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Implementation of parallel logic elements in a quasiperiodically driven Murali–Lakshmanan–Chua circuit system

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Abstract. We investigate the effect of deterministic input signals on a quasiperiodically driven Murali–Lakshmanan–Chua (QPDMLC) circuit, which exhibits strange nonchaotic attractors (SNAs). We show that if one uses two-square waves in an aperiodic manner as an input to a QPDMLC circuit, the response of the circuit can produce a logical output, controlled by such forcing. We also demonstrate that one of the variables of the circuit exhibits one logic element, while the other variable shows its complementary logic operation. It is further shown that these logical behaviors persist even for an experimental noise floor. Thus we confirm that SNA is an efficient tool for computation as in the case of the quasiperiodically driven Duffing oscillator.

Keywords. Strange nonchaotic attractors; logical strange nonchaotic attractor; Murali–Lakshmanan–Chua circuit.

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1. Introduction

It is well known that strange nonchaotic attractors (SNAs) possess fractal geometry but exhibit no sensitive dependence on initial conditions. SNAs were originally described by Grebogi *et al.* [1], who found that quasiperiodically driven dynamical systems admit them in parameter regions of positive Lebesgue measure. Since then, many theoretical as well as experimental studies pertaining to the existence and characterization of SNAs in different quasiperiodically driven nonlinear dynamical systems have shown many major features of these exotic but important class of attractors [2–8].

It is well known that the approaching of physical limits on Moore's law has led to the development of alternative methods to perform more number of computations out of limited number of hardwares [9–13]. In this direction, in 1998, the important work of Sinha and Ditto paved a new avenue of using chaos for computation [9, 10]. They have proposed a chaos-computing scheme based on the thresholding method to achieve a controlled response from a chaotic system.

Although, a nonlinear dynamical system can be a processor of the flexibly configured and reconfigured devices to emulate different logic gates, it was shown that the manufacturing nonidealities and ambient noise make it difficult to obtain different logic functions in these systems [14, 15]. In fact chaotic systems are highly sensitive to initial perturbations and thus a small amount of noise can completely change the system dynamics. As a result, special attention needs to be paid in choosing the appropriate nonlinear dynamics-based computing systems which are robust against noise.

Recently, we proposed a simple approach to encapsulate computations and noise robustness at the dynamics level. In particular, we have presented a route to logical SNAs in quasiperiodically driven Duffing oscillator systems [16]. In the present paper, we show how by using robust SNAs, including even noise, one can emulate different logic functions in a simple electronic circuit, thereby providing a sound nonlinear dynamics basis for computation.

2. Quasiperiodically driven Murali–Lakshmanan–Chua (QPDMLC) circuit

To illustrate our results, we consider the simplest second-order nonlinear dissipative nonautonomous circuit as shown in figure 1. In the absence of noise and square waves, the dynamics of the circuit has been studied in detail regarding different aspects of bifurcations, chaos and strange nonchaos numerically and experimentally [17, 18]. In the present work, we show that under the influence of square waves the circuit exhibits a logical strange nonchaotic attractor.

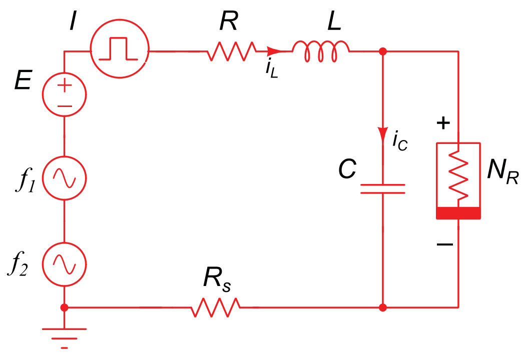

Figure 1. Circuit realization of the QPDMLC circuit including the logic pulse.

