Spectral analysis of financial threshold networks

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Abstract. This work involves spectral graph theory to study the properties of the financial threshold networks derived from the cross-correlations between 30 global financial indices. A weighted threshold network from the financial time series is created, and network behavior is parametrized by graph spectral features. The spectral radius of network shows that the system is more correlated at the time of financial stress and hence more prone to failure compared with a calm period. The principal eigenvector of the weighted adjacency matrix shows that the European and American markets have higher control over the system which increases with an increase in the threshold. At a sufficiently higher threshold, the European and American markets completely dominate the system state in comparison with the Asia–Pacific markets. The sign of components of the second-largest eigenvector of the weighted adjacency matrix can cluster nodes based on their geographical location. The two main clusters are European–American markets and Asia–Pacific markets. The Fiedler eigenvector shows that at the time of financial stress the nodes within a close geographical location show higher interaction with each other than the members of the other cluster. The inter-cluster interaction increases at the time of less financial stress.

Keywords. Threshold networks; financial systems; networks; spectral analysis.

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1. Introduction

Financial systems are complex evolving systems characterized by various hidden interactions and factors. The past few decades have experienced huge growth in studies focusing on understanding the complex nature of financial systems [1–6]. Most of the studies are focused on the calculation of correlation between change of price of financial commodities [1, 3], use of random matrix theory (RMT) [1, 7, 8] and network theory [4, 5]. Complex network and RMT methods are not only used in the field of econophysics but also are actively used in various other fields such as biology [9, 10], chemistry [11, 12], wireless communication [13], social systems [14], water distribution systems [15] and many others. There is a huge surge in algorithms and mathematical tools in complex network theory to understand the structure and evolution of complex systems [3, 10, 14, 16]. The spectral analysis of the complex network is an important methodology, aimed at exploiting the topological properties of graph matrices and is applied to diverse areas with application ranging from computer vision, pattern recognition, internet search, linguistic, internet topology, biology, water distribution, epidemiology [15, 17–20]. Spectral graph theory is an approach that combines linear algebra with graph theory to exploit the properties of eigenvalues and eigenvectors of graph matrices. The graph spectrum carries information about local as well as global graph structures.

In this work, we perform a spectral analysis of the graph matrices of the weighted threshold network constructed from the correlation matrices of global financial indices. The system is analyzed by the spectral features of the weighted network and local as well as the global behavior of the system is analyzed. The system is studied under different periods of high and low financial stress. We give a method for efficient information extraction from the correlation matrices, filtering information with a threshold. We study a number of graph spectral properties such as spectral radius, network energy, spectral gap, community structure, and graph partitioning in the context of financial networks. This paper is organized into six sections. Section
describes the system and data. Section 3 discusses the construction of the network. Section 4 explains the matrices associated with the network. Spectral analysis is done in section 5, and conclusions are given in section 6.

2. System and data

In this work, we analyze the daily close prices of 30 global financial indices around the world from 1 January 2006 to 31 December 2015 for a span of 10 years. The daily close price data is downloaded from yahoo finance (https://finance.yahoo.com/). The global indices can be divided into three categories, the European, Asian, and American markets. The details of the indices are given in table 1. We divide the complete period from 2006 to 2015 into segments of world’s major financial events as

1. January 2006 to November 2007: time period just before the 2008 crisis
4. April 2010 to September 2012: European sovereign debt crisis
5. October 2012 to December 2015: calm period

We also perform our analysis on the full period from 2006 to 2015, termed as ‘complete period’. The stock data are first filtered according to the rule that for a particular day if less than 50% of the market is closed then that day is removed from our analysis. Therefore for our analysis, we take only those days for which more than 50% of the indices trade.

The data are filtered and processed through a series of steps to remove any numerical artifacts. For index $i$, with price $P_i(t)$ at time $t$, the daily logarithmic returns $R_i(t)$ is calculated as

$$R_i(t) = \ln(P_i(t + \Delta t)) - \ln(P_i(t))$$  \hspace{1cm} (1)$$

where $\Delta t = 1$ day is the time lag. The normalized logarithmic returns are defined as

$$r_i(t) = \frac{R_i(t) - \langle R_i \rangle}{\sigma_i}$$  \hspace{1cm} (2)$$

where $\langle R_i \rangle$ is the average of return $R_i(t)$ over time period and $\sigma_i$ is the standard deviation of $R_i(t)$ given by $\sigma_i = \langle R_i^2 \rangle - \langle R_i \rangle^2$.

