Entropy analysis of Hall current and thermal radiation influenced by cilia with single- and multi-walled carbon nanotubes

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Abstract. The present investigation explores the significance of creeping viscous nanofluids in an axi-symmetric channel influenced by metachronal waves containing magnetohydrodynamics and Hall current. Heat transport analysis is also performed to derive the impact of thermal radiation on internal heat source phenomena. The use of mathematical formulation resulted in a set of nonlinear coupled partial differential equations. The governed differential system is transformed into an ordinary differential system by considering similar variables. Exact solutions in the closed form have been derived for the temperature, momentum and pressure gradient. Moreover, entropy generation due to heat transfer, thermal radiation and magnetic effects has been measured. The graphical results have been presented to interpret sundry parameters of interest. Streamlines and isotherms are also plotted against the multi-walled carbon nanotube. For the validation of our results, a comparison table is presented. It is also seen that entropy of the system increases and the Bejan number decreases with an increase in the Brinkman number.

Keywords. Second law analysis; Hall current; magnetic field; thermal radiation; SWCNT and MWCNT.

1. Introduction

Carbon nanotubes (CNTs) are members of fullerene structural family with amazing carbon particles and a high length-to-diameter ratio. The notable properties of CNTs make them interesting. The structures of CNTs exhibit absolutely surprising electrical and physical properties. CNTs were first found by Iijima [1] and Baughman *et al* [2]. Moreover, CNTs are subdivided into two unique classes: single-walled CNTs (SWCNTs) and multi-walled CNTs (MWCNTs). Apart from their rare existence, CNTs find their utilization as an added substance in structural materials, for instance, golf stub, water-boat, airship and engine-bicycles. Also, CNTs are extensively used in compound goods [3,4], electrical gadgets [5–8], hydrogen stockpiling [9,10], bio-therapeutic designing [11–13] and so on. CNTs exhibit immense thermal properties. As far as electrical properties are concerned, CNTs can be either metallic or semiconducting depending upon their presence in the network with respect to the tube centre. The transport properties of nanofluids are not only susceptible to the volume fraction of nanoparticles, but also depends upon the particle shape, size, mixture combinations, etc. Earlier studies clearly clarify that the thermal properties and viscous nature of the fluid improve by considering nanoparticles instead of conventional fluids. The subject, heat transfer with nanofluids, has been investigated by Makinde and Aziz [14], Abrar *et al* [15], Abrar and Awais [16], Manna *et al* [17], Kamran *et al* [18], etc.

Numerous interesting engineering usages of MHD problems have drawn the attention of mathematicians and

engineers, e.g., blood pump engines, cancer tumour treatment, MHD power generators, bleeding reduction during surgeries, etc. The varying magnetic effects with mixed convection were investigated by Barletta *et al* [19]. Hussain *et al* [20] presented a numerical solution to MHD flow by assuming a hybrid nanofluid model with an adiabatic obstacle. Rashidi *et al* [21] provided an analytical solution to heat transfer in micropolar fluid through a porous medium with thermal radiation. Turkyilmazoglu and Pop [22] analysed the combined effects of heat and mass transport over an infinite flat plate. Das and Jana [23] figure out the MHD flow with thermal effects by considering three types of water-based nanofluids (i.e., copper, aluminium oxide and titanium dioxide). The consequences of a magnetic parameter with nanoparticles have been investigated by several authors [24–27].

To the best of our knowledge, there is no study in the literature presenting the effects of Hall current, magnetic field and thermal radiation on a duct flow influenced by metachronal waves. The next section provides the mathematical description of the study and formation of entropy. Section 3 presents the closed form exact solution to the energy, momentum and pressure gradient. Various plots and comparison tables are presented in section 4 and the last section discloses the final outcomes of the current investigation.

