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Melting of ice cubes under controlled conditions

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Abstract. Weight variations with time during the melting of suspended ice cubes under controlled conditions at various temperatures ranging from 10–45° C have been experimentally studied. The melting data have been used to determine certain parameters leading to the calculation of complete melting time, thickness of the water film on ice surface and the temperature at the interface of air and water films.

Keywords. Ice cubes; melting.

1. Introduction

Mason (1957) calculated theoretically the rates of melting of small ice spheres falling under specified atmospheric conditions. In a later investigation (Drake and Mason 1966), complete melting periods of small ice spheres and cones were measured. Making allowance for the heat content of water retained by the particle during the melting process, it was concluded that the theory which allows for the heat transfer to an ice particle by conduction through the air and condensation upon its surface, agrees quite well with the observations. Heat transfer to ice spheres and freezing of spongy hail (Goyer *et al* 1969) was studied and it was found that the heat transfer to a melting hailstone, whose surface is covered with a film of water, is described by the same equation as that for a freezing hailstone with a dry surface.

The studies referred to above are mainly connected with the melting behaviour of non-dripping ice samples. It is, therefore, desirable to have further information about the melting of ice samples which drip during the melting process. The present paper is an attempt in this direction.

2. Experimental

Ice cubes of doubly distilled water weighing about 15 g with a nylon thread embedded centrally in each of them were prepared in a freezing chamber. An ice cube thus prepared, was suspended from the pan of a sensitive chemical balance kept in a closed chamber with arrangements of controlling the temperature from 5° C to 50° C. In order to provide time for stabilising the experimental conditions inside the chamber, the suspended ice cube was allowed to melt till its weight reached 13·5 g. At this point of time the observations were started and the weight of the melting ice cube was noted at various time intervals till the last portion of the unmelted ice core fell down getting detached from the embedded nylon thread.

During the course of melting the temperature of the surroundings was kept constant with fluctuations limited to $\pm 0\cdot 5°$ C. The melting data thus collected at different temperatures of the surroundings ranging from 10° C to 45° C are tabulated in table 1.

Further, the density of the ice cubes, similar to those used for the experiment in each case, was determined by known methods (Bisque 1967).

3. Results and discussions

When a sizable ice cube, suspended freely in a controlled environment, is allowed to melt, it gets initially surrounded by a thin film of water. As the melting proceeds, the water film is no longer capable of retaining the melt which drops down due to gravitational pull. Keeping in view that the heat required for such melting is supplied by the surroundings only, the heat transfer may be considered to take place through a series combination of the films of air and water interposed between the surroundings at temperature T_a and ice cube at temperature

Table 1. Variation of weight with time at different temperatures for the melting of suspended ice cubes (density = 0·9201 g/cc) in a closed chamber.

w g	*a* cm	Melting time (seconds)							
		10° C	15° C	20° C	25° C	30° C	35° C	40° C	45° C
13·50	1·225	0	0	0	0	0	0	0	0
13·00	1·210	311	185	170	151	129	105	82	61
12·00	1·177	916	649	486	424	346	299	246	189
11·00	1·143	1537	1016	779	645	514	456	401	348
10·00	1·107	2154	1474	1124	923	732	654	556	504
9·00	1·069	2813	1918	1451	1188	953	829	729	652
8·00	1·028	3505	2425	1772	1446	1171	1030	898	801
7·00	0·983	4189	2873	2143	1749	1415	1241	1076	960
6·00	0·934	5011	3367	2516	2051	1656	1462	1262	1136
5·00	0·879	5802	3891	2915	2385	1924	1688	1468	1304
4·00	0·816	6715	4505	3418	2729	2236	1944	1704	1515
3·00	0·741	7618	5139	3790	3071	2531	2189	1910	1703
2·00	0·647	8544	5756	4264	3478	2825	2481	2156	1911
1·00	0·515	9845	6610	4876	3995	3258	2849	2481	2208

T_0. Even if the thickness of such films on all sides of the melting ice cube may not be uniform everywhere at any instant of time during melting, it is expedient to assume an average thickness for air film as well as the water film without much loss of accuracy (Mason 1957; Drake and Mason 1966). Also for the steady state conditions, the rate of heat flow q through the air and water films is constant. Thus, if the temperature at the interface of air and water films is T_s, the sidelength of the unmelted cubical ice core at any time t is $2a$, and the value of average surface areas of air and water films are taken as the respective geometric means of inner and outer surface area in each case, then according to Drake and Mason we have,

$$q = -24(a+d)(T_s - T_a)k_aC = -[24\,a(a+d)(T_s - T_0)k_w]/d \tag{1}$$

where d is the average thickness of the water film, k_a and k_w are the average thermal conductivities of the air and water films respectively, and C is the ventilation coefficient which takes into account any air movement inside the closed chamber

Further, the heat required for the weight of the portion of ice cube melted per second at time t is given by,

$$q' = 24\,L_f\rho_i a^2 \cdot (da/dt), \tag{2}$$

where L_f is the latent heat of fusion of ice and ρ_i is the density of ice. If we assume that the air in the immediate vicinity of the ice cube gets water-saturated, with neither condensation nor evaporation occurring at the ice surface, and further consider that the entire heat transfer through the water film takes place by conduction only, then $q = q'$, and accordingly eliminating T_s from equations (1) and (2), we get,

$$-(da/dt)^{-1} = L_f\rho_i\,[a^2 k_w + adk_aC]/[(a+d)\,k_wk_aC\,(T_a - T_0)]. \tag{3}$$

