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Suppression of extreme events under environmental coupling

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Abstract. Extreme events occur in the complex system without generic early warning signals. When the predictability of the extreme events in complex systems is not possible, we can design an environment, which suppresses the extreme event from the systems. In this paper, we report the annihilation of extreme events in the dynamical system under environmental coupling. We consider two systems such as CO_2 laser and FitzHugh–Nagumo neuron models to examine the suppression of extreme events. When the systems are coupled with an environment, depending on the coupling strength the probability for the occurrence of extreme events decreases and after a critical coupling strength, the extreme events are annihilated from the system.

Keywords. Extreme events; CO_2 laser; coupled neurons; controlling.

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1. Introduction

Understanding the dynamics of complex systems has been a more challenging topic over the past few decades. In particular, extreme and catastrophic events are ubiquitous in a wide range of complex systems including natural and man-made systems like turbulent fluid flows, nonlinear waves, large-scale networks and biological systems [1–5]. Because of their catastrophic impact, unpredictability and lack of mathematical tools to understand, the studies of extreme events have attracted substantial attention in various scientific fields and observed in superfluid helium [6], plasma [7], optical fibres [8] and lasers [9–12]. It would be a considerable achievement for humanity in predicting extreme events such as droughts, flooding, financial crashes, seasonal climatic changes and insurance industry [3]. The methods of effective and timely prediction of extreme events can enable governments and stakeholders to take appropriate actions for militating against the adverse impacts by the climate-related extreme events. The extreme events have been identified in linear [6, 7, 13] and nonlinear [14–17] dynamical systems, modelled by partial and ordinary differential equations. However, until now we do not understand many relevant mechanisms of extreme events in complex systems. The probabilistic prediction of extreme and catastrophic events needs accurate modelling. Significant progress has been made in the computation of extreme statistics, and even though these methods estimate the probability distribution of the extreme events, they fail to identify general mechanisms of extreme events and are not capable of predicting individual extreme events. Another way of avoiding extreme events is by identifying the controlling mechanisms of extreme events.

Similarly, laser contributes widely to the scientific communities. Laser technology emerged from hospital operating rooms to office practices, clinics and private enterprises. It would be important that regardless of applications it is a necessity to establish and maintain them safely to the patient and the user at all times. The literature has shown that lasers, such as a class-B modulated CO_2 lasers can produce extreme laser pulses [12]. Likewise, excitable system neurons also exhibit extreme events [18, 19]. For example, the FitzHugh–Nagumo (FHN) model is a widely used model to describe the dynamics of a neuron. The FHN neuron in a network is capable of generating extreme events with non-homogeneous parameters [18, 19]. The routes and their mechanisms for the generation of extreme events were reported in the literature [2, 3]. However, the anticipation of extreme events in the neuronal system is still a challenging task. Failing to predict extreme events leads to epileptic seizures.

Also, the coupling between the systems plays a crucial role in the dynamics of the system like controlling [20–22], synchronization, chimera states [23] etc. In literature many coupling schemes have been introduced in different contexts, to mention a few, linear augmentation [20, 21] has been used to achieve desirable states of the system, delay coupling [24] has been used to understand the delay interaction between the systems, environmental coupling [25] has been used to control the system states and achieve amplitude death states, conjugate coupling [26] has been used to study the mixed interaction between the systems and error coupling [27] has been introduced to understand the diffusive interaction between the systems. These coupling schemes can be used in different contexts, depending on the applications.

Moreover, it is important to establish a predicting or controlling procedure to control the extreme laser pulses and epileptic seizures. For the aforementioned reasons, it is needed to establish a predicting or controlling procedure to control the extreme laser pulses and epileptic seizures. However, in all cases, extreme events cannot be identified before its occurrence. On the other hand, it would be another option to control the extreme event by an external environment, since the parameter of the system may not be accessible to control the extreme event. In the nonlinear dynamics literature, many controlling mechanisms and methods are available to control chaos [21]. However, we find that no study is available to control the emergence of an extreme event. For the present study, we consider a CO_2 laser model and show how to control extreme laser pulses by implementing environmental coupling [28].