2.1 Correlations

Correlation between different financial indices is studied by calculating Pearson’s correlation coefficient from the normalized log returns. The Pearson correlation coefficient between two indices $i$ and $j$ with normalized log returns $r_i(t)$ and $r_j(t)$ is defined as

$$C_{ij} = \langle r_i(t) r_j(t) \rangle.$$  \hspace{1cm} (3)$$

The correlation coefficient lies in the range $-1 \leq C_{ij} \leq 1$ where $C_{ij} = 1$ represents perfect correlation, $C_{ij} = -1$ indicates perfect anti-correlation and $C_{ij} = 0$ corresponds to no correlation between index $i$ and $j$. The correlation matrix for the complete period 2006–2015 as well as each window is calculated. Figure 1 shows how the correlation structure between each pair of indices evolves with window (time). For example, the system is positively correlated at the time of 2008 financial crisis and European sovereign debt crisis, whereas during other time periods, the correlation coefficient between pairs of indices significantly decreases. To separate noise from the actual correlations in the system, a null model is created by random shuffling of time series for each index. An ensemble of correlation matrices is created from the random shuffled matrices. The random correlation matrices are known as Wishart matrices and their properties are well studied [21, 22].

We compare the probability density function (PDF) of $\sqrt{(N-1)/2}$ independent off-diagonal elements of the correlation matrix for different financial windows with an ensemble of random correlation matrices (Wishart matrices) as shown in figure 2. We find that the PDF

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Figure 1. Heat map of correlation matrix for the 30 global financial indices from different financial periods.

for all windows is a non-symmetrical distribution with a high positive mean as compared with the random distribution. Most of the mass of PDF is concentrated on the positive side of distribution which shows that the global financial indices are positively correlated with each other. At the time of the crisis (European sovereign debt crisis or global financial crisis 2008), the distribution shifts towards positive correlations and indicates that the market at times of crisis are highly correlated.

3. Network construction

We construct an undirected weighted network for each window based on the correlation matrix using the threshold method. The global financial indices are treated as nodes and the correlation between them as edge that connect two nodes. Since the network is undirected and weighted, each edge between nodes $i$ and $j$ carries symmetric weight ($w_{ij} = w_{ji}$). The strength of the link is given by the correlation coefficient between the two nodes. In the given construction, only those links which have a strength higher than the given threshold are retained in the network. Different thresholds give different networks of financial interaction with the same set of nodes but different set of edges. The variation of properties of the network with the window and threshold are studied to analyze the topology and evolution of the correlation network.

For a given window, the threshold network is given by a graph $G(N, E)$ with indices as nodes in the network. The edge $E_{ij}$ between two nodes $i$ and $j$ is given by

$$E_{ij} = \begin{cases} C_{ij} & \text{if } i \neq j, \quad |C_{ij}| \geq \theta \\ 0 & \text{otherwise} \end{cases}$$

Figure 2. The PDF of $C_{ij}$ of global financial indices for different windows.

where $\theta$ is the threshold value and $C_{ij}$ is the Pearson correlation coefficient between positions $i$ and $j$ for a given window. Threshold network for 30 global financial indices at different threshold is shown in figure 3. The network is shown for three financial periods, global financial crisis 2008, European sovereign debt crisis, and calm period. We observe that the indices are more closely connected during the time of financial crisis. During the calm period for a higher threshold, the network greatly reduces to only a few interacting nodes. The size of the giant component at 0.6 threshold is much bigger for the crisis period than the calm period.
4. Graph matrices

Given a network (or graph), it is possible to define various graph matrices associated with it, the most commonly used matrices are the adjacency matrix $A$, weighted adjacency matrix $W$, Laplacian matrix $L$, normalized Laplacian matrix $L_n$, incidence matrix, etc. In the current analysis, we use the adjacency matrix $A$, weighted adjacency matrix $W$, and the Laplacian matrix $L$ for the analysis.

