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Microtransitions in hierarchical and climate networks

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Abstract. The prediction of the critical point of a phase transition is a topic of great current interest, and is of utility in many practical contexts. Therefore, the identification of precursors, or early warning signals of the critical point, has become the focus of current interest. Recent model studies have shown that a series of small transitions, which have been called microtransitions, act as precursors to the percolation transition. Here, we identify the existence of microtransitions in two distinct networks, for two distinct processes.

The first case is the process of avalanche transmission on branching hierarchical networks. Here, typical realizations of the original lattice of this network exhibit a second order transition. We note that microtransitions in the variance of the order parameter are seen in this case. Additionally, the positions of the microtransitions follow a scaling relation. The scaling relation can be used to calculate the position of the critical point, which is seen to be in agreement with the observed result.

We also introduce this method of identifying the microtransitions occurring before the tipping point to a complex real world system, the climate system. We analyse the discontinuous first order phase transition occurring in the climate networks. We apply the percolation framework to these networks to analyse the structural changes in the network and construct an order parameter and a susceptibility. Microtransitions can be found in the behaviour of the susceptibility. These can be used to predict the tipping point in the system. We discuss possible applications of this.

Keywords. Complex systems; networks; branching hierarchical network; percolation; microtransitions; El Nino southern oscillation.

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1. Introduction

The identification of precursors of phase transitions has been a topic of current research interest. The existence of phase transitions in real life situations such as congestion in road and internet traffic [1], blackouts in power grids [2] and monsoon dynamics [3] has led to the realization that the prediction of phase transitions is of great practical utility. It is in these contexts that the identification of precursors to the transitions assumes great importance. Early warning signals of transitions have been found in diverse phenomena ranging from the medical sciences, to ecosystems and the climate system [4]. The climate system is a very crucial and complex system consisting of various subsystems such as the atmosphere, the ocean, the cryosphere, the biosphere and the lithosphere. The intricate dynamics in these subsystems leads to various climatic phenomena such as cyclones, the monsoon and oceanic currents. Each climatic phenomenon occurs at a different time scale. They can vary from a few seconds to a few days, e.g., in the case of precipitation and cyclones. For the ocean, these scales range from a few decades (e.g., in the case of surface ocean circulation) to hundreds or even thousands of years (e.g. deep ocean circulation). Thus understanding and finding precursors of these climatic phenomena becomes a challenging task. These precursors are of different kinds, depending on the context. In the case of the transition to an icehouse earth, the increased values of the autocorrelation in the temperature function as a precursor to the transition [5]. The formation of the largest cluster in a climate network and a decrease in the susceptibility can also serve as an indicator of an anomalous climatic event [6]. The prediction of the monsoon period can be based on a critical transition precursor by identifying geographic regions which function as tipping elements of the monsoon and using them as observation locations for predicting onset and withdrawal dates [7]. In the context of theoretical models, microtransitions, where functions of the order parameter show small, but abrupt changes, have been used as precursors and predictors of the phase transition in the case of the percolation transition [8].

The specific model considered for avalanche transmission, is a 2D load bearing hierarchical network which can serve as a model for diverse systems ranging from natural systems such as river networks [9] and granular media [10], as well as for social systems [11] and is also similar to models that arise in biological contexts as models of lung inflation [12]. Here we investigate the microtransitions in avalanche transmission for the 2D load-bearing hierarchical network, where the network shows a transition from a state where most of the transmissions are successful, i.e. all test weights get absorbed, to one where most of the transmissions fail. This transition has been seen to be continuous for typical realizations of the branching hierarchical lattice (which we call the original lattices). We see the presence of microtransitions, signalled by peaks in the relative variance of the order parameter in avalanche transmission on the original lattice. The positions of the microtransitions act as precursors to the transition point for this case. They follow a scaling law, and the critical point can be predicted with good accuracy, using the scaling behaviour.

Finally we discuss microtransitions for a climate network system constructed out of oceanic observations, a complex time series is observed at different geographic locations (as defined by latitude and longitude measures) which constitute the network nodes and links are constructed based on the similarities of the dynamics between the pairs of nodes. We calculate the normalized largest cluster and susceptibility of the network and analyze the microtransitions occurring just before the percolation phase transition. Microtransitions have also been seen in the case of avalanche transmission in branching hierarchical networks [13]. These microtransitions can be used to predict the phase transition point.

