Phase diagram of high temperature \( Y_1Ba_2Cu_3O_{7–\delta} \) superconductor by Bean’s model and experimental techniques

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MS received 25 March 2022; accepted 26 June 2022

Abstract. Phase diagram of \( Y_1Ba_2Cu_3O_{7–\delta} \) (Y-123) polycrystalline sample via Bean’s model and experimental techniques is depicted by analysing electrical resistivity and vibrating sample magnetometer (VSM). A home-made VSM was used to take \( M–H \) data. The resistively upper critical field \( H_{c2}(T) \) of the sample was deduced from resistivity measurements under magnetic field perpendicular to the current flow direction study. The upper critical field in zero temperature \( H_{c2}(0) \) was determined by extrapolating applied magnetic field \( (H) \) vs. temperature \( (T) \). The coherence length, lattice parameters, oxygen content, penetration depth and critical current density at absolute temperature are estimated. The critical state equation \( J_c = J_c(T,H) \), based on experimental results and Bean critical-state model, was proposed to calculate the critical current density \( J_c \) vs. applied magnetic field. Applied critical magnetic field \( H_c \) and critical current density \( J_c \) vs. temperature \( T \) are calculated and plotted. We finally plotted the phase diagram of the polycrystalline sample as an essential structure for the progress of comprehensive applications of high-temperature superconductors, where substantial critical current densities \( (J_c) \) in the vicinity of applied fields and temperatures are required.

Keywords. YBCO (Y123); critical superconducting parameters; phase diagram; Bean’s model.

1. Introduction

Scientific information about high-temperature superconductors (HTSCs) will be contingent censoredly on probable understandings either the band structure particular to YBCO or a graphical difference at a critical point in the phase diagram [1]. Phase diagram can be obtained experimentally as temperature–magnetic field \( (T–H) \), temperature–critical current density \( (T–J_c) \) and magnetic field–critical current density \( (H–J_c) \) graphs from the measurements using various techniques. They can also be calculated by the theoretical models. As far as our understanding, the phase diagram in HTSCs increases, while applications will become more practical. Upholding the superconducting state needs that temperature, the magnetic field and the current density stay lower the critical parameters, while all of them depend on the material [2]. Some techniques for measuring \( J_c, H_c \) and critical temperature \( T_c \) have been reported, for example, related to \( J_c \) the permanent magnet method [3], the third-harmonic voltage method [4] and Hall probe method [5]. Among them, it is difficult to measure the large \( J_c \) samples with the third-harmonic voltage method, and the Hall probe method is complicated. Although the permanent magnet method is more suitable to measure critical current density \( (J_c) \) of HTSCs thin films, for polycrystalline we used a vibrating sample magnetometer (VSM). The value of critical current density \( (J_c) \) is proportional to the magnetization width \( \Delta M \) for a flat plate [6] and a function of temperature; the superconductor is able to transport more current if it is kept colder [7]. To measure the critical temperature \( (T_c) \) resistivity, the standard four-point probe method is the most popular technique [8]. For the determination of critical magnetic field \( (H_c) \), some techniques such as Werthamer, Helfand and Hohenberg (WHH) typical theory [9] and the Ginzburg–Landau (GL) mean-field theory can be used. Numerous measurements of \( H_{c2}(0) \) on YBCO single crystals and thin films [10–12] have been carried out earlier, while for YBCO polycrystalline \( H_{c2}(0) \) can also be estimated from the Ginzburg–Landau (GL) mean-field theory [13].