For this purpose, after applying Kirchhoff's laws and rescaling the circuit, we consider the following dimensionless form of equations:

$$\begin{aligned}
\dot{x} &= y - h(x), \\
\dot{y} &= -\beta(1+\nu)y - \beta x + F_1 \sin(\theta) \\
&\quad + F_2 \sin(\phi) + I + \varepsilon + \sqrt{D}\xi(t), \\
\dot{\theta} &= \omega_1, \\
\dot{\phi} &= \omega_2, \qquad (1)
\end{aligned}$$

where

$$h(x) = \begin{cases} bx + (a-b), & x > 1, \\ ax, & |x| \le 1, \\ bx - (a-b), & x < -1. \end{cases} \qquad (2)$$

The rescaled circuit parameters correspond to the values $a = -1.02$, $b = -0.55$, $\nu = 0.015$, $\beta = 1.0$ and $\omega_1 = 1.0$ and also we fix one forcing parameter $F_1 = 0.15$ and vary the other forcing parameter F_2. To investigate the system under quasiperiodic forcing, the frequency of the second harmonic force is kept as $\omega_2 = \sqrt{5} - 1/2$ to meet the requirement that the ratio of the frequencies ω_1/ω_2 is irrational.

3. Effect of three-level square waves on the QPDMLC circuit

Next, we analyze numerically the response of the quasiperiodically driven nonlinear system (1) to deterministic logic input signal I, consisting of two square waves (of different time scales) in the absence of noise. Specifically, for two-logic inputs, we drive system (1) with a low/moderate amplitude signal $I = I_1 + I_2$ with two-square waves of strengths I_1 and I_2 encoding two-logic inputs. The inputs can be either 0 or 1, giving rise to four-distinct logic input sets (I_1, I_2) : $(0,0), (0,1), (1,0)$ and $(1,1)$. For a logical '0', we set $I_1 = I_2 = -\delta$, whereas for a '1', we set $I_1 = I_2 = +\delta$, where δ represents a small/moderate intensity input signal. Now the input sets (0,1) and (1,0) give rise to the same input signal I. As a result, the four-distinct input combinations (I_1, I_2) reduce to three-distinct values of I, -2δ, 0, $+2\delta$, corresponding to the logic inputs $(0,0)$, $(0,1)$ or $(1,0)$, $(1,1)$, respectively. The output of the system is determined by the state $x(t)$ of system (1); for example, the output can be considered as logical '1' if it is in one particular state and logical '0' if it is in a different state. Specifically, the output corresponding to this 2-input set (I_1, I_2) for a system with the state values $x > 0$ is taken to be '1' and with $x < 0$, it is taken to be '0'. So, when the system switches between these two states, the output toggles from logical '0' to logical '1' and vice versa. Here we will explicitly show that one indeed observes for a given set of inputs (I_1, I_2) a logical output from the above quasiperiodically driven nonlinear system (1) in accordance with the truth table of the basic logic operations as given in table 1.

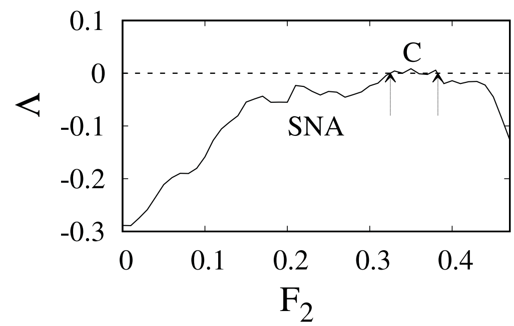

Figure 2. Maximal Lyapunov exponent Λ versus control parameter F_2: Solid curve corresponds to system (1) after giving the inputs I_1, I_2 and bias $\varepsilon = 0.05$ with control parameter $\delta = 0.05$ and without noise (D=0.0).

Table 1. Truth table of the basic logic operations

Input set I_1, I_2	OR	AND	NOR	NAND
(0,0)	0	0	1	1
(0,1)/(1,0)	1	0	0	1
(1,1)	1	1	0	0

3.1 *Transition to logical SNA*

Now, we consider the dynamics of (1) where both the inputs I_1 and I_2 take the values −0.05 when the logic input is 0, and value 0.05 when it is '1'. The input signal $I = I_1 + I_2$ is thus a three-level square wave form: −0.1 corresponding to the input set $(0,0)$, 0 corresponding to the input sets $(0,1)$ or $(1,0)$ and 0.1 corresponding to the input set $(1,1)$. Figure 2 shows that with increased

