signal. The information on phase is always correct. It is concluded that complex demodulation will give correct results for periodic variations like 11-year, 27-day, Sq or pulsations; but will give small values of amplitude for signals like sudden impulses or bays. The latter is not a limitation in conductivity studies where ratios like Z/H, Z/D or H/D are mostly used in calculations. It has been shown that even with the records of one magnetic storm the cause of anomalies can be accurately identified which otherwise would need a large number of events.

Keywords. Time series analysis; complex demodulation; geomagnetic deep sounding; conductivity anomalies.

1. Introduction

Isolation of signals of interest is the first step in many geophysical studies. In geomagnetism such variations range from periods of a few seconds to long-period phenomena such as the 27-day cycle, 11-year cycle, etc. The geomagnetic time-series consists of records of the components of the geomagnetic field (H, Z and D) digitised at the desired interval ($\triangle t$). The series may contain not only a finite number of oscillations but also all possible frequencies ranging from zero (linear trend) to the Nyquist frequency ($\frac{1}{2}\triangle t$). Several techniques developed by Lagrange, Buys-Ballot, Whittaker, Arthur Schuster and many others or the newer methods like Blackman-Tukey approach, fast-Fourier transform (FFT) and the concept of maximum entropy are used to obtain 'hidden periodicities' in a time-series. The most widely used techniques for the spectral analysis are the Blackman-Tukey and the FFT. These methods provide a measure of power at different frequencies ranging from zero (linear trend) to the Nyquist frequency. The spectral estimates thus obtained are essentially an average parameter for they give the integrated effect of a frequency component during the whole span of the analysed event.

Sometimes, we require the temporal changes in the amplitude and phase variations of the oscillation of interest during the period of the event. Band-pass filters are the common means of obtaining the variation of amplitude with time. For the same purpose Banks (1975) suggested a new and simple technique known as complex demodulation. The demodulates can be obtained more efficiently by the use of FFT. His method consists of first taking the spectrum through FFT of the original time-series. The frequency band around the peak of interest is then chosen which is later smoothened with a suitable discrete function. The centre of the band is shifted to zero and a new series formed after combining the positive and negative frequency components is subjected to inverse Fourier transform to obtain the demodulates. The advantage of demodulates over raw data, is that the former gives the uncontaminated amplitude and phase variations of a desired signal with time, whereas the raw data are a combination of several signals superimposed on noise due to which the correct amplitude and phase of any desired signal cannot be estimated. In the present paper we discuss the application of this method in the geomagnetic time-series and the geomagnetic deep sounding (GDS). The importance and limitations of the method of complex demodulation are highlighted by comparing results from this analysis with the results obtained from standard methods. Our results show that the present method is very useful for analysis of geomagnetic time-series.

2. Method

Complex demodulation technique mainly consists of three steps. First the time-series is reduced to zero mean, its ends are tapered using cosine bell weights such as $\frac{1}{2}\,[1-\cos(\pi(M-L)/J)]$ where $L = 0, 1, \cdots (J-1)$ and $J = M/10$ over the first and last 10% of the total data points M. The tapered series is extended by adding zeros towards the end to bring the number of data points N equal to 2^K where K is an integer. Then the FFT analysis is applied to obtain the spectrum of this time-series.

In the second step, a frequency band ($\triangle\omega$) around the peak of interest (ω') is selected. The series (both real and imaginary parts of the Fourier transform) in frequency domain is filtered by using a cosine bell weight of the following form :

$$W(\omega) = \tfrac{1}{2}\,(1 + \cos 2\pi(\omega - \omega')/\triangle\omega), \tag{1}$$

in the domain $(\omega' - \triangle\omega/2) \leqslant \omega \leqslant (\omega' + \triangle\omega/2)$ and zero elsewhere. The filtered series is now shifted to zero central frequency to have both positive and negative frequency components. The negative half of the frequency component is shifted to the positive side so as to avoid any overlap between the two halves. Finally, the new series thus obtained is subjected to inverse Fourier transform to obtain the demodulates in the time domain at the central frequency ω'. The sampling interval of the demodulated series is