In this research, extensive studies have been carried out on the critical current density \( (J_c) \), critical magnetic field \( (H_c) \) and critical temperature \( (T_c) \) associated with high-temperature YBCO bulk superconductor. Some of the critical current density \( (J_c) \) and critical magnetic field \( (H_c) \) data were calculated by theoretical work based on Bean critical-state model. The resistively upper critical field \( H_{c2}(T) \) of the sample, deduced from resistivity measurements under a magnetic field perpendicular to the current flow direction study. The upper critical field in zero temperature \( H_{c2}(0) \) was determined by extrapolating applied magnetic field \( (H) \) vs. temperature \( (T) \). The coherence length, the penetration depth and critical current density at absolute temperature are estimated. Eventually, we plot the phase

Published online: 18 October 2022
diagram to understand superconducting state and practical applications promptly.

2. Experimental

2.1 Sample preparation and X-ray diffraction

The polycrystalline Y$_1$Ba$_2$Cu$_3$O$_{7-\delta}$ sample was prepared by the conventional solid-state reaction method. According to the chemical formula the ratios of Y: Ba: Cu = 1:2:3, YBCO was synthesized by high-purity (99.9%) powders of Y$_2$O$_3$, Ba$_2$CO$_3$ and CuO (Merck, Germany). After formulation, the structure and stage ID of YBCO (Y-123) sensitized powder sample was stipulated by means of a Bruker D8 Advance diffractometer for X-ray diffraction (XRD). The Rietveld method and X Pert high score software were used to analyse XRD data. Oxygen content and lattice parameters of the sample were valued by XRD technique. Details on the sample preparation and XRD have been reported elsewhere [8].

2.2 Resistivity and resistively upper critical field

One of the most commonly used methods for studying the transport properties in high-$T_c$ superconductors, is electrical resistivity measurements. These measurements were done to report the none and superconducting states of the Y123 materials with and without applied magnetic fields. The measurements were undertaken using the standard four-point probe method with an AC lock-in technique and a PC data acquisition system. Interrelated to measure the resistively upper critical field $H_{c2}(T)$ of the sample, Physical Properties Measurement System was used (QD, San Diego, CA92121).

2.3 Vibrating sample magnetometer

To study critical current density ($J_c$) in high-$T_c$ superconductors, a vibrating sample magnetometer (VSM) is necessary. The magnetization of a sample in a uniform magnetic field is related to the vector equation;

$$B = \mu_0(H + M),$$

where $H$ is the magnetic field intensity in A m$^{-1}$, $\mu_0$ is the permeability of free space ($4\pi \times 10^{-7}$ H m$^{-1}$), $M$ is the induced moment per unit volume or magnetization, and $B$ is the magnetic induction or flux density measured in Tesla or Weber per square metre (in SI units). This equation is valid for all conditions. A VSM was constructed to enable measurements of magnetization vs. applied field for samples up to $4 \times 4 \times 2$ mm$^3$. The design of the instrument is related to the work of Smith [14] and Foner [15]. The sample is vibrated by an electromechanical driver. The sensor coils are wound in opposition and connected in series. The coils are axially located inside the stainless steel Dewar. A computer records data from the lock-in amplifier (Stanford SR 530) and magnet hall probe. A reference voltage is normally taken from the vibrator supply, although it can be taken from the reference coil. Sweep rates regarding the applied field can be varied from 0.0004 to $2 T$ min$^{-1}$. The use of the VSM allows samples to be measured under different applied field orientations. Samples measured using the VSM equipment was secured to the sample rod by ‘superglue’. Measurements can be made for two sample orientations, Ha || c-axis and Ha || ab-plane for single and melt-texture crystals, but only data for polycrystalline independent of the axis are reported due to the requirements of our concentration on polycrystalline. $M$–$H$ hysteresis loops were performed at 77 K in applied fields to 1.1 T using a 10-min sweep time. The critical currents were calculated using the extended Bean equations, which give the relationship between $J_c$ and the magnetization width $\Delta M$ for a flat plate as follows;

$$J_c = \frac{10\Delta M}{l_1\left(1 - \frac{l_2}{l_1}\right)},$$

where $l_1$ = 3mm and $l_2$ = 3mm are the lateral dimensions of the sample, and Ha \perp $l_1l_2$ plate. Among various techniques for measuring the critical current density ($J_c$) in superconductors, VSM is suitable for bulk superconductors with relatively high oxygen content samples such as our sample containing oxygen content 6.96 and lattice parameters 3.8183, 3.8843 and 11.7036 Å related to $a$, $b$ and $c$, respectively. All the measurements were performed under zero-field cooling (ZFC) conditions at the same sweep rate. This $J_c$ measurement technique that signifies the current aptitude of the wire, is also essential to whichever approximations of AC loss [16].