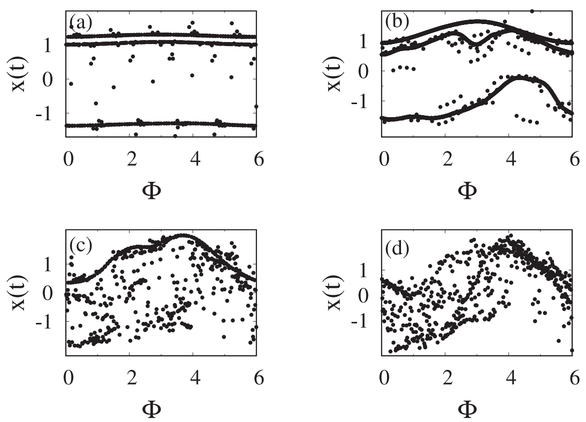

Figure 3. Projection of the numerically simulated attractors of eq. (1) in the (ϕ–x) plane for various values of F_2. (**a**) Three-quasiperiodic torus for $F_2 = 0.01$, (**b**) logical SNA for $F_2 = 0.1$, (**c**) standard SNA for $F_2 = 0.2$ and (**d**) chaos for $F_2 = 0.35$ with $\varepsilon = 0.05$.

forcing amplitude F_2 in (1), the maximal Lyapunov exponent also grows and that it changes sign (solid curve) at $F_2 = 0.3248$.

As F_2 is varied in the range $0 < F_2 < 0.001$, three-quasiperiodic torus is observed due to the effect of three-level square waves and one can observe three smooth branches in the Poincaré surface section plot in the (ϕ–x) plane as shown in figure 3a. As the value of F_2 increases further, the branches lose their smoothness and transit to an SNA (figure 3b) as the maximum Lyapunov exponent takes the value $\Lambda = -0.2890$ only. Here the fractal torus involves a kind of sudden widening of the attractors. In this range of F_2, $0.001 < F_2 < 0.1$, the output of the system is synchronized with the aperiodic input signal. If the aperiodic signal follows any kind of logic response of the system, the latter also follows the same. Therefore this kind of attractor can be called a logical SNA. On further increase in the value of F_2, $0.10 < F_2 < 0.3248$, the logical SNA loses its synchrony with the input signal and becomes the standard SNA (see figure 3c). Increasing the value further, $F_2 > 0.3248$, the attractor becomes chaotic (see figure 3d).

We observe that under optimal quasiperiodic forcing strength $0.001 < F_2 < 0.10$, the state $x < 0$ as logic output 0 and the state $x > 0$ as logic output 1 yield a clean stable logical OR gate SNA with $\varepsilon = 0.05$ (see figure 4 for full details).

Specifically, we note that the state variable x of the system mimics the input signals such that the logic high input is obtained at the output as $x > 0$ and logic low input is obtained as $x < 0$ at the output. Hence we observe that under optimal quasiperiodic forcing strength $0.001 < F_2 < 0.1$, the state $x < 0$ as logic output '0'