2. Branching hierarchical lattice and avalanche transmission process

The 2D load-bearing hierarchical network considered here, is based on a regular triangular lattice. Figure 1 shows a typical realization of the 2D load-bearing hierarchical network [14]. Here, the solid circles indicate the nodes and the solid lines represent the links of the network. Every node can connect with its nearest neighbours in the layer below with probability 1/2. Thus a site i in the layer L can connect to either of its nearest neighbours in the layer $L + 1$. Each node is assigned a number, which represents its capacity. Every node in the topmost layer has unit capacity. The capacity w_i^L of a site i in the layer L is the sum of the capacities of sites to which it is connected in the layer above and its own capacity one. The capacities obey the following equation:

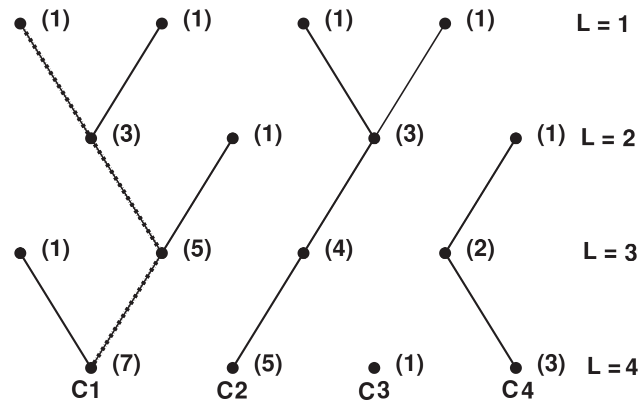

Figure 1. A typical realization of the network, the original lattice of size $M = 4 \times 4$. The solid circles are the nodes and solid lines are the links of the network. The capacity of each node is the number indicated in brackets next to the node. The beaded line is the trunk of the network. C1 and C2 are the clusters.

$$w_i^L = l(i_l^{L-1}, i^L)w(i_l^{L-1}) + l(i_r^{L-1}, i^L)w(i_r^{L-1}) + 1 \quad (1)$$

L=1, . . . ,N, where N is the total number of layers in the network. The link $l(i_l^{L-1}, i^L)$ takes value 1 if a left connection exists between the ith site of the layer L and the site to its left, viz. i_l^{L-1} in the layer $L - 1$. Otherwise it takes the value zero. The link $l(i_r^{L-1}, i^L)$ for the right connection has similar behaviour, with i_r^{L-1} being the site to the right in the $L - 1$ layer.

The network consists of many clusters, where a cluster is the collection of connected sites of the network. In figure 1 the clusters C1 to C4 are examples of typical clusters of different sizes. The size of a cluster is defined as the total number of connected sites in that cluster. The cluster having the largest number of connected sites is the maximal cluster, which is the cluster C1 here. The strongest path from the top-most layer to the bottom-most layer in the maximal cluster is called the trunk of the network. Any typical realization of the network as shown in figure 1, which has been called the original lattice [14], is the $q(0, 1)$ case of the Coppersmith model for granular media [10], is a model for river networks [9], is also similar to the voter model [11] and is a model of the lung inflation process [12].

The avalanche transmission process, or the process of weight or packet transmission along the connected paths of the network [14, 15] is defined as follows: when a weight W is deposited on a site in the first layer it retains a weight equal to its capacity W_c and transmits the rest $W - W_c$ to the site it is connected to in the layer below. Thus the weight is transmitted in the downward

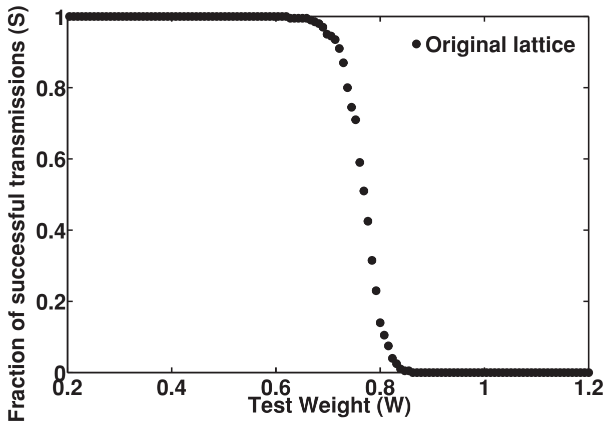

Figure 2. The order parameter as a function of test weight for the original lattice of size $M = 50 \times 50$, averaged over 200 realizations. The test weight (W) here is the fraction of the total number of test weight and the total capacity of the network.