Now putting $(a+d) = a'$, equation (3) after neglecting higher powers of d, reduces to

$$-(da/dt)^{-1} = L_f\rho_i a'/[k_aC\,(T_a - T_0)] - 2L_f\rho_i d/[k_aC\,(T_a - T_0)] \times [1 - k_aC/2k_w)]. \tag{4}$$

In order to evaluate a' from the melting data, we have,

$$w = 8\rho_i a^3 + 8\rho_w[(a+d)^3 - a^3], \tag{5}$$

where w is the weight of the melting ice cube with sidelength $2a$ of the unmelted cubical ice core at any time t and ρ_w is the density of the water in the surrounding water film at temperature T_s. Taking $\rho_w/\rho_i \simeq 1$, neglecting higher powers of d and rearranging, we get from equation (5),

$$(w/8\rho_i)^{1/3} \simeq [a + (d\rho_w/\rho_i)] \simeq (a+d) = a'. \tag{6}$$

3.1. *Determination of d, t_m and T_s*

Using the melting data of table 1, a' was calculated from equation (6) and plotted against t. From the curves so obtained, the values of (da'/dt) at various time intervals were determined by standard methods. The plot of $-(da'/dt)^{-1}$ vs a' (figure 1) gave a satisfactory straight line showing that,

$$-(da'/dt)^{-1} = B_1 a' + B_2 \tag{7}$$

where B_1 and B_2 are constants. Comparing equations (4) and (7) we get,

$$C = L_f \rho_i / [B_1 k_a (T_a - T_0)] \tag{8}$$

and,
$$d = -B_2 \Big/ \left\{ 2B_1 \left[1 - \frac{L_f \rho_i}{2B_1 k_w (T_a - T_0)} \right] \right\} \tag{9}$$

Determining the values of B_1 and B_2 from the slope and intercept respectively of the corresponding straight line in figure 1 while taking $\rho_i = 0{\cdot}9201$ g/cc (as determined experimentally) and $L_f = 80$ cal/g and further assuming that k_w does not vary appreciably from the thermal conductivity of water at T_0 and approximately equals 0·001324 cal. sec^{-1}cm^{-2}° C^{-1}cm (Gray 1957), the values of d can be calculated from equation (9). The approximation regarding k_w appears amply justified in view of the fact that the melting in the present investigation was allowed to occur with the melt continuously dripping down such that T_s does not differ widely from T_0.

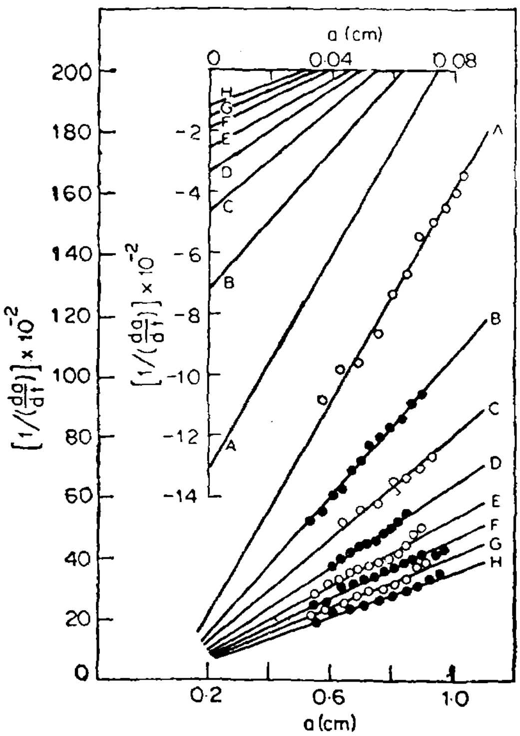

Figure 1. Plot of $[1/(da/dt)]$ vs a at different temperatures. (A) 10° C, (B) 15° C, (C) 20° C, (D) 25° C, (E) 30° C, (F) 35° C, (G) 40° C, (H) 45° C.

Table 2. Values of the parameters for the melting of suspended ice cubes in a closed chamber at different temperatures.

T_a° C	$B_1 \times 10^{-2}$	$B_2 \times 10^{-2}$	d cm	$t_m \times 10^{-2}$ Sec	T_s° C
10	175·00	13·00	0·0440	115·78	0·215
15	114·00	7·20	0·0377	74·83	0·295
20	84·50	4·60	0·0326	57·87	0·337
25	67·75	3·35	0·0296	46·80	0·383
30	55·55	2·45	0·0264	38·76	0·412
35	48·20	1·90	0·0236	33·87	0·429
40	42·30	1·50	0·0212	29·92	0·438
45	36·36	1·15	0·0180	25·89	0·468

In order to determine complete melting time t_m equation (7) was integrated with the boundary conditions $a' = b$, for $t = 0$, and $a' = d$, for $t = t_m$, giving

$$t_m = \tfrac{1}{2} B_1 (b^2 - d^2) + B_2 (b - d). \tag{10}$$

Equating q with q' from equations (1) and (2) and integrating with the boundary conditions as used for the integration of equation (7), we have,

$$T_s = [L_f \rho_i d / t_m k_w] [(b - d) - d \ln (b/d)]. \tag{11}$$

Finally, substituting the value of k_w from equation (9) in equation (11) we get,

$$T_s = 1/t_m [(b - d) - d \ln b/d] (2B_1 d + B_2)(T_a - T_0). \tag{12}$$

The values of B_1 and B_2 as determined graphically from figure 1, d from equation (9), t_m from equation (10) and T_s from equation (12), for various values of T_a in the range of $10° \text{C} < T_a < 45° \text{C}$ are included in table 2. As expected the values of d and t_m decrease whereas those of T_s increase with the increase of T_a.

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