2.4 The critical state

The concept of the critical state shows an important part in understanding the irreversible magnetic manner in HTSCs. Therefore, by way of simplification of the boundary conditions, we neglect the influence of the minor critical field and either possible outcome of an exterior current or a stability magnetic mode. We use a generalized critical state equation of the form

$$J_c(B,T) = \frac{J_0(T)}{1 + \frac{B}{B_c}}^n.$$

Choosing $n = 0$ and then 1, equation (3) simplicities to Bean’s archetypal and Kim’s equation due to a critical state, respectively. The critical state equation $J_c = J_c(T,B)$ is, in general, very complicated. In practice, various basic versions have been proposed. It was originally assumed by
Bean that $J_c(T,B)$ was a constant in the critical state. Later, based on experimental results, Kim et al [17] proposed the following critical state equation:

$$J_c(B, T) = \frac{J_0(T)}{1 + \frac{B}{B_0}}.$$  \hspace{1cm} (4)

In the above equation, critical current density without any field with a particular temperature such as $T$ is defined $J_0(T)$. Local flux density is denoted by $B$ and a model-dependent parameter is symbolized by $B_0$. We assumed $B_0$ for polycrystalline was the same as a single crystal and is obtained by the dignified magnetic hysteresis loops (MHLs) from the prolonged Bean critical-state standard report [6]. The MHLs were controlled over the temperature range $60 \leq T \leq 86$ K operating a marketable superconducting quantum interference device magneto measure (SQUID) ‘San Diego, CA92121, Model MPMS7 and Model XL’ equipped with $\pm 5$ T magnet [2].

3. Results and discussion

Figure 1a represents $M$–$H$ loops measurement of the polycrystalline YBCO (Y123) sample at 77 K temperature. According to this figure, rising magnetic field intensity decreases the magnetization width $\Delta M$, which cause the reduction in critical current density values.

Figure 1b shows the derived critical current density $J_c$ values and their fit curve for the polycrystalline sample YBCO. The zero-field $J_c$ value for the polycrystalline YBCO sample is very low (754.5 A cm$^{-2}$) in comparison with the other samples due to a large number of grain boundaries. For MTG (Nd/Y)BCO (neodymium/yttrium barium copper oxide), crystals are $2\times$ larger than those obtained for the YBCO single crystal, but almost $2\times$ smaller than the MTG (Y123) value, the last result being affected by the amount of Y211 (12%) in the case of MTG (Y123). Compared to the undoped Nd (123), the $J_c$ of the Nd (422)-doped sample is very high (>10000 A cm$^{-2}$) [18].

The shape of critical current density $J_c$ vs. the applied magnetic field and its fit curve are consistent with the data reported for thin-film single-crystal YBCO when the field was oriented perpendicular to the c-axis at 77 K [19]. It is clear that the value is much lower for our polycrystalline ($\approx$ 754 A cm$^{-2}$). Comparing with this value and the average $J_c$ value belong to the sample prepared from another method such as the slurry containing 0 wt% polyvinyl alcohol (PVA), which demonstrates that the $J_c$ value of the polycrystalline sample without PVA was about two times smaller than that of the polycrystalline sample [20]. This statement can be clarified by the disparity of the density. The average density of the sample with the polycrystalline and without PVA sample was $6.37 \pm 0.4$ and $4.6 \pm 0.3$ g cm$^{-3}$, respectively. Since the superconducting path in the

Figure 1. (a) $M$–$H$ loops measurement of YBCO (Y123) sample at 77 K temperature. (b) VSM data for polycrystalline (Y123) sample, showing critical current density $J_c$ and its fit vs. applied magnetic field Ha. (c) $M$ vs. $T$ curves of YBCO (Y123) sample.
sample is proportional to the density, increasing density cause rise in $J_c$, therefore the critical current density value of the polycrystalline sample enhanced compared with the sample prepared from the slurry without PVA.

