Indian Academy of Sciences Conference Series (2019) 2:1
DOI: 10.29195/iascs.02.01.0013

Reviving oscillations due to the memory in coupled nonlinear oscillators

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Abstract. Experimental and numerical observations of reviving oscillations from amplitude death (AD) in two-coupled mismatched nonlinear electronic circuits are described. The inclusion of processing delay in the coupling node of the coupled Chua's oscillators creates a memory of the past state for a finite time duration. Similarly, the fractional order dynamical system has a memory function and remembers the past state of the system. We show that these memory functions are responsible for the reviving and sustained oscillations in the coupled oscillators.

Keywords. Amplitude death; processing delay; reviving oscillations.

PACS Nos 05.45.Tp; 05.45.Xt; 05.45.−a

1. Introduction

Coupled nonlinear oscillators have been extensively studied in recent years because of their use in understanding various complex behaviours in many physical, chemical and biological systems. Among the various phenomena, quenching of oscillations [1–7] is a well known emergent behaviour in coupled nonlinear oscillators when they drive each other to a stable equilibrium. This phenomenon was evidenced first as an unexpected silencing of two side-by-side organ pipes [8] and, later demonstrated in chemical oscillators [9, 10]. This strange phenomenon was, at first explained [3] as an effect of large parameter mismatch on the coupled oscillatory systems. Later, it was also observed in two identical oscillators when a critical time delay is introduced in coupling [11]. Different coupling forms were also found to be able to induce the quenching of oscillations and two distinct classes of the quenching effect have been identified [2] as amplitude death (AD) and oscillation death (OD). The AD always emerges via reverse Hopf bifurcation in coupled systems [1], while the OD originates via different bifurcation routes, pitchfork, transcritical or saddle-node bifurcation [2] that depends on the coupling form as well as the dynamical system. The AD has advantages in some real-world applications such as stabilisation of DC micro-grids [12], laser systems [13] and synthetic genetic networks [14, 15]. While many other real systems have sustained oscillatory behaviour even they are coupled with another system.

Recently, a few investigations have been performed on the methods to avoid AD in nonlinear dynamical systems [16–21]. Among them, introducing a processing delay in the coupling annihilate the AD, in a network of coupled oscillators. The processing delay is generated in dynamical systems due to a finite response time required for processing the input signal. In other words, the coupled systems remember the past state of the system for a finite time. This property of the coupled system with processing delay analogues the fractional order dynamical systems (FOSs) with a memory of past. The FOSs are generic in nature and the integer order systems are the special case of FOSs. Since the fractional order enhances the oscillatory behaviour in the dynamical system [22], it would be of practical importance to understand the memory properties of FOSs. In this letter, we present the reviving of oscillation induced by processing delay and fractional order in two-coupled mismatched Chua's oscillators. We propose that in both the cases the memory function of the system brings the oscillatory behaviour from the AD state.

2. Electronic analogue of Chua's oscillator and reviving of oscillations

The Chua's oscillator is considered for examining the reviving oscillations with processing delay and fractional order. Instead of inductance–capacitance–resistance (LCR) circuit-based Chua's oscillator, we use the electronic analogue concerning the circuit, because any dynamical equations can be constructed circuitry using analogue simulation and it is straightforward to implement the concept processing delay through analogue simulation. The design of the analogue circuit is based on the methodology described in Ref. [23]. The dynamics of the Chua's oscillator is described by a set of three first order differential equations, and they are constructed using the analogue inverting weighted integrators. The parameters of the oscillator are described by the resistors, which are determined by the base resistor R_B and integrator capacitor, C [23]. We consider two analogue Chua's circuits and couple them bidirectionally between the variables y_1 and y_2. The equations for the coupled oscillators are given in eq. (1).