$$\triangle t' = N \times \triangle t/p,$$

where N is the number of data points in original time-series after adding zeros, $\triangle t$ the data interval of digitised series and p, the number of data points in the frequency domain. The above process can be adopted for any number of peaks in the frequency spectrum. We have tested our computer programme by forming a time-series having four cycles, two cycles, one cycle and half-cycle of a sinusoidal variation of angular frequency of $\omega_0 = 2\pi/16$ and the total number of data points was extended to 512. The data interval ($\triangle t$) was taken as one and initial phase as zero. The series is shown in figure 1. We have also added zeros between blocks of cycles to check the effectiveness of the method. The series was subjected to the above method for obtaining the demodulates. The sampling interval of the demodulated series is 16 and the total number of data points in the frequency domain was 32. The amplitude and phase variations of the demodulated series are shown in figure 1. It is seen that whenever there are changes in amplitude and phase in the original series, corresponding changes are also seen in the demodulated series as suggested by Banks (1975). The maximum amplitude in the demodulated series is half of that in the original time-series only when the number of cycles are four or more. An important result of our analysis is that for the regions in the original

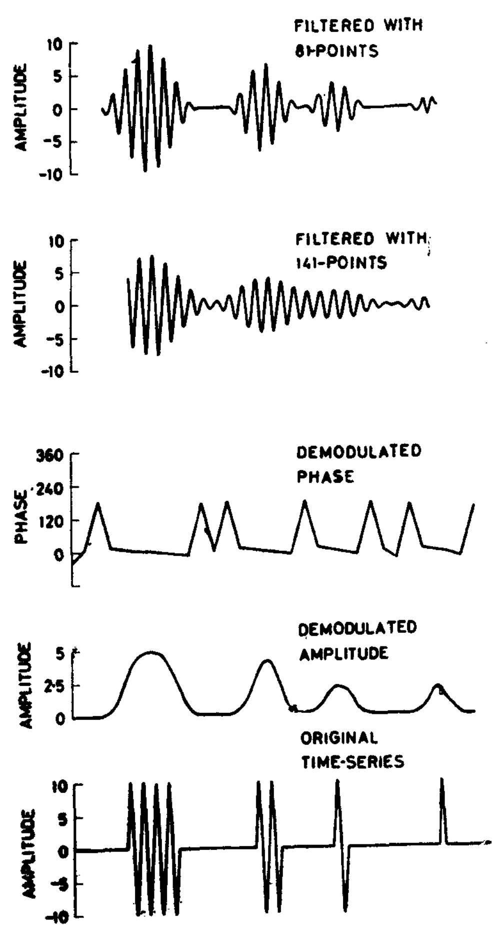


Figure 1. Amplitude and phase variations obtained through the use of complex demodulation together with the amplitude variations of the original time-series. The amplitude variations obtained by the use of two band-pass filters having 81-points and 141-points filter weights are also shown.

series having one or half cycle, the amplitude estimate is highly reduced. As far as the phase is concerned the method gives the correct estimate in all cases. In order to ascertain whether a smaller amplitude estimate, when only one or half cycle of the event is present, is a limitation with the method or is more physical in nature, we analy ed the series by subjecting it with two band-pass filters (Behannon and Ness 1966). The two band-pass filters were designed to provide a maximum gain of unity at 16 min and the number of filter weights was 81 and 141. The amplitude variations obtained after filtering the time-series are also shown in figure 1. It is seen that in the filtered series also the amplitude estimate is greatly reduced in the region of one or half cycle. In essence the amplitude is being smeared over a length of the period of the data both in complex demodulation or in the filtered series. This smearing reduces the amplitude of the signal.