The adjacency matrix $(A)$ is a square $N \times N$ matrix such that its element $A_{ij}$ is equal to one if there is a link between nodes $i$ and $j$ and zero if $i$ and $j$ are not connected. Since the network is un-directed, the adjacency matrix $(A)$ is symmetric such that $A_{ij} = A_{ji}$.

If the network is a weighted network, then the link connecting two nodes contains weight. The matrix of these weights is called the weighted adjacency matrix. The edge weights are represented as the weight matrix $W$ such that $w_{ij}$ is the strength of the link between nodes $i$ and $j$. For the threshold network, for a given threshold ($\theta$) and correlation matrix $C$, the element of weighted adjacency matrix $W$ is given by

$$w_{ij} = \begin{cases} C_{ij} & \text{if } A_{ij} = 1 \\ 0 & \text{otherwise.} \end{cases}$$

Since the network is un-directed, the weighted adjacency matrix is symmetric. The number of nonzero elements in $W$ decreases with an increase in the threshold. The adjacency matrix is a binary approximation of the weighted adjacency matrix. The unweighted graphs are a special case of weighted networks with weights either 0 or 1.

Given an undirected graph $G(N, E)$, the Laplacian matrix $(L)$ is defined as $L = D - A$ where $D = \text{diag}(d)$
is a diagonal matrix created with degree $d_i$ of node $i$ at diagonal element $D_{ii}$. For a weighted network, $L = D - W$, where $W$ is the weighted adjacency matrix. The off-diagonal element of $L$ takes value $-W_{ij}$ for weighted network ($-1$ for un-weighted network) for a pair of connected nodes with the diagonal elements equal to degree $d_i$. Laplacian matrix is widely used in spectral graph theory [23].

For a graph, we can also define the normalized Laplacian matrix as $L' = D^{-1/2}LD^{-1/2}$ [24, 25]. Normalized Laplacian is widely used to describe the dynamical processes since the effect of neighbors are equally normalized.

5. Spectral analysis

The spectrum of a network is the set of eigenvalues of the matrices (adjacency and Laplacian) associated with the network. Since adjacency and Laplacian are real symmetric matrices of size $N \times N$ with $N = 30$, they have a spectrum of $N$ eigenvalues with corresponding $N$ orthonormal eigenvectors. The spectrum of the adjacency matrix is a graph invariant. Let $\lambda_1^w \leq \lambda_2^w \leq \cdots \leq \lambda_N^w$ be the eigenvalues of the weighted adjacency matrix $W$. For each window the eigenvalues $\lambda_i^w$ (where $i = 1 \ldots N$) and the corresponding eigenvectors $\phi_i$ are determined for the weighted adjacency matrix. The largest eigenvalue ($\lambda_N^w$ or $\lambda_{\text{max}}^w$) of the adjacency (weighted) matrix is known as the principal eigenvalue or spectral radius of the network. The eigenvector associated with this eigenvalue ($\lambda_N^w$) is known as the principal eigenvector.

Laplacian matrix is another important matrix that is used in the spectral graph theory. The eigenvalues of the Laplacian matrix $L_i$ are all real and non-negative with bounds $0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N \leq 2d_{\text{max}}$, where $d_{\text{max}}$ is the maximum degree in the network. The smallest eigenvalue $\lambda_1 = 0$ is a trivial eigenvalue of $L$, with the constant eigenvector components. The eigenvalues of normalized Laplacian matrix are bounded by $0 \leq \lambda_i' \leq 2$. For a regular graph i.e. a graph with all nodes having the same degree $d$, the eigenvalues of Laplacian matrix and normalized Laplacian matrix are related by a constant such that $\lambda_i = d\lambda_i'$. Normalized Laplacian matrix has been aggressively used in various systems due to their connection with the random walks [24, 25].