2. Periodically modulated CO_2 laser

To show the controlling of extreme events by an environment, we consider the example of a nonlinear system such as a single-mode CO_2 laser modelled by the following equations [12]

$$\dot{I} = \tau^{-1}(N - k(t))I,$$
$$\dot{N} = (N_0 - N)\gamma - IN. \quad (1)$$

Here, I is proportional to the radiation density and N_0 and N are the unsaturated gain and the gain in the active medium, respectively. γ is the gain decay rate and τ is the transit time of light in the resonator. Here, $k(t) = k_0(1 + \alpha \cos 2\pi f t)$ is the modulated cavity loss, k_0 is the constant part of the loss and α and f are the driving amplitude and frequency, respectively. We use the following values of parameters for the present study: $\tau = 3.5\times10^{-9}$ s, $\gamma = 1.978\times10^5$ s^{-1}, $N_0 = 0.175$, $k_0 = 0.17$, $\alpha = 0.19$ and $f = 208.25$ kHz. In the absence of external modulation ($\alpha = 0$) the CO_2 laser exhibits a damped oscillation and the trajectory of the laser converges to a stable equilibrium point ($I_S = \gamma(N_0/k_0 - 1)$ and $N_S = k_0$), while the external modulation α induces periodic and chaotic oscillations in the system. It is preferable that the CO_2 laser operates in the periodic regime. However, the fluctuation in the amplitude of the external modulation makes the system to be chaotic. The presence of chaotic behaviour and extreme events have been reported by many researchers [9, 10, 12]. Figure 1 shows the presence of chaotic behaviour and extreme events in the CO_2 laser model [12]. Figure 1 is drawn by measuring the maximum of the intensity of the laser pulse ($\max(I_A/I_S)$). The colours of the image plot show the amplitude of the laser pulse.

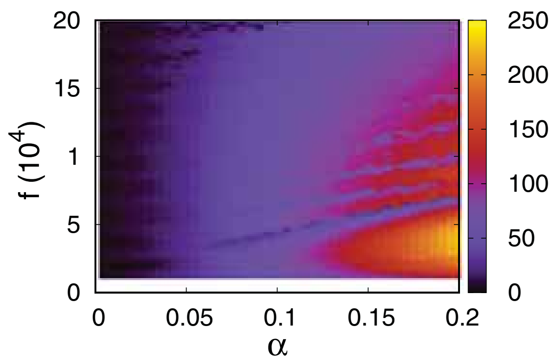

Figure 1. Two-parameter bifurcation diagram as a function of frequency (f) and amplitude (α) of external modulation. Colour in the image plot shows the maximum intensity of the CO_2 laser at a given parameter.

To characterize the extreme event/extreme light pulses we used the following criterion. The critical value of the extreme events is defined as the intensity of the laser exceeding four times standard deviations σ_{I_A/I_S} over the average intensity of the laser. The extreme event in I_{EE} is be defined as $I_{EE} = \langle I_A/I_S\rangle + n\sigma_{I_A/I_S}$. The threshold n is defined as :

$$n = \frac{\max(I_A/I_s)_{EE} - \langle I_A/I_s\rangle}{\sigma_{I_A/I_S}}. \quad (2)$$

Here, $\max(I_A/I_S)_{EE}$ is the maximum intensity of the extreme light pulse I_A/I_S for a given value of the amplitude of the external force α.

In figure 2, we have plotted the time series and probability distribution function (PDF) of the intensity of the laser in the regimes namely no extreme events and extreme events, as a function of the external forcing amplitude α. Figures 2a(i) and a(ii) show the PDF and time series of the laser pulse for $\alpha = 0.145$ which have no extreme events. Figures 2b(i) and b(ii) show the extreme event for the modulation amplitude $\alpha = 0.190$. The horizontal green lines show the average $\langle I_A/I_S\rangle$ of the intensity of the laser pulses. The red line is drawn for the threshold $n = 4$ (extreme event).