$$\frac{dx_{1,2}^{q_1}}{dt} = \alpha[y_{1,2} - x_{1,2} - h(x_{1,2})],$$
$$\frac{dy_{1,2}^{q_2}}{dt} = x_{1,2} - y_{1,2} + z_{1,2} + \epsilon_{1,2}(y_{2,1}(t-\tau) - y_{1,2}(t-\tau)),$$
$$\frac{dz_{1,2}^{q_3}}{dt} = -\beta_{1,2}y_{1,2}, \quad (1)$$

where

$$h(x_{1,2}) = bx_{1,2} + 0.5(a-b)[|x_{1,2}+1.4| - |x_{1,2}-1.4|].$$

Figure 1a shows the schematic diagram of the coupled analogue Chua's oscillators. The processing unit in figure 1b contains two processes: (i) giving coupling function and (ii) generating processing delay (while considering the case of fractional order we consider the delay $\tau = 0$). The coupling function can be anything like diffusive, delay and conjugate coupling, etc., for instance here we consider diffusive coupling. The processing unit takes time τ to process the input signal, and depending on the coupling scheme, it passes the signal to the respective system. To implement the processing delay the active first order namely all pass filter (APF) is used. The APF have the following transfer function, $T(s) = -g_1 s - w_0/s + w_0$, where it has flat gain at $g_1 = 1$, which is determined by $R_1 = 10$ kΩ and $R_2 = 10$ kΩ. Because of the linear phase response, the APF generates a delay $\tau(= T_d) \approx 2RC$.

For the numerical study, the system parameters are taken as constant and having the values as $\alpha = 10$, $a = -8/7$, $b = -5/7$, $\beta_2 = 20$ and β_1 is taken as a variable. We use the parameter β_1 to make parameter mismatch ($\Delta\beta = \beta_2 - \beta_1$) between the two systems. In the numerical study the parameter mismatch $\Delta\beta$ can be quantified, while in the real time hardware

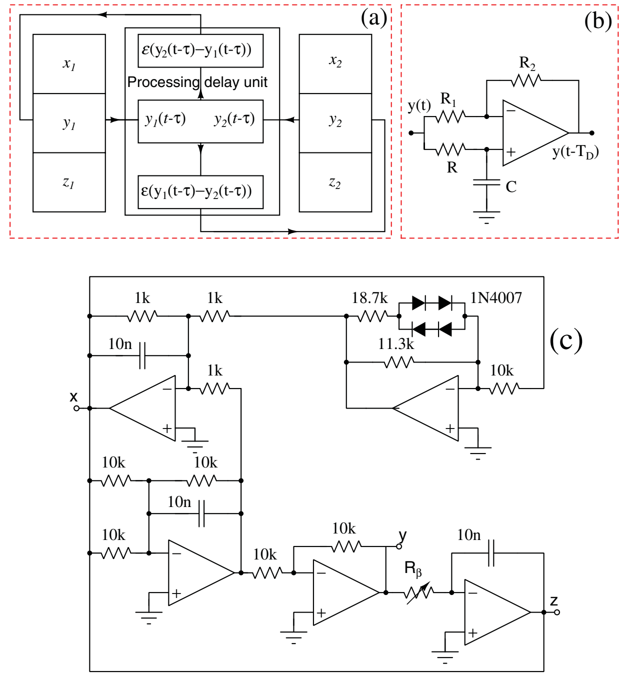

Figure 1. (**a**) Connection scheme of the delay-coupled Chua's circuits, (**b**) the all pass filter (APF) for generating the processing delay and (**c**) circuit of analogue Chua's circuit.

experiment, it is difficult to quantify the total mismatch between the coupled systems because of the circuit components (each system having 12 resistors, three capacitors and four diodes and they have the tolerance). To overcome this difficulty, we consider the frequency difference ($\Delta f = f_1 - f_2$) as the parameter mismatch. $\Delta R_\beta = R_{\beta_1} - R_{\beta_2}$ in the experiment essentially creates frequency difference between the systems, and it can be quantified. Hence, we use the frequency difference (Δf) as the mismatch in the experimental study.