5.1 Energy of a graph

The most important application of spectral graph theory is in chemical field due to a correspondence between the graph eigenvalues and molecular orbital energy levels in hydrocarbons [12]. The energy of a molecule corresponds to the sum of the absolute values of eigenvalues of the molecular graph [20]. This sum of the absolute values of the eigenvalue of a network is called graph energy [19, 20] determined from the eigenvalues $\lambda_i^w$ of the adjacency matrix $W$ as

$$ E(G) = \sum_{i=1}^{N} |\lambda_i^w|. $$

Although the graph energy is primarily defined in a chemical context, it is relevant to all types of graphs which led to the development of the theory of graph energy [20]. The energy of a graph $E(G)$ is a graph invariant.

The variation of network (graph) energy with threshold and different financial periods for the global financial indices is studied and shown in figure 4. We find that the energy of the graph is higher for the period of financial stress and decreases for a calm period. The 2008 global financial crisis shows the highest value for graph energy for all thresholds followed by the European debt crisis. The period before the 2008 crash and after 2008 crash shows a small difference in energy but the difference in energy decreases with an increase in the threshold. The calm period shows the lowest energy and indicates a stable market. We find that network energy can be used as a measure to estimate the severity of a financial crisis.

5.2 Robustness of network: Largest eigenvalue

The largest eigenvalue of the adjacency matrix, also known as the spectral radius, is related to the information flow, and the robustness of the network implies the ability of the network to preserve its functionality while facing external perturbations which may lead to failure of nodes or links. In the present correlation network, the node failure means either the country is removed from the network or is in a state of malfunction. The link failure between two countries in the correlation network implies cutting the connection (destroying correlation) between them. The spectral radius quantifies how the
node failure can propagate to affect the whole financial network.

In recent years, the huge interest in the study of the spectral radius is motivated by the relation of the spectral radius with the epidemic threshold $\tau_c$ for the infection rate of a SIS-type network [26]. The epidemic threshold is the inverse of spectral radius i.e. $\tau_c = 1/\lambda_{max}$, for the spread of a contagion (virus) in a network [16]. Smaller value of $\lambda_{max}$ implies a more robust network. Spectral radius not only considers the interaction with neighbors but also accounts for the next nearest neighbors interactions. The information flow or communication in the network is also given by $\lambda_{max}$, which can be understood in terms of the walk. For an unweighted adjacency matrix $\lambda_{max}$ is proportional to the number of walks in the network. A high number of walks implies better communication between nodes in the network, which indicates an efficient flow of information or the substance in the network but at the same time also indicates a less robust network that is prone to the spread of contagion.

Figure 5 shows the variation of $\lambda_{max}$ of weighted adjacency matrix for different financial windows. We find that in a given network, the connectivity and flow are maximum for the 2008 financial crisis, followed by the European sovereign debt crisis for nearly all thresholds. At higher threshold the European sovereign debt crisis and 2008 financial crisis behave in a similar way with nearly identical $\lambda_{max}$. The crisis period symbolizes a more correlated network and hence the information exchange between nodes is higher. These two periods are less robust and are very prone to failure as indicated by a smaller value epidemic threshold ($\frac{1}{\lambda_{max}}$) compared to other time frames. The calm period shows the lowest value of $\lambda_{max}$, therefore, it is more robust against the targeted or random failures. The difference in $\lambda_{max}$ between different windows decreases with an increase in the threshold. Figure 6 shows the variation of epidemic threshold ($\tau_c$) with the network threshold for different financial windows. It is evident from figure 6 that the calm period is more robust against the spread of contagion in a network whereas the crisis period (European sovereign debt crisis and 2008 financial crisis) are weak against the node failures. A smaller perturbation is needed during the crisis period for a system-level crash compared to the calm period.

5.3 Spectral gap $\Delta \lambda$

The difference between the first and the second eigenvalue of the adjacency (weighted) matrix is called the spectral gap. The spectral gap represents the strength of graph connectivity [15] and measures the robustness of network connections. A higher $\Delta \lambda$ means a more robust network in terms of connections. $\Delta \lambda$ also quantifies the bottlenecks and links which act as a bridge between two parts of the network. The significance of $\Delta \lambda$ lies in the fact that targeting bridges in the network will split the network into parts and result in a disconnected network, at the same time correcting the bottlenecks will have a less congested information flow.