Another important point is the resolution of the demodulated series. This can be improved slightly by adding zeros in the centre of the two halves of the negative and positive frequency components in the frequency domain. The addition of zeros improves the resolution through a change in the Nyquist frequency or the sampling interval of the demodulated series. We have studied this aspect by analysing the storm-time variations. Natural data form a better series to test methods because of the presence of noise and sometimes strong signals having frequencies different from the frequency of interest. We selected H, Z and D elements of the magnetic records at Jaipur and Kodaikanal (JPR and KOD) stations for the period from 0000 UT of 10 January 1976 to 0700 UT of 11 January 1976. The variations were digitised at 3-min interval yielding 620 data points. It was filtered with a 141-point high-pass filter having a maximum gain of unity around 188 min and below. The FFT spectra of the filtered series for all three components (H, Z and D) showed a peak around 49·5 min. Corresponding to this peak, the demodulates of H, Z and D elements were obtained by choosing the frequency band from $2\pi/102\cdot4$ to $2\pi/32\cdot7$ min around the central frequency of $\omega' = 2\pi/49\cdot5$, i.e., 16 points on either side of the centre frequency. The sampling interval of the demodulated series was 49·5 min. Figure 2a shows the amplitude variation of H, Z and D elements at both JPR and KOD stations. The same frequency band consisting of 16 points on either side of the centre frequency ω' was also lengthened to 64 and 128 data points in the frequency domain by adding zeros in the centre of frequency band 32 and 96 respectively. Thus the two independent data sets in the frequency domain were subsequently inverse Fourier-transformed to obtain the complex demodulates in the time domain. Figure 2b shows the demodulated series of H, Z and D for the first case where the sampling interval of the demodulated series becomes 24 min. We find that increasing the series two-fold in frequency domain by adding zeros reduces to half the sampling interval. Figure 2c shows the demodulates of H, Z and D for JPR and KOD in the second case where the sampling interval of demodulated series reduces to 12 min. It is seen that the subsequent effect of adding zeros in the frequency domain does not degrade the information contained in the demodulated series as is evident from figures 2a to 2c. However, the addition of zeros does show some improvement in resolution. The other alternative for better resolution could be to increase the number of points by widening the frequency band, i.e., by including more number of points on either side of the central frequency ω' ; but in this case the demodu-