The numerical phase diagram is drawn between the coupling parameter ϵ and $\Delta\beta$ as shown in figure 2a. The diagram is drawn by measuring the amplitude of the time series y. We considered 500000 data points and removed 300000 data points as transient. When the amplitude of the variable y is below 10^{-5} then we considered it as a AD state. β_2 is fixed at 20 and β_1 is varied. The coloured regime corresponds to the AD and the white regime corresponds to the oscillatory regime. In the regime marked I and II, the system has anti-phase and in-phase synchronisations respectively. The red regime is corresponding to the AD when the delay (τ) is zero. While including the delay in the coupling, the AD regime decreases. For $\tau = 0.03$, the shrunk AD is marked in blue colour. Further increasing the delay ($\tau = 0.05$), the reduced AD regime is shown in magenta colour. When the delay reached $\tau = 0.07$, the AD regime completely disappears from the system.

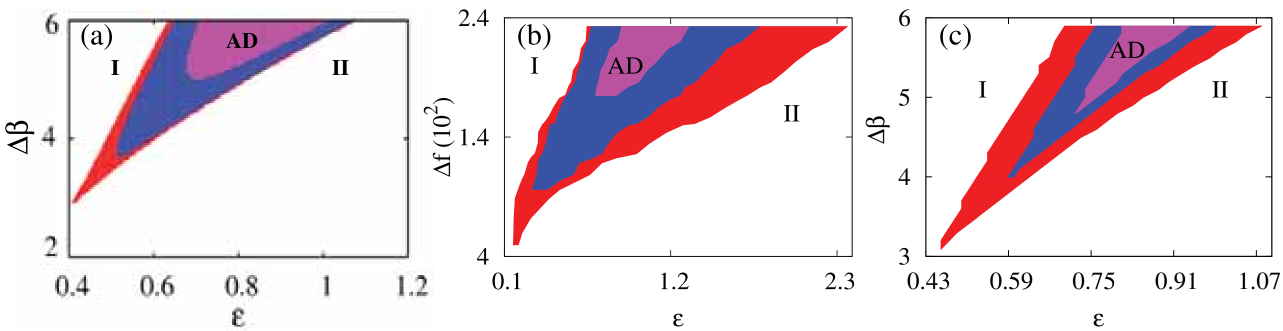

Figure 2. Phase diagrams of the coupled Chua's oscillators. (**a**) Numerical study in the (ϵ–$\Delta\beta$) plane and (**b**) experimental observation in the (ϵ–Δf) plane. (**c**) For the coupled fractional order Chua's circuit in the (ϵ–$\Delta\beta$) plane as a function of order q_2 of the system.

In the experiment, we created the frequency mismatch similar to that of the numerical study. To identify the AD regime in the system when the delay becomes zero ($\tau = 0$), the resistance R in the filter circuit is kept at zero. Hence, the filter circuit produces no delay, and the system becomes mismatched oscillators. The coupling resistors between the systems are varied in such a way to identify the transition from the oscillatory state to the AD state. The plot in figure 2b shows the boundary between the AD state and oscillatory state as a function of coupling strength ϵ and the frequency difference Δf. To enable the delay in the coupled variables, the resistance R in the filter circuit is increased to 500 Ω. The delay τ produced by the filter is $T_D = 2RC = 2 * 500 * 10\ n = 10\ \mu$s. The AD regime for $T_D = 10\ \mu$s is shaded in blue colour in figure 2b. It shows the shrunk regime of AD compared with the regime of AD without delay. Further, the resistance R is increased to 1000 Ω; the filter circuit produces delay $T_D = 20\ \mu$s and the corresponding AD regime is shown in figure 2b in magenta colour, which has a smaller regime of the AD state. The AD regime is completely disappearing when R is increased to 1350 Ω ($T_D = 27\ \mu$s). Figures 3a(i) and (ii) show the phase space and time series of the oscillators in the anti-phase regime and figures 3b(i) and (ii) correspond to the AD state. Figures 3c(i) and (ii) are the phase space and time series of the reviving of the oscillation in-phase regime.