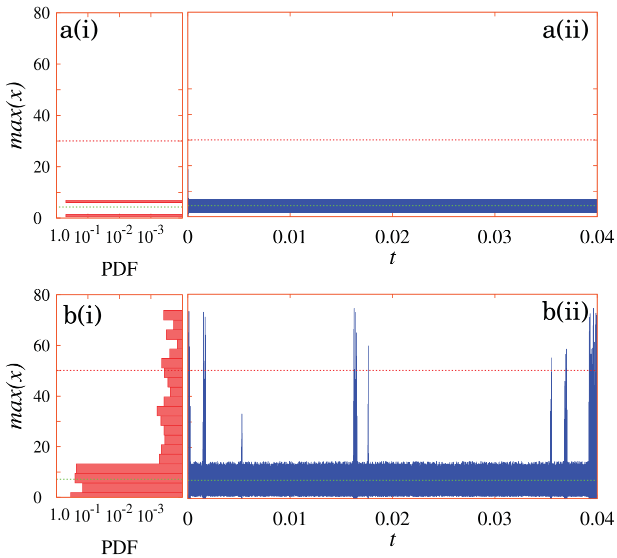

Figure 2. (**a**) (left) PDF and (**b**) (right) time series of laser intensity. The red line shows the threshold value for $n = 6$ and the green line represents average intensity.

3. Controlling an extreme event through environmental coupling

As we have seen from the previous section, the CO_2 laser can emit extreme laser pulses for a wide range of system parameters. In order to control the extreme events in the laser, an environment can be designed as given in the following equations:

$$\dot{I}_i = \tau^{-1}(N_i - k(t))I_i,$$

$$\dot{N}_i = (N_0 - N_i)\gamma - I_i N_i + \epsilon E,$$

$$\dot{E} = -dE - \frac{\epsilon}{M}\left(\sum_i^M N_i\right). \tag{3}$$

All the parameters for the CO_2 laser are taken as given in the previous section and the value of decay constant $d = 0.1$. We have coupled the gain of the two lasers with an environment. The extreme intensity of the laser can be understood with the help of time taken by the system to gain population inversion. If the laser system has taken more time for population inversion then the laser intensity will be high. In order to reduce the number of populated atoms in the higher energy state, we have coupled two CO_2 lasers with its gain. This means that the gains of the two systems are shared with their mean values. Also, we design an environment (E), which reduces the population inversion with delaying constant d. By changing the value of this delaying constant, we can reduce the gain (N) of the coupled CO_2 laser. In figure 3, we have plotted two-parameter bifurcation diagrams as a function of interaction strength between two lasers. Figure 3a shows the regime of extreme events when the coupling strength $\epsilon = 0$. Increasing the interaction strength to $\epsilon = 0.0001$, the extreme events in the coupled laser systems started decreasing, which is shown in figure 3b. Similarly, figure 3c shows the decrease of extreme events as the coupling strength is increased further. Figure 3c is plotted for $\epsilon = 0.1$.