direction and the sites involved in this process constitute the path of connection. If there is still excess weight left at the bottom-most layer of the network it is then transmitted to a randomly chosen site of the first layer. Let P_L be the site on such path P. We can write

$$W^{\mathrm{ex}}(P_L) = W - \sum_{k=1}^{L} W_c(P_k). \tag{2}$$

If a test weight transmitted in this way encounters a fully saturated site, and also has no alternate path to take, then the transmission is considered to have failed. If the transmitted weight is absorbed at some sites in the network then the transmission corresponds to a successful transmission. The order parameter in an avalanche process on the 2D load-bearing hierarchical networks is defined as the fraction of transmissions that are successful. The order parameter for the typical realizations, i.e. the original lattice varies continuously with the test weight (figure 2) [16].

We note that the transmission of messages on these base substrates shows a percolation transition on the original lattices [16].

3. Microtransitions in avalanche transmission

We now study microtransitions in the avalanche transmission on the 2D load-bearing hierarchical network. The microtransitions are signalled by microscopic changes in the order parameter. The global transition in the system is the transition of the order parameter at the critical point, from values of order zero to values of order one, whereas the microtransitions are small changes in the order parameter well before the transition point. The study of the avalanche transmission on the network shows a transition of the network from the state where all avalanches are transmitted, to a state where almost all transmissions fail, as the test weight, which is placed on the top layer, increases. In order to study the microtransitions which occur before the transition, we look at the variance and relative variance of the absorbed weight, which is the weight absorbed by the occupied nodes of the network for a given test weight. The absorbed weight has some non-zero value in the free flow state and it is zero in the state of failure of avalanche transmissions. The variance V and relative variance RV of the absorbed weight is defined as

$$V = \left(\langle O^2\rangle - \langle O\rangle^2\right) \quad \text{and} \quad RV = \frac{\left(\langle O^2\rangle - \langle O\rangle^2\right)}{\langle O\rangle^2}, \tag{3}$$

where O is the weight absorbed by the occupied nodes of the network for some given test weight. The average is taken over the total number of nodes which are occupied. If a node is partially occupied we consider that as an occupied node.

Here, we calculate the average value of the variance of the absorbed weight. We always deposit the test weight at a randomly chosen site in the top-most row and the weight transmits in the network according to the process defined in section 3. We calculate the variance and relative variance of the absorbed weight for different realizations of the network. The average over the realizations reduces the sharpness of the peaks in the variance plot for the original lattice compared to a single realization of the original lattice. The variance and relative variance are shown in figure 3a for a 50×50 original lattice, averaged over five realizations. The scaling relation for the peak positions here is $P_c - P_{i+1}/P_c - P_i$ and turns out to be a constant (figure 3b). We take P_c to be the test weight that corresponds to the maximum value of susceptibility, where the susceptibility is defined as $\chi = M\sqrt{\langle S_1^2\rangle - \langle S_1\rangle^2}$, where S_1 is the fraction of the occupied sites and is a function of the test weight (figure 3c) [16]. The average is taken over the realizations of the original lattice.

Using the scaling relation plotted in figure 3b we write

$$\frac{P_c - P_{i+1}}{P_c - P_i} = 0.9942. \tag{4}$$

In the case of the 50×50 lattice, we find that $P_c = 0.7646$ using the values $i = 36, P_{36} = 0.1439$ and $P_{37} = 0.1475$. The value of P_c from the scaling relation is close to the value of P_c from the order parameter plot [13].