Figure 7 shows the change in the spectral gap with the threshold for all financial windows. At a low threshold, all windows show nearly the same value of spectral gap which changes sharply with the change in threshold. At a higher threshold (0.7) financial window corresponding to the European debt crisis shows the highest value of $\Delta \lambda$. The calm period shows the lowest value of $\Delta \lambda$, whereas the crisis period (European debt crisis and 2008 financial crisis) are weak against the node failures.
Δλ whereas the 2008 crisis shows the lowest value. This implies that the network during the European debt crisis is more robust in terms of link failure but less robust against the node failure as shown by spectral radius and epidemic threshold (figures 5 and 6). The calm period is robust against the node failure at high $\tau_c$ but shows a low value of $\Delta \lambda$, which means it is easy to split the network by targeting links than nodes.

5.4 Principal eigenvector

Principal eigenvector is the eigenvector corresponding to the largest eigenvalue $\lambda_{\text{max}}$ (or $\lambda_N$) of the (weighted) adjacency matrix of a graph. The components of the principal eigenvector also rank a node based on the number of paths passing through the node that connects two distinct nodes in the network [15]. It is a centrality measure with the score of a node equals the corresponding component of the principal eigenvector. The principal eigenvector has all the components with the same sign.

In the context of global indices, principal eigenvector represents the market mode or global financial conditions. Figure 8 shows the principal eigenvector components of weighted adjacency matrix for all financial periods (windows) at a different threshold. There are two states of principal eigenvector, one with all components positive and the other with all components negative. Figure 8 shows that at 0.1 threshold the network at high financial stress is in the state where all components are negative whereas the network during the less stress has principal eigenvector components in a positive direction. As principal eigenvector represents the state of the global economy, we assess the contribution of each index toward the overall state of the system. We find that the overall state of the system is highly controlled by the American and European markets whereas the countries belonging to the Asia–Pacific region has only a small contribution. The contribution of the American and European markets becomes more dominated with an increase in threshold, and at a sufficiently high threshold ($\approx 0.5$), American and European markets fully dominate global economic conditions with countries from Asia–Pacific region having almost negligible contribution. From figure 8, at 0.5 threshold, the main contributions for all windows are from Argentina (MERV), Brazil (BVSP), Mexico (MXX), USA (GSPC), UK (FTSE), Spain (IBEX), Belgium (BFX), Euro Zone (STOXX50E), Hungary (BUX), Canada (GSPTSE), Austria (ATX), Germany (GDAXI), France (FCHI), Netherland (AEX), Switzerland (SSMI), Russia (RTS.RS) and Israel (TA100). There are only two countries [Russia (RTS.RS) and Israel (TA100)] from the Asia–Pacific region (total 14 countries in Asia–Pacific) which contributes to the global economic state whereas 15 out of 16 countries (except Greece) from the American and European markets contributes to the state of the system. This tells us that market is dominated by the American and European markets.

5.5 Clusters based on eigenvectors

In a network, the community is defined as a group of nodes with higher communicability among them than the rest of the nodes in the network. If we assume that the nodes are connected with springs as edges then each eigenvector represents a mode of oscillation of the complete network and the corresponding eigenvalue indicates the strength of that mode [20]. The eigenvector corresponding to the second largest eigenvalue (will be called as the second largest eigenvector) of the weighted
The adjacency matrix $W$, has both positive and negative components. This breaks nodes into two clusters, one with components having a positive sign and the other with a negative sign. The nodes in the first cluster move coherently in one direction representing a mode of oscillation. Similarly, the nodes in the second cluster move in the reverse direction giving another mode of oscillation. The sign of the product of the two components of the second eigenvector gives whether the nodes are in the same cluster or not. If the product of two-component is positive then both nodes belong to the same cluster, and they are in opposite clusters if the sign of the product of components is negative.