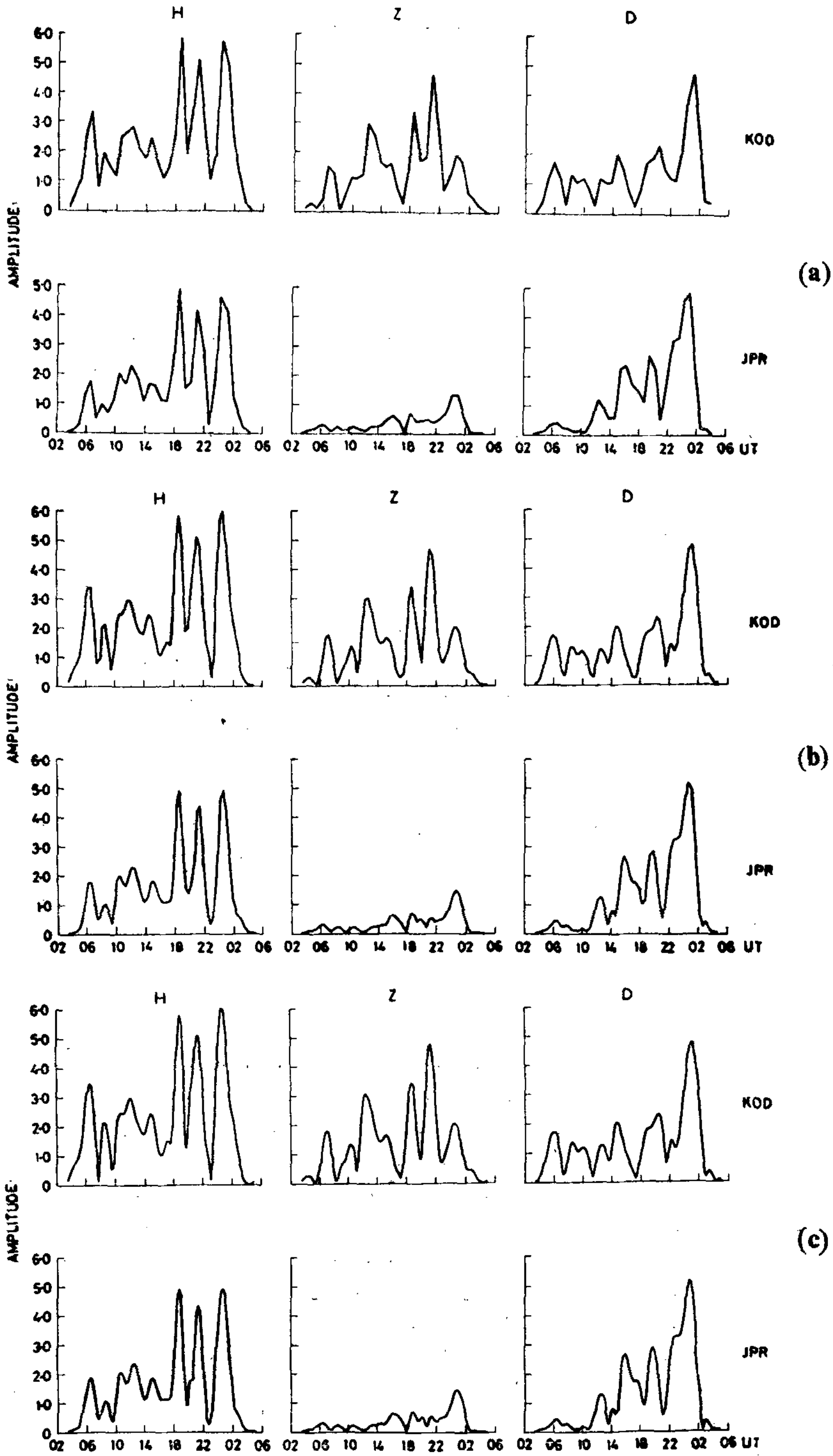


Figure 2. Amplitude variations of *H*, *Z* and *D* elements of 10 January 1976 for the period around 49·5 min at Jaipur and Kodaikanal. The total number of data points at the frequency domain are **a**, 32 **b**, 64 and **c**, 128.

lates will be contaminated with unwanted frequencies. In general, a compromise has to be made between the resolution and the band-width desired.

3. Application and importance

The amplitude and phase estimates are found to be sufficiently accurate if the original series contains four or more cycles of the oscillation. When the number of cycles is less than four, the amplitude estimate is reduced but the phase is still estimated correctly. The method thus is more suitable for continuous oscillations like pulsations, Sq-variations, 27-day variations, 11-year variations, etc. It will give small values for amplitudes when applied to events like bays and storm time-variations. This does not limit the use of the technique if only ratios like Z/H, Z/D or H/D are of importance in computations since the amplitude estimates for all the elements are likely to be reduced equally. The above is the case when induction vectors are to be estimated. These vectors are defined through the relation $Z = AH + BD$ and a reduction of the estimates of H, Z and D amplitudes by the same factor will not affect the result. In the conventional method one would need at least five to six (if not more) strong events before any reliable estimate of this vector could be made and this means a rather long recording period at each site. The induction vectors are important because they point towards the current concentrations causing the anomalies and as such are widely used in identifications of sub-surface conductivity contrasts. In the method used here just from the analysis of a single magnetic storm, the induction vectors can be estimated accurately for a wide range of frequencies. Another important use of this method could be the estimation of the direction of polarisation of the source field as a function of time. Polarisation of the source fields is important both in the study of the external current system and in the conductivity studies of the sub-surface geology.

If we assume that the magnetic variations H and D can be written as

$$D = a_1 \cos \omega t,$$

$$H = a_2 \cos (\omega t + \delta),$$

the point (H, D) will trace out an ellipse with passage of time. The nature and orientation of the polarisation ellipse depend on a_1, a_2 and δ. However, in the present analysis, the polarisation characteristics are defined by introducing three parameters H_ψ, ψ and R_ψ. H_ψ is the magnitude of the major axis, ψ is the azimuth of the major axis and R_ψ the ratio of the lengths of minor and major axes. The relevant formulae, to calculate these in terms of a_1, a_2 and δ, are given in Lilley and Bennett (1972). The new polarisation parameters (H_ψ, ψ and R_ψ) have been introduced keeping in mind their utility in the study of conductivity anomalies.

In the anomalous areas the vertical component Z strongly depends upon the direction of polarisation (ψ) of the inducing field. The dependence of the ratio Z/H_ψ on the azimuth ψ can be of use in locating the strike of conductivity contrast. The vector angle ψ for which Z/H_ψ is maximum will define the possible direction of conductivity contrast.

So far we have assumed that H and D do not contain anomalies. The anomalies in these horizontal components can also be identified if data from closely spaced observatories are used in the study of trend in the variation of polarisation parameters (H_ψ, ψ and R_ψ). Any deviation in these parameters from smooth longitudinal or latitudinal dependence can throw light on internal anomalies in H and D. The anomalies in horizontal components are quantified (Schmucker 1969) through the relation:

$$|B_A| = (H_A{}^2 + D_A{}^2)^{1/2}, \tag{2}$$

where H_A and D_A are the anomalous part of H and D. The parameter $\mathbf{B}_A$ of late has become important in the study of sub-surface geology.