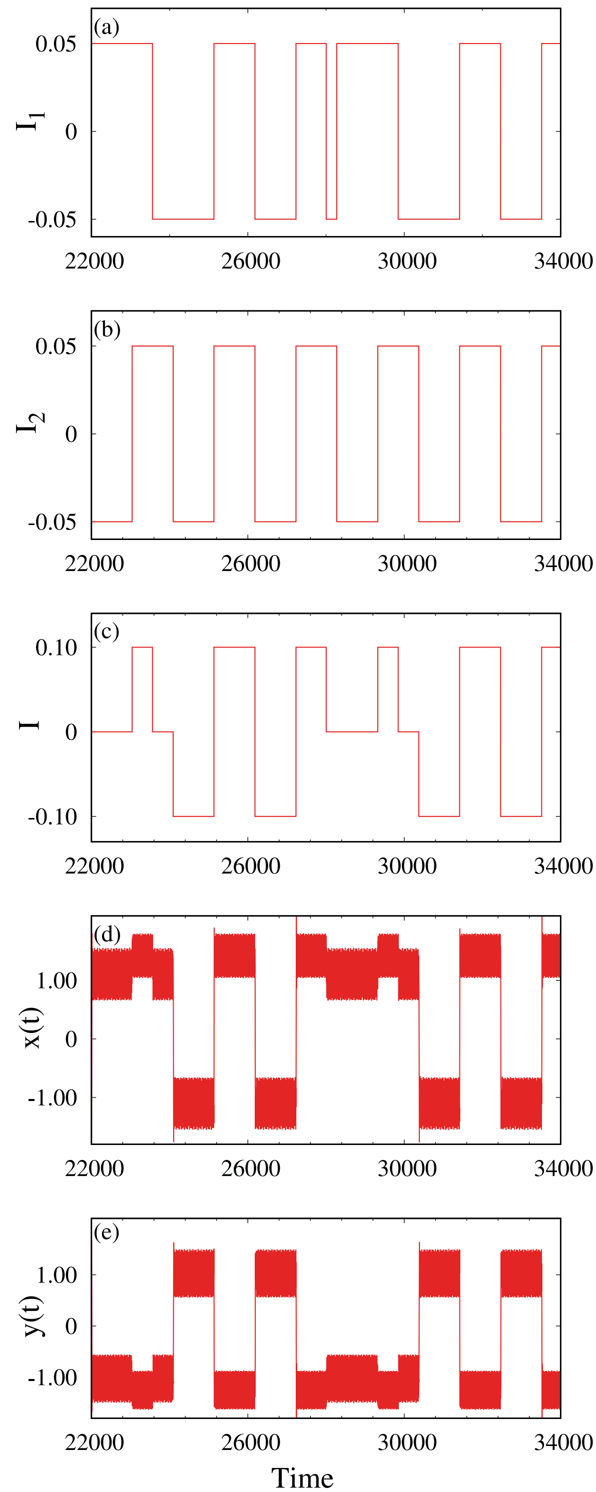

Figure 4. From top to bottom panels: (**a**)–(**c**) show a stream of input signals I_1, I_2 with $I_1 = I_2 = -0.05$ when the logic input is '0' and $I_1 = I_2 = 0.05$ when the logic input is '1'. The '3' level square waves with -0.1 correspond to the input set (0,0), 0 for the (0,1)/(1,0) set and 0.1 for the (1,1) input set. Panels (**d**) and (**e**) represent the dynamical response of the system $x(t)$ and $y(t)$ under quasiperiodic forcing $F_2 = 0.05$, where one obtains the desired OR and NOR logic outputs for $\varepsilon = 0.05$ (see table 1).

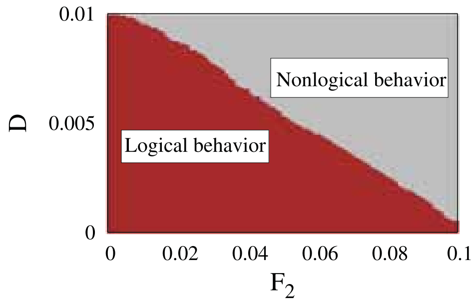

Figure 5. Two-parameter diagram for logic operation as a function of the noise intensity (D) and the amplitude of forcing F_2.

and the state $x > 0$ can be considered as logic output '1' yield clean logical OR gate SNA with $\varepsilon = 0.05$. On the other hand, the corresponding state variable y of the same system mimics the inverted output signal of x and yields a clean logical NOR gate SNA (figure 4). Thus in this circuit the single QPDMLC gives not only the logical OR via 'x' variable but also prove that its complement, that is NOR through 'y' variable simultaneously. Similarly, we observe that as the value of bias changes from $\varepsilon = 0.05$ to $\varepsilon = -0.05$ the response of the system morphs from OR/NOR gate SNA to AND/NAND gate SNA logic behavior.

3.2 *Effect of noise in the logic gates*

Finally, we consider the behavior of system (1) in the presence of noise. For the sake of definiteness, we choose the noise intensity to be comparable to that of a weak internal noise which originates in electronic components that may model system (1). Such noise essentially originates in the analog components and is usually $\sim 1\,\mu$V. It is observed that the behavior of the largest Lyapunov exponent of the forcing amplitude F_2 in the presence of noise is found to be practically coincident with that of the noise-free case. Hence, in the presence of noise, the logical SNA in the system retains its negative Lyapunov exponents and the fractal structure. It is clear from figure 5 that the logical SNA can be implemented over a range of parameter values F_2 and D where it can emulate and morph the AND and OR operation in the presence of noise. Our study confirms that the logic behavior remains when the noise strength is below the mV range. Hence the logic nature in our system persists even when the noise originates due to analog electronic components of the system.