2. Mathematical analysis

In this paper, the peristaltic transport of Newtonian axisymmetric flow of CNTs in a duct of finite length is

Figure 1. Geometry of the problem.

considered. Metachronal waves are developed due to the collective beating of cilia, which directs the flow. The magnetic field of strength B_0 is applied in the transverse direction. The fluid is a water-based nanofluid containing CNTs, where the host fluid and considered nanoparticles are thermally stable. We introduce the cylindrical coordinate system $(\bar{R}, \bar{Z})$ to present the geometry of the problem. Figure 1 shows the systematic view of the governed model and coordinate system. The shape of cilia tips is supposed to have the following pattern:

$$\begin{aligned}\bar{R} &= e + \varepsilon \cos \left[\frac{2\pi}{\lambda} (\bar{Z} - c\bar{t}) \right], \\ \bar{Z} &= e + \varepsilon \alpha \sin \left[\frac{2\pi}{\lambda} (\bar{Z} - c\bar{t}) \right],\end{aligned}\quad (1)$$

where e the mean radius of the tube, ε the wave amplitude, Z_0 the reference position, λ the wavelength, α the measure of the eccentricity of the elliptical motion and c the wave speed. After employing no slip condition, the velocities of the transporting fluid are given as follows:

$$\begin{aligned}\bar{W} &= \left(\frac{\partial \bar{Z}}{\partial \bar{t}} \right)_{\bar{Z}_0} = \left(\frac{\partial \bar{g}}{\partial \bar{t}} \right) + \left(\frac{\partial \bar{g}}{\partial \bar{Z}} \right) \left(\frac{\partial \bar{Z}}{\partial \bar{t}} \right) = \left(\frac{\partial \bar{g}}{\partial \bar{t}} \right) + \left(\frac{\partial \bar{g}}{\partial \bar{Z}} \right) \bar{W}, \\ \bar{U} &= \left(\frac{\partial \bar{R}}{\partial \bar{t}} \right)_{\bar{Z}_0} = \left(\frac{\partial \bar{f}}{\partial \bar{t}} \right) + \left(\frac{\partial \bar{f}}{\partial \bar{Z}} \right) \left(\frac{\partial \bar{Z}}{\partial \bar{t}} \right) = \left(\frac{\partial \bar{f}}{\partial \bar{t}} \right) + \left(\frac{\partial \bar{f}}{\partial \bar{Z}} \right) \bar{U},\end{aligned}\quad (2)$$

together equations (1 and 2), the radial and vertical velocities of the cilia are given as follows:

$$\begin{aligned}\bar{W} &= \frac{\left(\frac{-2\pi}{\lambda} \right) [\varepsilon \alpha c e \cos \left(\frac{2\pi}{\lambda} (\bar{Z} - c\bar{t}) \right)]}{1 - \frac{2\pi}{\lambda} [\varepsilon \alpha c e \cos \left(\frac{2\pi}{\lambda} (\bar{Z} - c\bar{t}) \right)]}, \\ \bar{U} &= \frac{\left(\frac{-2\pi}{\lambda} \right) [\varepsilon \alpha c e \sin \left(\frac{2\pi}{\lambda} (\bar{Z} - c\bar{t}) \right)]}{1 - \frac{2\pi}{\lambda} [\varepsilon \alpha c e \sin \left(\frac{2\pi}{\lambda} (\bar{Z} - c\bar{t}) \right)]}.\end{aligned}\quad (3)$$

The following variables are used to convert the flow from a fixed frame to a wave frame:

$$\begin{aligned}\bar{Z} &= \bar{z} + c\bar{t}, \quad \bar{R} = \bar{r}, \quad \bar{U} = \bar{u}, \quad \bar{W} = \bar{w} + c, \\ \bar{P}(\bar{Z}, \bar{R}, \bar{t}) &= \bar{p}(\bar{z}, \bar{r}, \bar{t}).\end{aligned}\quad (4)$$