We discuss below a few applications of the method of complex demodulation to establish the points mentioned above.

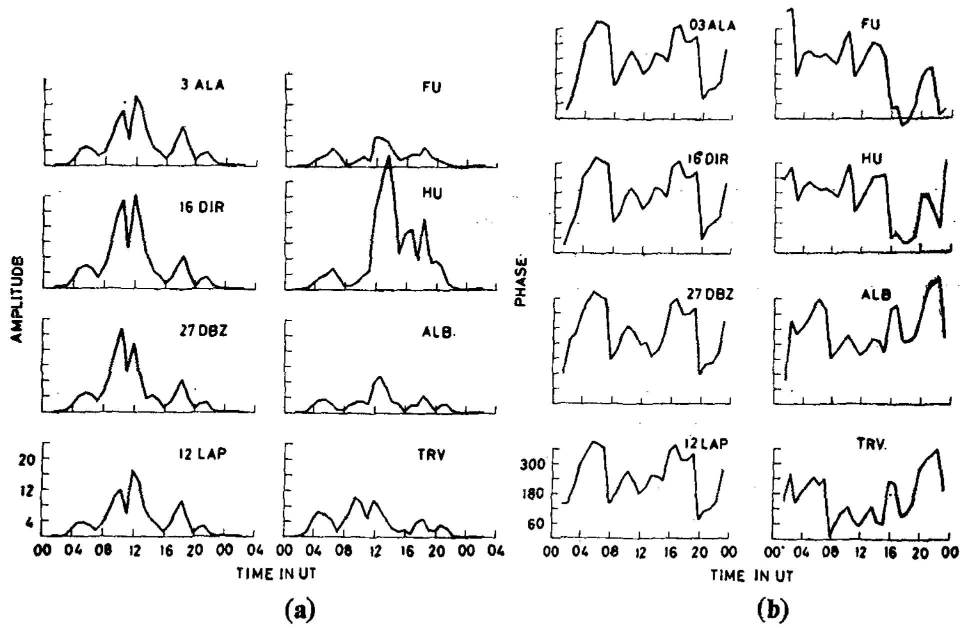
3.1. *Analysis of geomagnetic time-series*

In the first example, we have analysed the series generated from the variations in the H-element during the magnetic storm of 9 April 1971. The record was digitised for the period 0000 UT to 2400 UT at 3 min interval yielding a total 480 data points. Stations selected are Alibag and Trivandrum from Indian region; Huancayo and Fuquene from American region and 03ALA, 16DIR, 27DBZ and 12LAP a set of temporary stations from the Ethiopian region. Each series first filtered with a 61-point high-pass filter (Behannon and Ness 1966) having a cut-off period of about 150 min (at 0·5 gain). These series were then subjected to FFT analysis and this showed a strong peak around 49·5 min in all the records. A narrow band consisting of 16 points on either side of the central frequency $\omega' = 2\pi/49{\cdot}5$ was selected to cover the period range of 102·4 to 32·68 min. This series in the frequency domain was smoothened with the weights defined by equation (1) in the domain $(\omega' - \triangle\omega/2) \leqslant \omega \leqslant (\omega' + \triangle\omega/2)$ and zero elsewhere. The resultant band of frequencies afterwards was shifted to a zero central frequency. The negative half of the frequency components, thus obtained, was moved to the positive side to make a new series. It may be mentioned here that no zeros were added in the centre of the series. The new series of 32 data points so obtained in the frequency domain was inverse Fourier-transformed to give the demodulates of the H-elements. The sampling interval of the demodulated series is 48 min in this case. Figures 3a and 3b show the amplitude and phase variations at different stations in different regions. The day-time events between 0130 UT to 1100 UT for the Indian region and between 1100 UT to 2100 UT for the American region show an expected equatorial enhancement in each region (figure 3a). The enhancement in Indian region (TRV/ALB) is found to be about 2·0 whereas in American region (HU/FU) it is about 4·0. The results corroborate the earlier finding of Rastogi (1962) for daily Sq-ranges that the equatorial enhancement in American zone is much larger than that of Indian zone.