3.3 *Characterization of logical SNA*

To characterize strange nonchaotic attractors, we utilize the finite-time Lyapunov exponents and Fourier spectra. The distributions of finite-time Lyapunov exponents for logical SNA is shown in figure 3.1a. One can note that for SNA, the finite-time Lyapunov exponent remains negative for most of the times but it can be positive intermittently. It is clearly shown in figure 3.1a where the distribution of finite-time Lyapunov exponents in the positive side is apparent. To substantiate the results of logical SNA we compute the time-dependent Fourier transform $X(\Omega, N) = \sum_{n=1}^{N} x_n e^{i2\pi n\Omega}$, for $\Omega = \sqrt{5} - 1/2$. The scaling relation $|X(\Omega, N)|^2 \sim N^{\beta}$ holds, and for SNA this scaling exponent takes the value $1 < \beta < 2$. This behavior is shown in figure 3.1b for SNA, where we observe a relatively robust power-law behavior with $\beta = 1.9$. The inset in this figure also demonstrates the fractal walk of the trajectory in the complex *(ReX, ImX)* plane, as required for SNAs.

3.4 *Effect of frequency (ω_1) on the optimal window of logic gates*

Finally, we also examine the effect of frequency (ω_1) on the logical behavior region. Since the system is driven by quasiperiodic forcing (that is the ratio of ω_1/ω_2 is always irrational), we vary the value ω_1 in the logical behavior regime. It is observed in figure 7 that the logical behavior is exhibited in a wide range of ω_1 values, $0.7 < \omega_1 < 5.0$. It confirms that the logical behavior persists even when we change ω_1 over a range.

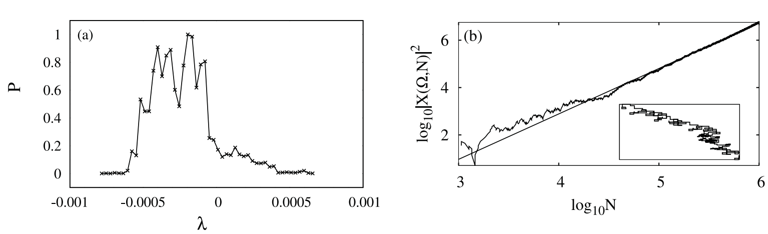

Figure 6. (**a**) Finite-time Lyapunov exponents for QPDMLC circuit including the effect of two-aperiodic square waves: logical SNA for $F_2 = 0.05$. Finite-time Fourier spectra $|X(\Omega, N)|^2$ vs N^{β} on the logarithmic scale for (**b**) logical SNA for $F_2 = 0.05$ with $\beta = 1.9$. The inset in (**b**) shows a fractal walk in the complex plane (ReX, ImX).

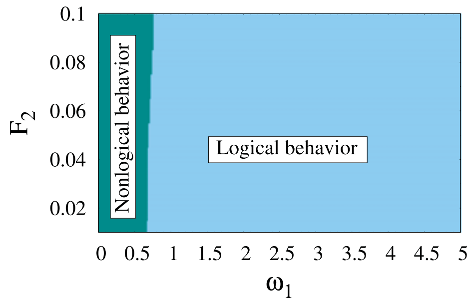

Figure 7. Two parameter phase diagram for frequency ω_1 vs. amplitude of forcing F_2 indicating the logical behavior region.

4. Conclusion

In this work, we have analyzed the response of a QPDMLC circuit to deterministic input signals. We have included the two-square waves in an aperiodic manner as input signals to the MLC circuit, where the response of the system can produce a logical SNA output controlled by the quasiperiodic forcing. Changing the biasing the system can change the 'OR/NOR' logical SNA output to the 'AND/NAND' logical SNA output. In our present work, we have shown that with a low/moderate quasiperiodic forcing, logic operations can be obtained in nonlinear dynamics subjected to two-aperiodic square waves. The dynamical behavior in the logic operation region is SNA. Consequently the dynamics is robust under weak noise. Therefore an efficient computational process can be designed.

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