Figure 1c exhibits the $M-T$ measurements, which have been achieved using PPMS technique. The $M-T$ measurement of the YBCO (Y123) superconducting specimen has shown that there is a significant consistency with the critical temperature that is 91 K.

Figure 2a includes the resistivity vs. magnetic field data for YBCO (Y123) sample. One might observe that the data for temperature less than 90.6 K show some linear background before the resistivity speedily increases. The reason for this linear background has not been identified, but $H_{\text{onset}}$ for this specimen have been determined with the deviation from the lined background. The onset field $H_{\text{onset}}$ is simply the magnetic field at which the primary resistivity is found to differ from zero in the resistivity vs. magnetic field graph. To describe the other part of the resistive transition in the magnetic field vs. temperature plane, those magnetic fields at which the resistivity equals 50% of the 'normal-state' resistivity were determined and symbolized as $H_{50}$ by arrows in figure 2a. In conventional superconducting materials, $H_{50}$ is frequently related to the mean-field upper critical field $H_{c2}$. This property might be used as an effective tool for determining of the $H_{c2}^0(T)$ in the HTSCs. However, one usual assignment of $H_{50}$ and consequently $H_{c2}^0(T)$ can be achieved by the connection of two straight line extrapolations from the sharpest slope in the transition region and the normal state resistivity area. Parameters such as $(\rho_n)$ resistivity in normal-state, $(\rho_{50})$ resistivity in the midpoint of the transition $H_{50}$, and $(H_{\text{onset}})$ the onset magnetic field, correlated to the temperatures are listed in table 1. $(\rho_n)$ is estimated to be 0.506 m $\Omega$ – cm at 87.2 K. As a result, the data shows a very broad resistive transitions associated with YBCO (Y123) polycrystalline superconductor.

Figure 2b shows the resistive transition in the applied magnetic field $(H)$ vs. temperature $(T)$, deduced from the nominated tinges of resistivity $\rho$ vs. applied magnetic field $H$ data for the cuprate YBCO. The temperatures for the tinges are from 87.2 to 91 K. Illustrative flaw rods are joined to the figure. This resistive transition regarding YBCO sample proposes such a conception due to the ‘resistive higher critical field’ where none of the superconducting state resistivity is refurbished and could remain an uncertain idea related to particular YBCO superconductors. Furthermore, it can be revealed that resistivity vs. temperature plot stays to be non-linear and starts at 91.5 K in pseudogap phase. This display the Fermi-liquid behaviour in the resistivity graph at temperature greater than $T_c$. Therefore, the two properties in diverse temperature areas about $T_c$ are fairly contrasting: Non-Fermi liquid behaviour in the resistivity at interval temperature 90–91 K and Fermi-liquid at temperature greater than critical temperature. Although temperature dependency of these dissimilar properties in the resistivity have been identified in another composites [21,22], but the basis of these behaviour is still unclear [23,24]. To confirm the information related to the magnetic field with further conservative data, the intermediate contact of the resistive transition can be compared with the $H_{c2}(T)$ lines gritty from the resindable magnetization for YBCO [25]. As a result, the $H_{c2}$ line obtained from the reversible magnetization by Welp et al [25] is in a good agreement with the midpoint fit to the resistive transition. The magnetic field vs. temperature diagram shown in figure 2b obviously designates that, for the YBCO superconductor considered, the eccentric upward is normal for throughout the whole temperature range measured here. Though, because of the very sizeable higher