The derived fundamental system of equations after introducing the above variables is of the following form [15]:

$$\frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\bar{u}}{\bar{r}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0, \quad (5)$$

$$\begin{aligned}\rho_{nf} \left[\bar{u} \frac{\partial \bar{u}}{\partial \bar{r}} + \bar{w} \frac{\partial \bar{u}}{\partial \bar{z}} \right] &= -\frac{\partial \bar{p}}{\partial \bar{r}} + 2\mu_{nf} \left[\frac{\partial^2 \bar{u}}{\partial \bar{r}^2} \right] \\ &+ \mu_{nf} \frac{2}{\bar{r}} \left[\frac{\partial \bar{u}}{\partial \bar{r}} - \frac{\bar{u}}{\bar{r}} \right] + \mu_{nf} \left[\frac{\partial}{\partial \bar{z}} \left(\frac{\partial \bar{u}}{\partial \bar{r}} \right) + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2} \right], \\ &- \frac{\sigma B_0^2}{1+m^2} [\bar{u} - m(\bar{w} + c)],\end{aligned}\quad (6)$$

$$\begin{aligned}\rho_{nf} \left[\bar{u} \frac{\partial \bar{w}}{\partial \bar{r}} + \bar{w} \frac{\partial \bar{w}}{\partial \bar{z}} \right] &= -\frac{\partial \bar{p}}{\partial \bar{z}} + 2\mu_{nf} \left[\frac{\partial^2 \bar{w}}{\partial \bar{z}^2} \right] \\ &+ \mu_{nf} \left[\frac{1}{\bar{r}} \left(\frac{\partial \bar{u}}{\partial \bar{z}} + \frac{\partial \bar{w}}{\partial \bar{r}} \right) + \frac{\partial}{\partial \bar{r}} \left(\frac{\partial \bar{u}}{\partial \bar{z}} + \frac{\partial \bar{w}}{\partial \bar{r}} \right) \right] \\ &- \frac{\sigma B_0^2}{1+m^2} [(\bar{w} + c) + m\bar{u}] + \rho_{nf} g \alpha (\bar{T} - \bar{T}_0),\end{aligned}\quad (7)$$

$$\begin{aligned}(\rho c_p)_{nf} \left[\bar{u} \frac{\partial \bar{T}}{\partial \bar{r}} + \bar{w} \frac{\partial \bar{T}}{\partial \bar{z}} \right] &= k_{nf} \left[\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\bar{r} \frac{\partial \bar{T}}{\partial \bar{r}} \right) + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2} \right] \\ &+ Q_0 + \frac{16\sigma^* T_\infty^3}{3k^*} \left[\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\bar{r} \frac{\partial \bar{T}}{\partial \bar{r}} \right) \right],\end{aligned}\quad (8)$$

where $\bar{u}$ and $\bar{w}$ are the velocity elements in the $\bar{r}$ and $\bar{z}$ directions, B_0 the total magnetic field, σ the electric conductivity,

α the coefficient of linear thermal expansion of the fluid, g the acceleration due to gravity, Q_0 the internal heat source, m the Hall parameter, ρ_{nf} the effective density, μ_{nf} the effective dynamic viscosity, $(\rho c_p)_{nf}$ the heat capacitance, k^* the Rosseland mean absorption coefficient, α_{nf} the effective thermal diffusivity, σ^* the Stefan–Boltzmann constant and k_{nf} the

effective thermal conductivity, given as:

$$\left. \begin{aligned} \alpha_{nf} &= \frac{k_{nf}}{(\rho c_p)_{nf}}, \quad \rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s, \\ \mu_{nf} &= \frac{\mu_f}{(1 - \phi)^{2.5}}, \quad (\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_s, \\ k_{nf} &= k_f \left(\frac{(1 - \phi) + \frac{2\phi k_{CNT}}{k_{CNT} - k_f} \log\left(\frac{k_{CNT} + k_f}{2k_f}\right)}{(1 - \phi) + \frac{2\phi k_f}{k_{CNT} - k_f} \log\left(\frac{k_{CNT} + k_f}{2k_f}\right)} \right), \end{aligned} \right\} \quad (9)$$