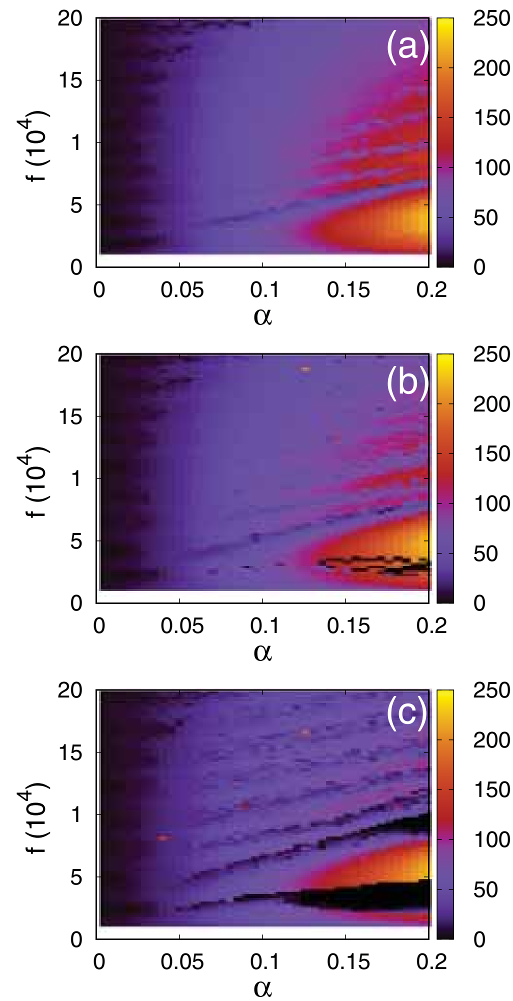

Figure 3. Two-parameter bifurcation diagram for the intensity of the CO_2 laser as a function of amplitude (α) and frequency (f) of the external modulation.

4. Controlling extreme events in FHN neuron model

To generalize the mechanism used for controlling the extreme events in the CO_2 laser, we test the controlling mechanism for another system, namely FHN neuronal model. The equation describing the coupled-FHN

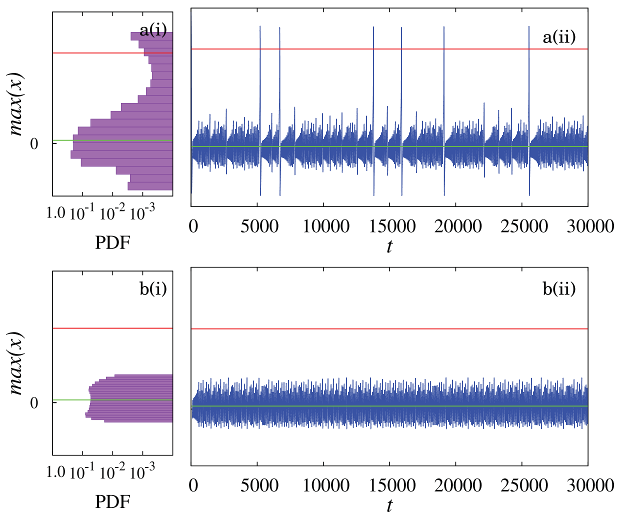

Figure 4. (**a**) PDF and time series of the two-coupled FHN neuron with extreme events. (**b**) PDF and time series of the two-coupled FHN neuron when they are connected with an environment.

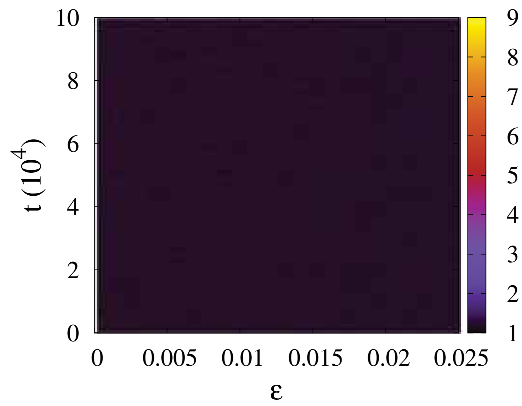

Figure 5. Dynamics of the FHN neuron as a function of coupling strength ϵ with the environment. The system has no extreme events when they are strongly coupled with the environment.

model is given in eq. (5).

$$\dot{x}_i = x_i(a_i - x_i)(x_i - 1) - y_i + \sum_{j=1}^{N} A_{ij}(x_j - x_i),$$
$$\dot{y}_i = b_i x_i - c_i y_i, \tag{4}$$

where x and y are the activator (the membrane potential) and inhibitor (recovery) variables, respectively. a_i, b_i and c_i are the system parameters, k is the coupling strength between the neurons and $A \in \{0, 1\}^{n \times n}$ is the symmetric adjacency matrix (when $A_{ij} = A_{ji} = 1$, if the units i and j are coupled).