In our analysis, the second eigenvector for global financial indices split the nodes into two categories based on the sign as shown in figure 9. For the global financial indices, these clusters physically represent the geographical regions to which the index belongs. The first cluster includes countries belonging to the Asia–Pacific region whereas the other cluster with the opposite sign is the countries in the European and American markets. There is no change in these two clusters with time and countries belonging to each cluster remains the same for all financial windows. The sign of all countries gets changed simultaneously with the change in the financial period (window). The first cluster of the Asia–Pacific region contains 14 countries including India, China, Russia, Japan, Australia, Israel, South Korea, Hong Kong, Singapore, Taiwan, Malaysia, New Zealand, Indonesia, and Pakistan. The second cluster of European–American region has 16 countries which are USA, UK, Spain, Belgium, Euro Zone, Hungary, Austria, Germany, Mexico, Greece, Brazil, France, Canada, Netherland, Argentina, and Switzerland.

At 0.1 threshold (figure 9), the financial window belongs to time period before the 2008 crisis (window 1), European debt crisis (window 4), and the calm period (window 5) show the same sign for components whereas the 2008 crash (window 2) and period after 2008 crash (window 3) have opposite signs. With a change in the threshold, there is a change in groups, i.e. at 0.2 threshold, windows 4 and 5 are in one group whereas windows 1, 2, and 3 in another group which further change with a threshold. With an increase in threshold it is hard to separate the network into partitions.

The analysis similar to the second-largest eigenvector can be applied to other eigenvectors. For example, the third eigenvector will have different values of components, and can be used to break the network into three clusters, after an appropriate selection procedure. As the second-largest eigenvector can be used for the network partitioning into two clusters, the third-largest eigenvector can be used to partition a network into triants, fourth largest partitions network into quadrants and so on [20]. The partitions created by these eigenvectors may or may not be independent of each other.

5.6 Fiedler eigenvalue and eigenvector

The second smallest eigenvalue $\lambda_2$ of Laplacian matrix is also known as Fiedler eigenvalue which gives the algebraic connectivity in a network. It quantifies the robustness of the network in terms of the link failures and measures the difficulty or cost to cut the network into independent components [17]. A higher algebraic connectivity implies a more robust network against the link failure, which means that it will be difficult to break the network into independent non-connected components.

The eigenvector corresponding to the second smallest eigenvalue of the Laplacian matrix (Fiedler eigenvalue)
is known as Fiedler eigenvector. Fiedler [15, 18] shows that the Fiedler eigenvector is an approximate solution for the bi-partitioning of the graph. The graph is partitioned into two groups based on the sign of the component of Fiedler eigenvector, the first group contains all nodes with a positive value of Fiedler eigenvector component and a second group with a negative sign of Fiedler eigenvector. Figure 10 shows the variation of the components of Fiedler eigenvector with the threshold for different financial periods. The partitioning is difficult at low thresholds but with the increase in threshold (0.5) the graph bi-partitioning gives two clear groups for the crisis period (2008 financial crisis and European debt crisis). The groups are based on the geographical location of the index and are based on Asia–Pacific markets or American–European markets. This also shows that at the time of crisis the market within the geographical location shows greater interactions than the markets within the other group. At the time of low stress (calm period), the market is more diffused with interaction not related to the geographical location. The inter-cluster interactions are lower at the time of financial stress with a high concentration of intra-cluster interactions creating bottlenecks in the network. At the less stressful time, the inter-cluster interaction increases, and the bottlenecks are removed.

6. Conclusion

In this work, we use the spectral graph theory to study the network created from the financial time series of 30 global financial indices for a period of 10 years. The period is split into windows indicating different financial regimes. A weighted threshold network is created from the correlation matrix of global financial indices. The spectral properties of weighted adjacency matrices and Laplacian matrices are calculated and compared for different financial regimes. The spectral radius of network shows that the system at the time of crisis is more prone to failure and less robust against the random or target attacks compared with a calm period. The principal eigenvector of the weighted adjacency matrix shows that the state of the global economy is mainly controlled by the American and European markets compared to Asia–Pacific region. The sign of the components of the second-largest eigenvector can partition nodes into two clusters representing either American–European markets or Asia–Pacific region. The second smallest eigenvalue of the Laplacian matrix (Fiedler eigenvector) shows that at the time of crisis, nodes show higher interaction with the other nodes belonging to the same geographical cluster. Inter-cluster interaction is minimum at the time of crisis whereas during the calm period both geographical clusters show high inter-cluster interaction and Fiedler eigenvector is not able to separate into two clear clusters.

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