and introducing the following non-dimensional variables:

$$\begin{aligned} r &= \frac{\bar{r}}{e}, \quad z = \frac{\bar{z}}{\lambda}, \quad w = \frac{\bar{w}}{c}, \quad u = \frac{\lambda \bar{u}}{ec}, \quad p = \frac{e^2 \bar{p}}{c\lambda\mu_f}, \\ \delta &= \frac{e}{\lambda}, \quad \theta = \frac{(\bar{T} - \bar{T}_0)}{\bar{T}_0}, \quad t = \frac{c\bar{t}}{\lambda}, \quad M = \frac{\sigma B_0^2 e^2}{\mu_f}, \\ Gr &= \frac{\rho_{nf} g \alpha e^2 \bar{T}_0}{c\mu_f}, \quad \Omega = \frac{Q_0 e^2}{k_f \bar{T}_0}, \quad m = \frac{\sigma B_0}{an_a}, \end{aligned} \quad (10)$$

$$+ \frac{M}{1 + m^2} (w + 1) - Gr\theta = 0, \quad (12)$$

$$\left(\frac{d^2\theta}{dr^2} + \frac{1}{r} \frac{d\theta}{dr} \right) \left(\frac{k_{nf}}{k_f} + \frac{4}{3} Rn \right) + \Omega = 0, \quad (13)$$

where ϕ is the solid volume fraction, Gr the Grashof number, Ω the internal heat source parameter and $Rn = (4\sigma^*/k^*k_f)T_\infty^3$ the radiation parameter. The boundary conditions are given as follows:

$$\left. \begin{aligned} w'(r) &= 0, \quad \theta'(r) = 0 \quad \text{at } r = 0, \\ w &= \frac{-2\pi\epsilon\alpha\beta \cos(2\pi z)}{1 - 2\pi\epsilon\alpha\beta \cos(2\pi z)} - 1, \quad \theta = 0 \quad \text{as } r = \hbar(z) = 1 + \epsilon \cos(2\pi z). \end{aligned} \right\} \quad (14)$$

where a is the electric charge and n_a the number density of electrons. The assumptions of long wavelength and low Reynolds number are as follows:

- The length of the metachronal wave is assumed to be very long ($\lambda \rightarrow \infty$).
- Inertial forces are minute when compared to viscous forces, therefore, a very low Reynolds number ($Re \ll 1$) is considered.

The set of equations (5–8) after dropping the dashes reduces to the following set of equations:

$$\begin{aligned} \frac{dp}{dr} &= 0, \\ \frac{dp}{dz} - \frac{1}{(1 - \phi)^{2.5}} \left(\frac{1}{r} \frac{dw}{dr} + \frac{d^2w}{dr^2} \right) \end{aligned} \quad (11)$$

A non-equilibrium situation emerges as a result of the exchange of momentum, temperature and magnetic effects within the fluid and at the walls, which generates a continuous entropy. The volumetric entropy generation term (S_G) can be derived as follows:

$$\begin{aligned} S_G &= \frac{1}{\theta_0^2} \left(k_{nf} \left(\frac{\partial \bar{T}}{\partial \bar{r}} \right)^2 + \frac{16\sigma^* T_\infty^3}{3k^*} \left(\frac{\partial \bar{T}}{\partial \bar{r}} \right)^2 \right) \\ &+ \frac{\sigma B_0^2}{\theta_0(1 + m^2)} \left((\bar{w} + c) + m\bar{u} \right)^2, \end{aligned} \quad (15)$$