The two-coupled FH neurons exhibit extreme events when we consider the system parameters as: $a_1 = a_2 = a = -0.025794$, $b_1 = 0.0065$, $b_2 = 0.0135$, $c_1 = c_2 = c = 0.02$ and the coupling strength ϵ=0.128. Figures 4a(i) and a(ii) show the dynamics of the coupled neuron when the neurons are not connected with any environment. In other words, the coupled neuron exhibits extreme events when they are isolated from the environment. However, neurons are always covered by a chemical environment. Neurons are responsible for the computation and communication that the nervous system provides. The neurons are electrically active and through chemical signals, they communicate with other neurons. However, in the nervous system, neurons are surrounded by several types of glial cells. In the central nervous systems, astrocytes, microglia, oligodendrocytes and ependymal cells perform different functions to support neurons. Astrocytes control the chemical environment around neurons and are crucial for regulating the blood–brain barrier. Oligodendrocytes form a myelin sheath in the nervous system to allow nerve impulses to move more quickly and microglia act as phagocytes and play a role in immune surveillance. Ependymal cells filter blood to produce cerebrospinal fluid. Generally, they are supporting cells for the neurons in the central nervous system. These cells support neurons in the central nervous system by maintaining the concentration of chemicals in the extracellular space, reacting to tissue damage, removing excess signalling molecules and contributing to the blood–brain barrier. Hence, our model reflects the environment around the neuron which controls the chemical environments [29]. The coupled neuron with the controlling environment is given in the equations.

$$\dot{x}_i = x_i(a_i - x_i)(x_i - 1) - y_i + \sum_{j=1}^{N} A_{ij}(x_j - x_i) + \epsilon E,$$
$$\dot{y}_i = b_i x_i - c_i y_i,$$
$$\dot{E} = -dE - \frac{\epsilon}{M}\left(\sum_{i}^{M} x_i\right) \tag{5}$$

Here, we consider the value of the parameter $d = 3$. As we discussed earlier, the coupled neurons exhibit extreme events when it has no interaction with the environment and the corresponding plots are shown in figures 4b(i) and b(ii). When we allow the neurons to interact with the environment the probability of exhibiting extreme events decreases and for a critical value of the interaction strength ϵ the extreme events are completely annihilated from the system. The effect of coupling with the environment on controlling extreme events is shown in figure 5. We can see that in the lower coupling strength the number of extreme events is high (probability of extreme events is high) and while increase in coupling strength decreases the number of extreme events (probability of extreme events is low). We can see that when the coupling strength is higher than 0.02, the coupled neurons exhibit no events. The plots clearly show that the probability of extreme events can be controlled and even annihilated from the system with the environment. In the present study we consider that the environment is coupled with the membrane potential between the neurons. However, one can couple another variable (recovery-variables) with the environment and control the extreme event in different coupling strengths. However the same is not true for other systems, we found that coupling with the environment is system dependent.

From the above results, it has been shown that environmental coupling can be used to control extreme events in the dynamical system. From the literature, we can understand that the environmental coupling can reduce the amplitude of oscillation to the amplitude death state [28]. In a similar way in the present work, the environment coupling reduces the amplitude of oscillation and controls extreme events.

5. Conclusion

In the present work, we have shown the controllability of extreme intensity laser pulses of the CO_2 laser and coupled-FHN neuron with an environmental coupling. The environment is designed in such a way with a decay constant. In the case of the CO_2 laser model the environment reduces the probability of population inversion. Also, we made mean field coupling between the two systems, and since the gain of the two lasers is not same at a particular time, they interact with each other with their mean value of the gain. These decay constants of the environment and mean field interaction reduce the gain of the laser and hence the laser has low-intensity pulses. In the case of the coupled neuron, the environment coupling controls the chemical environment around the neuron. By doing this we can control the electrical activity and the chemical signal between the neurons, which eliminates the extreme events from the system.