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**Figure 2.** (a) Nominated traces of resistivity vs. magnetic field data for YBCO compound. $H_{50}$ and $H_{\text{onset}}$ are shown by arrows on the traces. (b) The resistive transition data for polycrystalline (Y123) sample, showing applied magnetic field $(H)$ vs. temperature $(T)$. The representative error bars are attached to the data.
Table 1. Parameters $H_{c_{0.0}}$, $(\rho_{50})$ resistivity at $H_{c0}$ and $(\rho_{n})$ resistivity on normal-state at temperature varying in the range 87.2–90.6 K related to Y123 superconductor in the vicinity of different (0–5 Tesla) applied magnetic field.

<table>
<thead>
<tr>
<th>Temperature (K)</th>
<th>90.6</th>
<th>90.2</th>
<th>89.8</th>
<th>89.5</th>
<th>89.1</th>
<th>88.7</th>
<th>88.4</th>
<th>88</th>
<th>87.6</th>
<th>87.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\rho_{50})$ Resistivity at $H_{c0}$ (m $\Omega$ – cm)</td>
<td>0.591</td>
<td>0.548</td>
<td>0.505</td>
<td>0.462</td>
<td>0.426</td>
<td>0.385</td>
<td>0.365</td>
<td>0.315</td>
<td>0.275</td>
<td>0.253</td>
</tr>
<tr>
<td>$(\rho_{n})$ Resistivity at normal-state (m $\Omega$ – cm)</td>
<td>1.188</td>
<td>1.096</td>
<td>1.01</td>
<td>0.924</td>
<td>0.852</td>
<td>0.77</td>
<td>0.73</td>
<td>0.63</td>
<td>0.55</td>
<td>0.506</td>
</tr>
</tbody>
</table>

Critical fields, investigations on the resistive transition in the YBCO have been mainly restricted to samples in which $T_c$ and $H_c$ are significantly repressed, either in intensely underdoped [26] or overdoped samples [27]. Our data denote that determination of the curve on $H_n$ and $T$ YBCO have been mainly restricted to samples in which critical fields, investigations on the resistive transition in the YBCO single crystal when the field was oriented perpendicular to a–b plane and $H_{c2}(0) = 130$ T.

To estimate the order of magnitude of $J_c$, we assume $\lambda = 140$ nm, which is in excellent agreement with data for $\lambda$ [36] and data for $\kappa = \lambda / \xi$ [37]. The following equations for different coherence lengths $\xi$ and penetration depth $\lambda$, at different temperatures were used respectively.

$$\xi_{GL}(T) = \frac{\xi_L(0)}{\sqrt{1 - T/T_c}}$$  \hspace{1cm} (7)

$$\lambda_L(T) = \frac{\lambda_L(0)}{\sqrt{1 - T/T_c}}$$  \hspace{1cm} (8)

Figure 4 shows critical current density vs. temperature curve for the polycrystalline Y(123) sample when applying numerical factor, which was obtained by comparing VSM data with a theoretical value of $J_c$ at 77 K, given by the equation as follows:

$$J_c(0) = \frac{\varphi_0}{\sqrt{3} \pi \xi^2(0)} \frac{\xi_L(0)}{\sqrt{1 - T/T_c^c}} \frac{2}{\sqrt{5} \pi \lambda^2(0) \xi(0)} H_{c2}(0)$$  \hspace{1cm} (9)

We obtained a value of 39813.894 A cm$^{-2}$ for critical current density close to absolute zero. This value is less than the most recent report [38] by almost a factor of 2.85 ($1.13668 \times 10^7$ A cm$^{-2}$) and 2.62 for YBCO polycrystalline (104310 A cm$^{-2}$) [37]. Comparing these values

Figure 3 shows the applied critical magnetic field $H_c$ vs. temperature $T$ for the polycrystalline (Y123) sample. The critical field at absolute zero temperature for the sample is 8% less than the data, as reported by Welp et al [25] for
of $J_c(0)$, and $H_{c2}(0)$ revealed poorer flux pinning properties in the Y123 polycrystalline sample related to the last data, but better local flux density at zero temperature compared to the other report [39].