Equation (15) reflects the contribution of three different factors to entropy generation. These factors are heat transfer (E_H), thermal radiation (E_{Rn}) and magnetic field (E_M). Entropy basically gives the degree of disorder of the system and its surroundings and the rate of dimensionless entropy formation ($E_G = S_G/S_g$) is as follows:

$$E_G = \left(\frac{k_{nf}}{k_f} + \frac{4Rn}{3} \right) \left(\frac{\partial \theta}{\partial r} \right)^2 + \frac{\Lambda Br M}{1 + m^2} (w + 1)^2, \quad (16)$$

where

$$S_g = \frac{k_f \bar{T}_0^2}{\bar{\theta}_0^2 e^2}, \quad \Lambda = \frac{\bar{\theta}_0}{\bar{T}_0} \quad \text{and} \quad Br = \frac{c^2 \mu_f}{k_f \bar{T}_0}, \quad (17)$$

where Λ is the dimensionless temperature difference and Br the Brinkman number. To find out the irreversibility distribution, another parameter Bejan number (Be) was introduced by Bejan [29], which is the ratio of the heat transfer irreversibility to the total irreversibility and is given as follows:

$$\theta(r, z) = \frac{3(h^2 - r^2)}{4(3\Psi_1 + 4Rn)} \Omega, \quad (19)$$

$$w(r, z) = \frac{1}{\left(\frac{M}{1+m^2}\right)} \left(\Psi_2 + \frac{\left(\frac{M}{1+m^2} + \left(\Psi_3 + \frac{dp}{dz}\right) \Psi_4\right) {}_0F_1\left[\left(\frac{M}{1+m^2}\right) \frac{r^2}{4(1-\phi)^{2.5}}\right]}{\Psi_4 {}_0F_1\left[\left(\frac{M}{1+m^2}\right) \frac{h^2}{4(1-\phi)^{2.5}}\right]} \right), \quad (20)$$

the flow rate is described as follows:

$$Be = \frac{E_H}{E_H + E_{Rn} + E_M}. \quad (18)$$

$$Q = 2 \int_0^{h(z)} r w(r) dr, \quad (21)$$

3. Exact solutions

This segment presents the closed form exact solutions to the ordinary differential equations (12 and 13) together with

on substituting equation (20) into equation (21), we obtain a solution to dp/dz

$$\frac{dp}{dz} = \frac{\left(\Psi_5 32 \left(\frac{M}{1+m^2}\right)^2 (F + h^2) \Psi_7 + 8h^2 {}_0F_2\left[\left(\frac{M}{1+m^2}\right) \frac{h^2}{4(1-\phi)^{2.5}}\right] (4\Psi_8 + 3\Psi_9)\right)}{\left(\frac{M}{1+m^2}\right)^2 h^4 \Psi_4 ((1-\phi)^{2.5})^2 {}_0F_3\left[\left(\frac{M}{1+m^2}\right) \frac{h^2}{4(1-\phi)^{2.5}}\right] (3\Psi_1 + 4Rn)}, \quad (22)$$

boundary condition (equation 14). The constitutive boundary layer equations for the considered flow analysis incorporates continuity, momentum and energy equations. These equations are coupled nonlinear partial differential equations. To convert the governing system of partial differential equations into a system of ordinary differential equations, similar variables

have been introduced. The obtained system is a linear, second-order inhomogeneous ordinary differential system. The exact solution to the ordinary differential system is presented in equations (19, 20 and 22) derived by the program designed in the Mathematica software called 'DSolve'. The general solution to inhomogeneous differential equations comprises the complementary and particular parts. For a complementary solution, 'DSolve' chooses the Bessel functions, because the homogeneous part of equations (12 and 13) corresponds to the standard form of Bessel equation of order zero. Once, we have successfully obtained the complementary solution, 'DSolve' straightforwardly proceeds to the particular solution. The interpretation of temperature, velocity and pressure gradient is computed as follows:

the mean flow rate and local Nusselt number [28] can be calculated as follows:

$$F = Q - \left[0.5 + \frac{\varepsilon^2}{4}\right], \quad Nu = -\left(\frac{\partial \theta}{\partial r}\right)_{r=h}, \quad (23)$$

where the expressions for $(\Psi_i, i = 1-9)$ are given as follows:

$$\left. \begin{aligned} \Psi_1 &= \left(\frac{(1-\phi) + \frac{2\phi k_{\text{CNT}}}{k_{\text{CNT}} - k_f} \log\left(\frac{k_{\text{CNT}} + k_f}{2k_f}\right)}{(1-\phi) + \frac{2\phi k_f}{k_{\text{CNT}} - k_f} \log\left(\frac{k_{\text{CNT}} + k_f}{2k_f}\right)} \right), \\ \Psi_2 &= \left(\frac{M}{1+m^2} \right) + \frac{3(\hbar^2 - r^2)\Omega}{4(3\Psi_1 + 4Rn)} Gr - \frac{dp}{dz}, \\ \Psi_3 &= \frac{M}{1+m^2} - \frac{3(\hbar^2 - r^2)\Omega}{4(3\Psi_1 + 4Rn)} Gr, \\ \Psi_4 &= (1 - 2\pi\alpha\beta\varepsilon \cos(2\pi z)), \\ \Psi_5 &= -\Psi_4 {}_0F_1 \left[\left(\frac{M}{1+m^2} \right) \frac{h^2}{4(1-\phi)^{2.5}} \right], \\ \Psi_6 &= Rn(1-\phi)^{2.5} - 3Grh^2 \left(-8 + \left(\frac{M}{1+m^2} \right) h^2(1-\phi)^{2.5} \right), \\ \Psi_7 &= \Psi_6 \Omega 24 \left(\frac{M}{1+m^2} \right)^2 (F + h^2) (1-\phi)^{2.5} \Psi_1, \\ \Psi_8 &= \left(\frac{M}{1+m^2} \right)^2 Rn (-1 + \Psi_4) (1-\phi)^{2.5} + 3Gr\Psi_4\Omega, \\ \Psi_9 &= \left(\frac{M}{1+m^2} \right)^2 (-1 + \Psi_4) (1-\phi)^{2.5} \Psi_1. \end{aligned} \right\} \quad (24)$$

4. Results

This section discloses the physical aspects of various significant parameters on streamlines, isotherms, Nusselt number, thermal conductivity, temperature field, velocity field and pressure gradient.

4.1 Streamlines and isotherms

A very attractive phenomenon in peristaltic motion, called trapping, is shown with respect to change in various parameters. Figure 2a and b clearly shows that with an increase in the magnitude of eccentricity of the elliptical motion (α), the size of bolus decreases, but the number of trapped boluses increases. Figure 3a and b shows that the size of bolus increases rapidly, but the number of trapped boluses starts decreasing with the increase in flow rate (Q) from 0.2 to 0.4. To visualize the isotherms, figure 4a and b shows two different values of amplitude ratio (ε). It can be seen that by increasing the amplitude ratio from 0.1 to 0.2, the size of bolus increases, but the number of trapped boluses decreases rapidly. Hence, for small amplitude ratios, the isotherms carry more heat. The impact of thermal radiation parameter (Rn) is shown in figure 5a and b. It is clear that with an increase in the magnitude of Rn , the size of bolus reduces, but the number of trapped boluses increases instantaneously. Physically, for

higher Rn , the fluid energy increases immediately as the fluid molecules are heated up sharply, which increases the fluid temperature.

4.2 Nusselt number

The Nusselt number (Nu) represents the enhancement of heat transfer through a fluid layer as a result of convection over the conduction across the same fluid layer. Basically, a Nu close to one indicates pure conduction (laminar flow), $Nu > 1$ indicates more effective convection (turbulent flow) and negative Nu is not possible, because convection only increases heat transfer. The influence of the thermal radiation parameter (Rn) on the Nusselt number is shown in figure 6a and b for both cases (pure water-SWCNT and pure water-MWCNT). It is seen that the magnitude of Nu decreases within the boundary layer region, when Rn increases from 1.0 to 10. Hence, for smaller values of Rn , the waves carry more energy during this flow; therefore, Rn plays a vital role in the development of the boundary layer flow region.