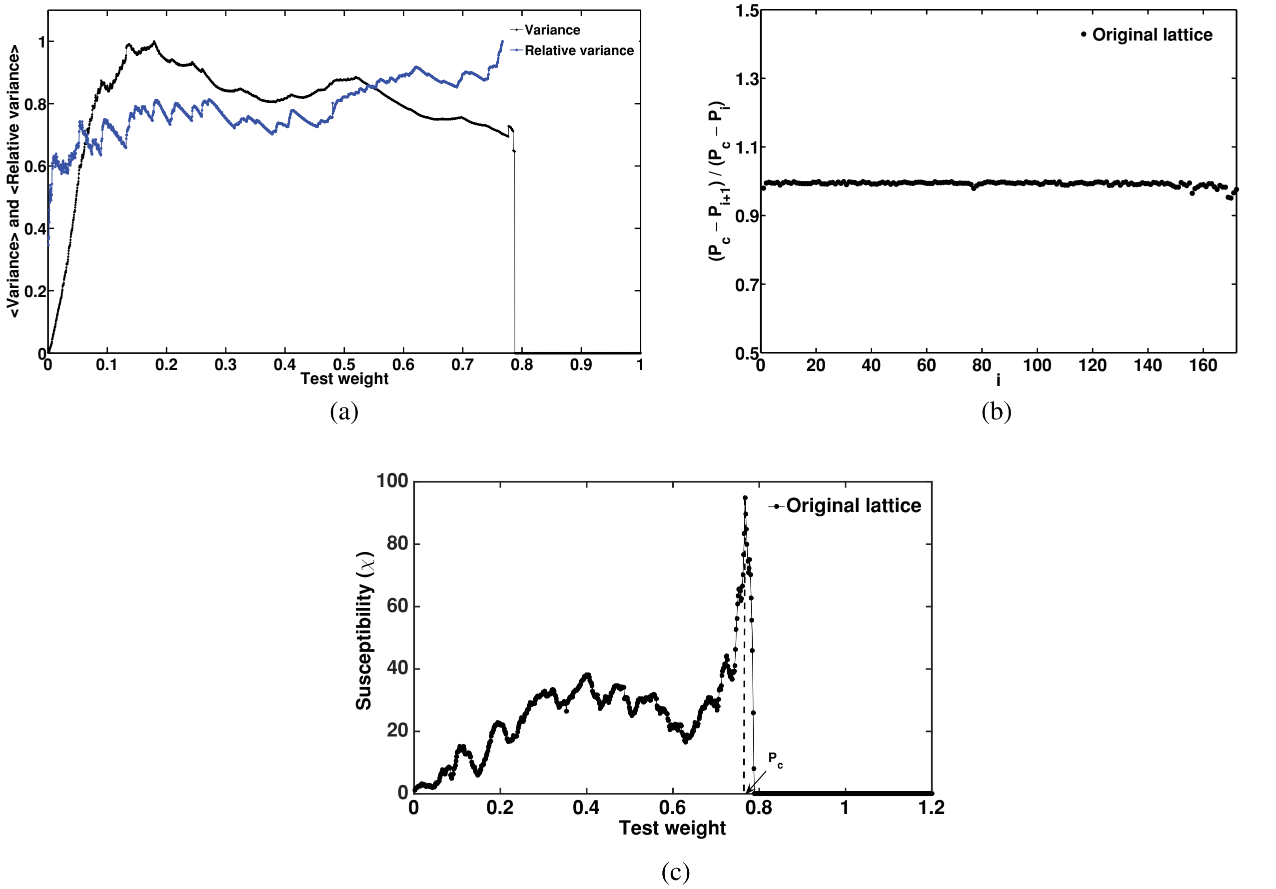

Figure 3. (**a**) The variance and the relative variance of total absorbed weight as a function of test weight for the original lattice of size $M = 50 \times 50$, averaged over five realizations. (**b**) The scaling relation for the original lattice behaves like a constant function. (**c**) The susceptibility for the 50×50 original lattice averaged over five realizations. Here, P_c is the test weight corresponding to the maximum value of susceptibility.

We note that the microtransitions act as precursors of the phase transition in avalanche transmissions on the original lattice. Here, the scaling relation $(P_c - P_{i+1})/(P_c - P_i)$ predicts the critical point with good accuracy. This scaling relation does not depend on the realizations of the network and the choice of channels of transmission.

4. Climate networks

In this section, we construct a climate network which is based on near surface air temperature data collected at the grid points (nodes) of a geographic grid based on latitude and longitude values. The datasets analysed consist of daily near surface (1000 hPa, nearly equal to one atmospheric pressure) air temperature data of ERA-Interim reanalysis (The European Centre for Medium-Range Weather Forecasts (ECMWF)) [17]. This dataset contains the information about daily near surface (1000 hPa) air temperature $[\tilde{T}^y(d)]$ within the period 1979–2017 at 1, 15, 680 geographical nodes whose location is fixed by a pair of latitude and longitude. We extracted a data for $N = 726$ nodes (i.e., longitude–latitude grid point) for 7.5° grid resolution from this huge data file. We constructed the climate network based on the similarity of the dynamics between the nodes [6] and studied the evolution of clusters.