Figure 3b shows the similarity of phase variations in the three regions. The gross features of the phase variations are similar but phase shifts are noticed from one region to another. The results suggest that the source causing these variations is the same for all regions but has some phase difference, which is natural to expect.

3.2. *Estimation of horizontal disturbance vectors* ($\mathbf{B}_A$)

The second example illustrates the use of this programme in conductivity anomaly studies. The horizontal disturbance vectors $\mathbf{B}_A$ (as defined by equation (2)) were



Figures 3a, b. Amplitude and phase variations of H-element of 9 April 1971 at two Indian stations (Alibag and Trivandrum), two American stations (Huancayo and Fuquene) and four Ethiopian stations (03 ALA, 16 DIR, 27 DBE and 12 LAP, the temporary stations).

calculated from the demodulates for the two Indian stations Annamalainagar (ANR) and Trivandrum (TRV) taking Kodaikanal (KOD) as a normal station (Agarwal *et al* 1979). We have used the storm-time variations in H, Z and D elements for the period from 0000 UT of 10 January 1976 to 0700 UT of 11 January 1976. Here also, the series was digitised at 3 min interval and it yielded a total number of 620 data points. A 141 point high-pass filter having unit gain for periods of 188 min and below (Behannon and Ness 1966) was applied to the data. The fast Fourier analysis of this time-series also showed a peak around 49·5 min at all stations. A similar process was adopted to make a new series in the frequency domain for the same frequency band consisting of 16 points on either side of the central frequency $\omega' = 2\pi/49 \cdot 5$ as in the previous case. Then the series was lengthened to 128 data points by adding zeros in the centre of the positive and negative halves of the frequency components. It may be noted that with this change the sampling interval of the demodulated series reduced to 12 min. The complex demodulates of H, Z and D so obtained for the 49·5 min central period are shown in figure 4a. It is seen that the large Z-variations at ANR, TRV and KOD indicate the presence of a strong regional anomaly in the area (figure 4a). Otherwise one would expect a small Z-variations at all these observatories because they fall under low latitude regions. The results confirm the earlier studies of Nityananda *et al* (1975) and Singh *et al* (1977) on anomalies in this area. The comparison of the polarisation ellipses (figure 4b) at these stations, calculated for four events (at 0630, 1840, 2130 and 0400 UT), does not show any systematic trend either in H_ψ, the major axis ψ, the direction of polarisation or in R_ψ, the ratio of minor to major