4.3 Thermal conductivity

Thermal conductivity is the property of a material to conduct heat. It strongly depends on the material structure, porosity, density and moisture content. We can clearly see a meaningful variation in the thermal conductivities of these base fluids,

Figure 2. Streamlines for (a) $\alpha = 1.0$ and (b) $\alpha = 2.0$.

Figure 3. Streamlines for (a) $Q = 0.2$ and (b) $Q = 0.4$.

where engine oil has the maximum thermal conductivity and pure water has the minimum thermal conductivity in both cases (SWCNT and MWCNT), see figure 7a and b.

4.4 Second law analysis

Figure 8a shows the behaviour of entropy on increasing the values of Brinkman number (Br). It is seen that the entropy of the fluid system increases with an increase in

the magnitude of the Br. Physically, as Br increases, heat transfer influences the fluid viscosity within the tube, therefore increases the entropy of the system. Maximum entropy is observed at the centre of the tube (i.e., at $r = 0$). Figure 8b shows that an increase in Br reduces the Bejan number at the centre of the tube, while the Bejan number increases at the ciliated walls. Figure 9a shows the influence of the dimensionless temperature difference (Δ) on the entropy. It is seen that the entropy of the system increases

Figure 4. Isotherms for (a) $\varepsilon = 0.1$ and (b) $\varepsilon = 0.12$.

Figure 5. Isotherms for (a) $Rn = 1.0$ and (b) $Rn = 2.0$.

for higher values of Λ . Bejan number is the decreasing function of Λ , see figure 9b. It is due to the fact that the heat transfer irreversibility is small when compared with the total irreversibility.

4.5 Temperature

The influence of the thermal radiation parameter (Rn) on the temperature field is shown in figure 10a. It is seen that the fluid

temperature decreases constantly as there is an increase in Rn . Physically, an increase in the radiative parameter Rn increases the mean absorption coefficient (k^*), which reduces the fluid temperature significantly. Figure 10b depicts the behaviour of internal heat source parameter (Ω) on $\theta(r, z)$. It is found that Ω is directly proportional to $\theta(r, z)$, moreover, the fluid exhibits maximum energy at the centre of the tube (i.e., $r = 0$) and minimum near the walls. Furthermore, no temperature variation is seen, when Ω is zero.

Figure 6. Variations of Rn on (a) SWCNT and pure water against Nu and (b) MWCNT and pure water against Nu .

Figure 7. Variations of three different base fluids on (a) SWCNT and (b) MWCNT.

Figure 8. Response of Br at (a) E_G and (b) Be .

4.6 Velocity

The influence of magnetic parameter (M) on $w(r, z)$ is shown in figure 11a. It is shown that as M increases there is a constant decrease in velocity. From a physical point of view, an increase in the magnetic field accelerates the strength of Lorentz force, which is a resistive force, therefore, more resistance is offered to the fluid motion, which consequently reduces the velocity. Figure 11b shows the effect of the flow rate (Q) on the $w(r, z)$.

It is found that Q is directly proportional to the flow field, moreover, the fluid exhibits maximum velocity at the centre of the tube (i.e., $r = 0$) and minimum near the ciliated walls.

4.7 Pressure gradient

Figure 12a and b shows the influence of the pressure gradient dp/dz on the axial direction upon increasing the values of

Figure 9. Responses of Λ at (a) E_G and (b) Be .

Figure 10. Responses of (a) Rn at $\theta(r, z)$ and (b) Ω at $\theta(r, z)$.

Figure 11. Responses of (a) M at $w(r, z)$ and (b) Q at $w(r, z)$.

α and β , respectively. The pressure gradient is a physical quantity that characterizes in which direction and at what rate the pressure increases most rapidly. Figure 12a and b shows a non-uniform oscillating behaviour on increasing the values of α and β , respectively. However, it is very clear that the SWCNT holds more pressure gradient than the MWCNT.