In figure 5a and b we showed experimental model-dependent parameters for 1–5 tesla and average of the parameters with its linear fit vs. temperature curve for the polycrystalline Y(123) sample, respectively. The maximum value of $B_0$ in 60 K is 2.2 T, where the magnetic field is 5 T, while the minimum value of $B_0$ is zero where magnetic field is 1 T in the temperature 86 K. We assume all values of $B_0$ and a model-dependent parameter for polycrystalline was the same as single crystal and melt textured growth (MTG) crystal and decreased in the temperature range 60 K $\leq T \leq$ 86 K. The average value of $B_0$ is almost linear and decreases from 1.3 T in 60 K to zero in 86 K. As the temperature increases the magnitude of $B_0$ changes linearly with a negative slope. This process is shown schematically in figure 5b for the fit curve.

The equation of fit curve in figure 5b and its extrapolation is necessary to plot and extrapolate the magnetic field vs. critical current density using the critical state equation $J_c = J_c(T,B)$ based on experimental results [17] (equation 4). In this equation, $J_0(T)$ symbolizes as the critical current density without magnetic field at temperature $T$. The local magnetic induction and an archetypal-dependent factor are $B$ and $B_0$, respectively. $J_c = J_c(0,0)$ has been proposed to be 39813.894 A cm$^{-2}$. The local flux density at zero temperature has also been proposed to be 121.028 Tesla. Figure 6 shows the numerical relationship between applied magnetic flux density $B$ and critical current density $J_c$ for polycrystalline (Y123) sample when we apply the critical state equation. We consider that $J_c$ depends exponentially on $B$, where the average of $B_0$ depends linearly on temperature according to figure 6.
Figure 7 depicts phase diagram of high-temperature superconductor polycrystalline YBCO, which is obtained experimentally and theoretically by a combination of figures 3, 4 and 6.

Bearing in mind, when electrons form Cooper pairs in superconductors, they can allocate the same level of energy state or quantum wave-function that causes a lower energy state for the superconductors. The connectivity of electron pairs breakdown when the temperature and magnetic field state for the superconductors. The connectivity of electron state or quantum wave-function that causes a lower energy superconductors, they can allocate the same level of energy figures 3, 4 and 6.

4. Conclusion

Phase diagram of the polycrystalline Y(123) sample containing oxygen contents 6.96 for quickly measuring magnetization curve and critical field \(H_c\), the critical current density \(J_c(0)\) and critical temperature at zero fields \(T_c\) as 121.028 T, 39813.894 A cm\(^{-2}\) and 91 K, respectively. We obtained experimentally \((T vs. H_c)\), \((T vs. J_c)\) and \((H_c vs. J_c)\) diagrams from the measurements using resistivity, VSM and magnetic fields applied perpendicular to the current flow direction techniques accompanied with some theoretical work, such as Bean’s archetypal and Kim’s equation due to critical state models. We also theoretically investigated the possibility of predicting by the theoretical models, which are based on the experimental measurements, and are particularly important because of the technological applications of the thermodynamic and electrodynamics quantities, such as the thermal expansion, critical magnetic intensity and the critical current density. We found that the method could judge not only the values of the critical parameters in YBCO polycrystalline but also could be useful for single crystal and melt-texture growth superconductors. This method will be useful in \(J_c\), \(H_c\) and \(T_c\) distribution evaluations for promoting technological seeds.

Acknowledgements

We acknowledge the support from the research centre, Shiraz Branch, Islamic Azad University, Shiraz, Iran. We are grateful to Dr Ghaffary and Mrs Jafarimehr from the Islamic Azad University-Shiraz branch for their help and valuable comments.

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