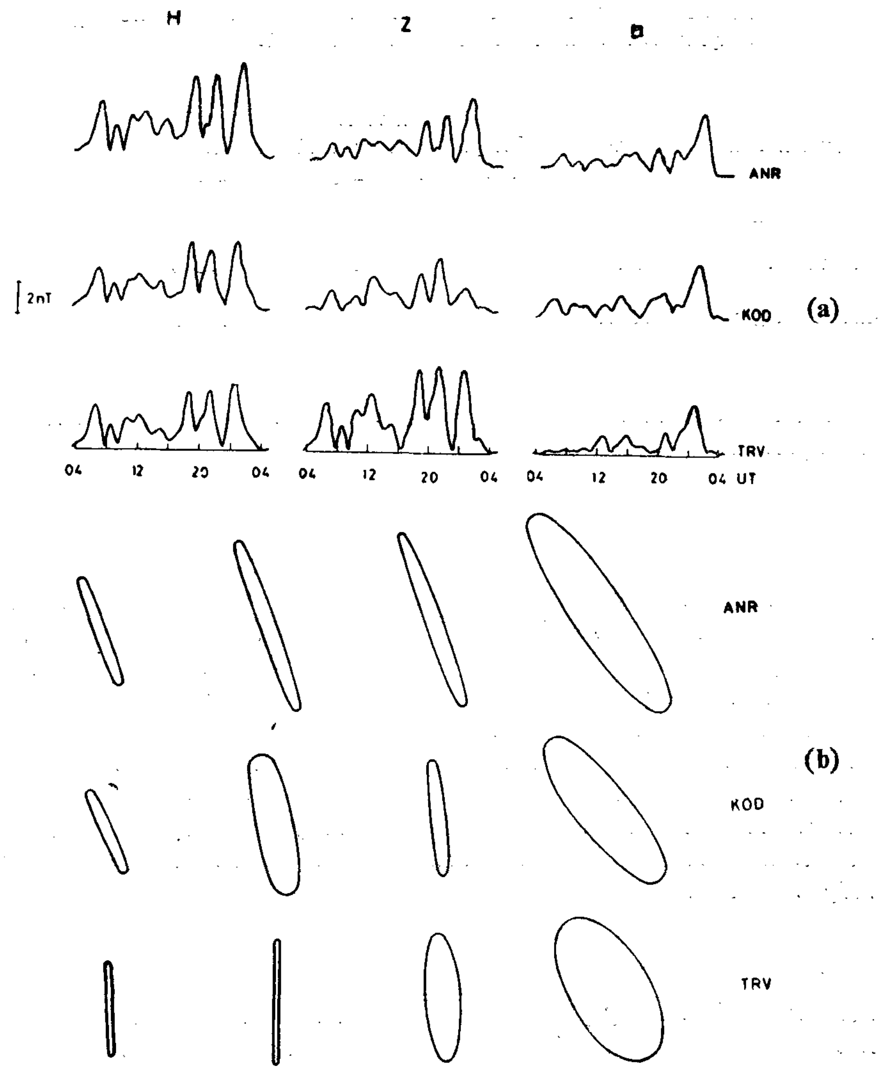


Figure 4 a. Amplitude variations of H, Z and D elements of 10 January 1976 for the period around 49·5 min. at Annamalainagar, Kodaikanal and Trivandrum **b.** polarisation ellipses drawn for the four selected events around 0630, 1840, 2130 and 0400 UT at these stations.

axes. In one of our earlier studies (Nityananda *et al* 1977) we have established that H and D at ANR and TRV are anomalous. The anomalies in H and D at these two stations are identified through the relation :

$$H_A = H_{\text{STN}} - H_{\text{KOD}}$$

$$D_A = D_{\text{STN}} - D_{\text{KOD}}.$$

These anomalies were estimated for the four selected events from the amplitude variations at 1500, 1840, 2130 and 0040 UT (figure 4a). The magnitude of the mean

Table 1. Direction of mean horizontal disturbance vectors ($\mathbf{B}_A$) obtained by complex demodulation and Nityananda *et al* (1977).

Station name	Code	$\mathbf{B}_A$ (full-circle bearings) Complex demodulation	Nityananda *et al* 1977 Bays	sscs
Trivandrum	TRV	116·0°	103·0°	95·0°
Annamalinagar	ANR	335·0°	327·0°	317·5°

horizontal disturbance vector $|\mathbf{B}_A|$ and its direction were then calculated through the equations:

$$|\mathbf{B}_A| = (H_A^2 + D_A^2)^{1/2}$$
$$\theta = \tan^{-1} D_A/H_A.$$

The mean directions of the disturbance vectors were found to be 335° ± 10° and 116° ± 2° at ANR and TRV respectively (Agarwal *et al* 1979). On a rotation of 90° in anti-clockwise direction these vectors point towards the internal currents causing the anomaly (Schmucker 1969). It is noteworthy that the location of the internal currents obtained by rotating the disturbance vectors ($\mathbf{B}_A$) agrees well with the directions found by Nityananda *et al* (1977). Table 1 shows the direction of mean vectors ($\mathbf{B}_A$) obtained by both methods. The result is significant in the sense that we can obtain the same result from the analysis of only one magnetic storm, whereas in the earlier method (Nityananda *et al* 1977) this was determined from the analysis of about 30 bays and storm sudden commencement.

4. Conclusions

The method gives a very good estimate of both amplitude and the phase of a signal as a function of time in a time-series. The estimates are quite accurate if four or more continuous cycles of the signal are present in the series. When the number of cycles is less than four the phase estimate is still correct but the amplitude is underestimated. A comparison of the demodulate and the results from conventional band-pass filter shows that both methods fail when the number of cycles of a signal in the series is less than four.

The method of complex demodulation is also highly effective in conductivity studies. Records of only one magnetic storm may be sufficient to identify the anomalous areas which otherwise would need about five to six strong events in conventional techniques. The phases estimated from this analysis are found to be highly useful in characterising the spatial variation of the source field and also in identifying the anomalies in horizontal components.

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