Tables 1 and 2 show the validated results obtained. An excellent comparison is made between the present and Bejan [29] results, by keeping $Rn = 0 = m$ and rest of the parameters are same as in ref. [29].

5. Conclusions

The key points of the examined flow analysis are the following:

- It is found that the size of bolus increases rapidly at high flow rate values.
- It is found that isotherms carry more heat at small amplitude ratios.

Figure 12. Responses of (a) α at dp/dz and (b) β at dp/dz .

Table 1. Numerical computation of the velocity against SWCNT with $\varepsilon = 0.1$, $\alpha = 0.1$, $\beta = 0.2$, $\Omega = 0.5$, $z = 0.5$, $\phi = 0.3$, $Rn = 0.0$, $m = 0.0$.

Q	M	Gr	$w(r, z)$, present solution			$w(r, z)$, Akbar and Butt [30]		
			$r = 0.0$	$r = 0.25$	$r = 0.5$	$r = 0.0$	$r = 0.25$	$r = 0.5$
0.3	2.0	0.5	0.448967	0.346254	0.030112	0.448954	0.346236	0.0301363
0.4			0.689396	0.569413	0.200434	0.689394	0.569486	0.200482
0.5			0.929843	0.792723	0.370835	0.929834	0.792735	0.370827
	1.0		0.968034	0.819956	0.660112	0.968022	0.819977	0.660109
	3.0		0.872516	0.751459	0.367037	0.872505	0.751464	0.367044
	5.0		0.729409	0.646107	0.354189	0.729404	0.646104	0.354192
		1.0	0.729441	0.646183	0.354199	0.729435	0.646127	0.354194
		3.0	0.729567	0.646209	0.354212	0.729559	0.646217	0.354204
		5.0	0.729698	0.646312	0.354209	0.729683	0.646307	0.354214

Table 2. Numerical computation of the velocity against MWCNT with $\varepsilon = 0.1$, $\alpha = 0.1$, $\beta = 0.2$, $\Omega = 0.5$, $z = 0.5$, $\phi = 0.3$, $Rn = 0.0$, $m = 0.0$.

Q	M	Gr	$w(r, z)$, present solution			$w(r, z)$, Akbar and Butt [30]		
			$r = 0.0$	$r = 0.25$	$r = 0.5$	$r = 0.0$	$r = 0.25$	$r = 0.5$
0.3	2.0	0.5	0.448945	0.346256	0.0301398	0.448957	0.346239	0.0301365
0.4			0.689378	0.569467	0.200425	0.689397	0.569488	0.200482
0.5			0.929824	0.792745	0.370882	0.929837	0.792738	0.370827
	1.0		0.968045	0.819945	0.372923	0.968026	0.819979	0.372998
	3.0		0.872512	0.751447	0.367068	0.872508	0.751466	0.367044
	5.0		0.729402	0.646104	0.354199	0.729407	0.646106	0.354192
		1.0	0.729439	0.646129	0.354189	0.729441	0.646131	0.354195
		3.0	0.729568	0.646219	0.354201	0.729576	0.646229	0.354205
		5.0	0.729715	0.646313	0.354209	0.729711	0.646327	0.354216

- The engine oil has the maximum thermal conductivity and pure water has the least thermal conductivity.
- Entropy is the increasing function of the dimensionless Brinkman number and temperature difference.
- Bejan number is inversely proportional to the dimensionless Brinkman number and temperature difference.
- It is noted that the thermal radiation reduces the energy of the flow system.
- It is seen that variation in temperature profile is higher in case of MWCNT than in SWCNT.
- Velocity field is accelerated with an increase in the flow rate parameter, while the flow velocity is reduced by a high magnetic parameter.
- It is observed that the pressure gradient exhibits non-uniform oscillating behaviour against the increasing values of α and β for both the cases (SWCNT and MWCNT).

- A excellent comparison is made between the present and already published paper.

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