We calculate the filtered daily near surface air temperature $T^y(d)$ and the cross-correlation function $C_{i,j}$ as follows:

$$T^y(d) = \frac{\tilde{T}^y(d) - \mathrm{mean}(\tilde{T}(d))}{\mathrm{std}(\tilde{T}(d))}, \tag{5}$$

$$C^y_{i,j}(\tau) = \frac{\langle T^y_i(d-\tau) T^y_j(d)\rangle - \langle T^y_i(d-\tau)\rangle\langle T^y_j(d)\rangle}{\sqrt{(T^y_i(d-\tau) - \langle T^y_i(d-\tau)\rangle)^2} \cdot \sqrt{(T^y_j(d) - \langle T^y_j(d)\rangle)^2}}, \tag{6}$$

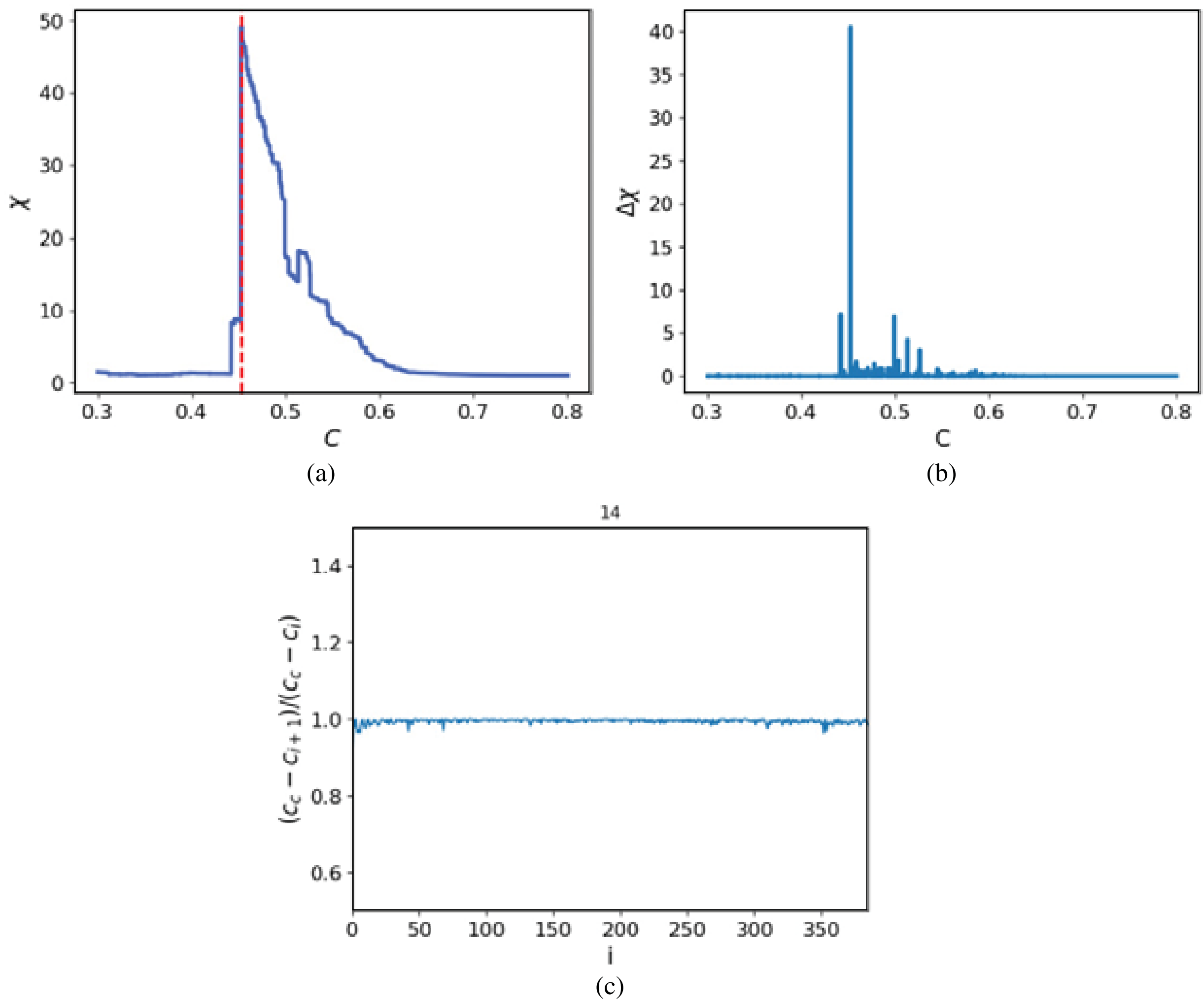

Figure 4. (**a**) The susceptibility χ as a function of the link strength C for the network one year before a very strong El Niño episode, December 2014. (**b**) $\Delta\chi$ as a function of the link strength C. (**c**) The scaling relation for the relative $\Delta\chi$ positions, $(C_c - C_{i+1})/(C_c - C_i)$ as a function of the index i.

where 'mean' and 'std' are the mean and standard deviation of the temperature on day d over all years and τ is the time lag between 0 and 200 days. Only the temperature data points prior to day d are considered. Averages ($\langle \quad \rangle_d$) are taken over 365 days. The link weight is defined as the maximum of the cross-correlation function $\max(C^y_{i,j}(\tau))$. Links were added one by one according to the link strength, i.e., we have first added the link with the highest weight (maximum of the cross-correlation function $C^y_{i,j}$), and continued selecting edges ordered by decreasing weight.

During the evolution of our network, we measured the size of the normalized largest cluster (s_1) and susceptibility (χ).

$$s_1 = \frac{S_1}{N}, \tag{7}$$

$$\chi = \frac{\sum_s' s^2 n_s(C)}{\sum_s' s n_s(C)}. \tag{8}$$

Here, S_1 is the size of the largest cluster and N is the total number of nodes in the network, $n_s(C)$ is the number of clusters of size s at edge's weight C, and the prime on the sums indicates the exclusion of the largest cluster S_1 in each measurement. The percolation transition in this climate network can be quantified via the order parameter viz. the existence of giant component (cluster) containing $O(N)$ nodes.