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References

[1] S Albeverio, V Jentsch and H Kantz, *Extreme Events in Nature and Society* (Springer, Berlin, 2006)
[2] C Nicolis, V Balakrishnan and G Nicolis, *Phys. Rev. Lett.* **97**, 210602 (2006)
[3] G Nicolis and C Nicolis, *Foundations of Complex Systems*, 2nd edn (World Scientific, Singapore, 2012)
[4] P Bak, K Christensen, L Danon and T Scanlon, *Phys. Rev. Lett.* **88**, 178501 (2002)
[5] S Kumarasamy and A N Pisarchik, *Phys. Rev. E* **98**, 032203 (2018)
[6] A N Ganshin, V B Efimov, G V Kolmakov, L P Mezhov-Deglin and P V E McClintock, *Phys. Rev. Lett.* **101**, 065303 (2008)
[7] H Bailung, S K Sharma and Y Nakamura, *Phys. Rev. Lett.* **107**, 255005 (2011)
[8] D R Solli, C Ropers, P Koonath and B Jalali, *Nature (London)* **450**, 1054 (2007)
[9] A N Pisarchik, R Jaimes-Reátegui, R Sevilla-Escoboza, G Huerta-Cuellar and M Taki, *Phys. Rev. Lett.* **107**, 274101 (2011)
[10] C Bonatto, M Feyereisen, S Barland, M Giudici, C Masoller, J R Rios Leite and J R Tredicce, *Phys. Rev. Lett.* **107**, 053901 (2011)
[11] M G Kovalsky, A A Hnilo and J R Tredicce, *Opt. Lett.* **36**, 4449 (2011)
[12] C Bonatto and A Endler, *Phys. Rev. E* **96**, 012216 (2017)
[13] A Mussot, A Kudlinski, M Kolobov, E Louvergneaux, M Douay and M Taki, *Opt. Express* **17**, 17010 (2009)
[14] F T Arecchi, U Bortolozzo, A Montina and S Residori, *Phys. Rev. Lett.* **106**, 153901 (2011)
[15] C Kharif and E Pelinovsky, *Eur. J. Mech. B* **22**, 603 (2003)
[16] P K Shukla, I Kourakis, B Eliasson, M Marklund and L Stenflo, *Phys. Rev. Lett.* **97**, 094501 (2006)
[17] F Baronio, M Conforti, A Degasperis, S Lombardo, M Onorato and S Wabnitz, *Phys. Rev. Lett.* **113**, 034101 (2014)
[18] R Karnatak, G Ansmann, U Feudel and K Lehnertz, *Phys. Rev. E* **90**, 022917 (2014)
[19] G Ansmann, K Lehnertz and U Feudel, *Phys. Rev. X* **6**, 011030 (2016)
[20] P R Sharma, A Sharma, M D Shrimali and A Prasad, *Phys. Rev. E* **83**, 67201 (2011)
[21] V Varshney, P R Sharma, M D Shrimali, B Biswal and A Prasad, *Int. J. Nonlinear Sci.* **26**, 1 (2018)

[22] N Punetha, V Varshney, S Sahoo, G Saxena, A Prasad and R Ramaswamy, *Phys. Rev. E* **98**, 022212 (2018)

[23] M J Panaggio and D M Abrams, *Nonlinearity*, **28**, R67 (2015)

[24] D V Ramana Reddy, A Sen and G L Johnston, *Phys. Rev. Lett.* **80**, 23 (1998)

[25] A Sharma, M D Shrimali and S K Dana, *Chaos*, **22**, 023147 (2012)

[26] R Karnatak, R Ramaswamy and A Prasad, *Phys. Rev. E* **76**, 035201(2007)

[27] T Stankovski, T Pereira, P V McClintock and A Stefanovska, *Rev. Mod. Phys.* **89**, 045001 (2017)

[28] V Resmi, G Ambika, R E Amritkar and G Rangarajan, *Phys. Rev. E* **85**, 046211 (2012)

[29] http://library.open.oregonstate.edu/aandp/chapter/12-2-nervous-tissue/