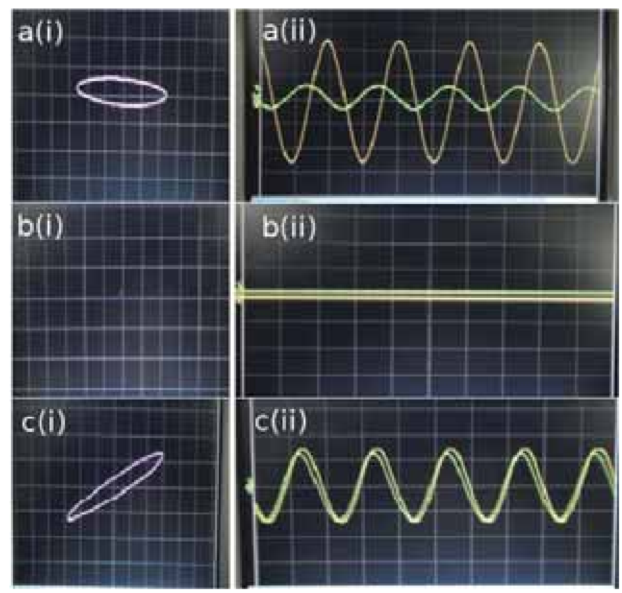

Figure 3. Experiment: (i) Phase space and (ii) time series of the three dynamical regimes: (**a**) anti-phase, (**b**) AD and (**c**) in-phase regimes.

3. Reviving of oscillations in coupled fractional order Chua's oscillators

In the previous section, we have seen that the processing delay or the memory of the system in the coupling leads to reviving of oscillations in coupled Chua's oscillators. Here, we show that the order of the system varies the coupled oscillators reviving the oscillatory state from the AD. We consider the delay ($\tau = 0$) and the order $q_2 < 1$ in eq. (1). Figure 2c shows the phase diagram of the coupled Chua's oscillators in the (ϵ–$\Delta\beta$) plane as a function of order q_2. In figure 2c the red coloured regime is plotted for the order $q_2 = 1.0$. Whilst the order $q_2 = 1.0$ the system behaves like an integer user system and the system has very low memory (remember only the previous state), hence the regime of the AD state is similar to the case when processing delay is zero. However, when decreasing the order q_2 to 0.97 the AD regime is decreased as shown in blue colour in figure 2c, because the system acquires memory due to the presence of fractional order. Similarly, when the order q_2 is decreased to 0.94, the AD regime decreases further, which is shown in magenta colour in figure 2c. We can compare this with the results of increasing of processing delay in the coupling, because when decreasing order q_2 increases the memory of the system (similar to the increasing of memory in the case of processing delay). Beyond a critical value of the order, say $q_2 = 0.91$, the AD state is annihilated from the systems. The above study shows that when the order q_2 is decreased the AD regime is gradually annihilated from the system. It shows that the fractional order in the dynamical system enhances the oscillatory behaviour of the oscillator.

4. Conclusion

We have numerically and experimentally demonstrated the processing delay induced reviving of oscillation from the AD in coupled nonlinear oscillators. In the presence of processing delay, the coupled system has the memory of the past state in the coupling node in the name of processing time. When the memory of the node increased to a critical value then the coupled system completely annihilate the stable states from the system and enhances the oscillatory state. Similarly, the system with fractional order remembers the past state for the infinite time. When the order is changed, the memory function of the fractional order also changes. After a critical value of the order q_2, the AD state is annihilated from the system. We found that in both the cases the memory behaviour of the system or the node of the coupled system enhances the oscillatory state. Most of the natural systems are in the oscillatory state, and the reason for the sustained oscillatory behaviour can be understood with the help of the fractional order system and processing time between the systems.

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