We note that the susceptibility shows a distinct pattern one year prior to a very strong El Niño episode. The red line indicates the value of the link strength C at which the climate network undergoes the percolation transition, i.e. the formation of a giant component (cluster) containing $O(N)$ nodes (figure 4a). In figure 4b we have plotted the magnitude of successive jumps in the susceptibility with respect to the link strength, which marks the position of the values of C at which the jump occurs. Using the scaling relation plotted in figure 4c we write

$$\frac{C_c - C_{i+1}}{C_c - C_i} = 0.9956. \tag{9}$$

Such a scaling relation enables us to predict the transition point at which the network undergoes the percolation transition. We have analysed these microtransitions for each network of each year within the period 1979–2017 and we found that for the indicator year 2014, $C_c = 0.4669$ using the values $i = 44, C_{44} = 0.6719$ and $C_{45} = 0.6710$. The value of C_c from the scaling relation is in reasonable agreement with the actual value of $C_c = 0.4522$ from the order parameter jump.

We note that 2014 is a pre-El Niño year. A similar pattern of activity is observed for all the pre-El Niño years. Thus we observe a distinct pattern of microtransitions before the percolation phase transition point for all the indicator years prior to an El Niño activity. This may be useful for the prediction of El Niño activity in the subsequent year.

5. Conclusions

We have seen that microtransitions appear in dynamical processes on two distinct networks. One is the process of avalanche transmission on 2D load-bearing hierarchical networks. Here, the scaling relation for the original lattice behaves as a constant function. The scaling behaviour is used to predict the critical point. The calculated value of the critical point from the scaling relation is close to the value of critical point obtained from the order parameter plot. Thus, the microtransitions behave as precursors of the transition for the typical realization of the branching hierarchical lattice, and can be used to predict the point of transition. We have also seen that microtransitions show scaling behaviour in a very complex real climate network system. This can also be used to predict the critical point. We hope to explore further whether the kind of microtransitions studied here can be used to predict complex climatic phenomena like the El Niño phenomenon, the onset of the monsoon, or cyclonic activity.

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