Analysis of fibre waviness effect through homogenization approach for the prediction of effective thermal conductivities of FRP composite using finite element method

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Abstract. In this study, homogenization approach is proposed to analyse the fibre waviness in predicting the effective thermal conductivities of composite. Composites that have wavy fibre were analysed by finite element method to establish equivalence between micro- and macro-mechanics principles, thereby, it is possible to minimize the computational efforts required to solve the problem through only micro-mechanics approach. In the present work, the influence of crest offset, wavy-span on the thermal conductivities of composite for different volume fractions and thermal conductivity mismatch ratios were also studied. It is observed that the homogenization results are in good agreement with minimal % error from those obtained through pure micro-mechanics approach at the cost of low computational facilities and less processing time for converged solutions.

Keywords. Effective thermal conductivity; polymer composites; fibre waviness; finite element method; homogenization.

1. Introduction

Composite materials are extensively used from Iron age applications to current space age applications such as aerospace, electronic packaging, reactor vessels, turbines, etc., due to their high strength to weight ratio, tailorable properties, long durability, stability against chemical reaction, etc. fibre-reinforced polymer composites not only used for structural applications, but also in heat transfer applications either for enhancement or insulation purposes. Heat transfer in the FRP composite depends on the thermal properties, volume fraction, orientation, etc., of each constituent of the composite. The effective thermal conductivity and other thermo-physical properties of composites have been a topic of considerable theoretical, experimental and numerical interest from the long time to tailor the composite as per the desired properties.

Composite materials are non-homogeneous and exhibit anisotropic response due to structural and thermal loads. Analysis of a composite structure as in a state of heterogeneity by providing the material properties of constituent materials is mathematically complex and therefore theories such as micro-mechanics and macro-mechanics are developed for the theoretical analysis. The homogenized properties of a composite lamina obtained from micro-mechanical theories are used in the macro-mechanical analysis of a composite made of several individual laminas stacked in a specified manner.

In the micro-mechanical approach, a particular portion of the composite known as ‘representative volume elements’ (RVE) is selected and the properties of RVE are found which are considered to be lamina properties. In this approach, there are many assumptions, such as arrangement of fibres in a particular pattern (square/hexagonal) in a matrix, no voids in the matrix, all fibres are of uniform cross-section, perfectly aligned, the interface between the fibre and matrix is perfectly bonded or fully debonded. These assumptions lead to too much deviations in theoretical and experimental results. Numerical approaches such as finite element method (FEM) are developed to overcome some of the assumptions of micro-mechanical theories. Though FEM is an approximate method, it can be effectively used after proper mesh-refinement and validation.

Generally aligned fibre composite laminates are frequently used in beam, plate or shell forms. For these composites, by simple rule of mixtures, the axial thermal conductivity (in the fibre direction) of each lamina is predicted satisfactorily. Prediction of through-thickness thermal conductivity is quite complex and more problematic. Yet, this is important, since heat sources on one side of the laminate often create a through-thickness temperature gradient.

Earlier, several researchers studied thermal conductivities of composites by experimental, theoretical and numerical approaches. Prediction of effective transverse thermal conductivity of fibre-reinforced composites was made for several models, such as experimental determination of effective thermal conductivity of aligned fibre composite [1], effect of fibre orientation on the thermal conductivity of

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unidirectionally aligned fibre composite [2], thermal conductivity of constituents of FRCL by back-out method [3], simple thermal resistance model [4], effective thermal conductivities of 2-D array circular- and square-cylinder composites [5], transverse thermal conductivity of a composite material with continuous unidirectional fibres packed in square array by finite element and statistical models [6], theoretical conduction models [7], filler size effects [8], interface resistance models [9–12], optimization of transverse thermal conductivity of aligned unidirectional fibres of elliptical shape [13], 2-D numerical model [14], 2-D thermal contact resistance model [15] and homogenized model for totally or partially debonded composite of Mahesh *et al* [16,17]. As per most of the literatures, effect of waviness on effective thermal conductivity of composite was not covered widely.

Several researchers developed theoretical and numerical models with the assumption that fibres are aligned perfectly straight, but during fabrication, they may take the wavy form rather than perfectly straight due to manufacturing errors. Results obtained with these assumptions will creep in errors in the prediction of the composite effective thermal conductivities in axial and through thickness transverse directions. In the present analysis, 3-D finite element method is proposed to study the effect of fibre waviness, crest offset, wavy-span on the effective thermal conductivities of composite by micro-level approach. (Crest offset is the deviation in the crest of wavy profile in the transverse direction to the fibre axis. The wavy-span is the spread of waviness in the total length of a composite). The study also aims at developing equivalence between micro- and macro-mechanical approaches, which is very much useful when the computational domain is large or domain is with more complexities.

2. Problem modelling

2.1 Geometric modelling

In micro-mechanics approach, the effect of fibre waviness, crest offset, wavy-span on the thermal conductivities of a composite are studied by considering an arbitrary curve in plane 1–3 (i.e., through thickness direction) with the dimensions as shown in figure 1. This curvature limits the maximum volume fraction to 0.6 and maximum crest deviation from the fibre axis to $L_o + R$ units. The crest offset effect is studied for 25, 50, 75 and 100% of L_o at different volume fractions. Also wavy-span effect is studied for different lengths of wave spread over 25, 50, 75, 100% of composite length. Micro- and macro-mechanics equivalence is established for composite with 1/3rd wavy-span. In all the above cases, due to the existence of symmetry in the model about through thickness direction (i.e., wavy plane 1–3), half the geometry is modelled for study.

Dimensions considered in the present analysis are: $a = 250$ units, $b = 125$ units in perpendicular direction to plane 1–3 and radius of fibres corresponds to volume fractions ranging from 0.1 to 0.6.

Figure 1. Composite with wavy fibre.

The process of macro-mechanics or homogenization approach is carried out in two stages. In stage I, 1/3rd wavy-span composite is modelled as straight segment (A) and wavy segment (B) separately as shown in figure 2, then the thermal conductivities of straight and wavy fibre composites are found by micro-mechanics approach. In stage II, due to skewness of wavy fibre, three blocks are modelled one behind the other with perfect contact between them, the first and third blocks represent a straight fibre portion and middle block represents a wavy portion of fibre. Homogenized properties evaluated in stage I are attributed to the corresponding blocks and effective thermal conductivities of composite are evaluated.

Homogenization is a useful technique for solving the problems involving large domain or with more complexities, which requires high end computational facilities and more processing time for obtaining converged solutions. Simplicity of homogenization approach can be understood easily from table 1. For example, the current 1/3rd wavy-span model by the micro-mechanics approach requires 116369 nodes at $V_f = 0.6$ for mesh converged solution, i.e. more computational resources and time to solve a large number of equations (116369 equations), whereas with the macro-mechanics approach at $V_f = 0.6$, converged solution requires maximum of 1310 nodes either in stage I or stage II. Maximum number of nodes at $V_f = 0.6$ for micro-mechanics approach is almost 89 times higher than the maximum nodes required for macro-mechanics approach for the current model. This may be very high, when the domain is larger or complexities are more.

2.2 Finite element modelling

2.2a Heat conduction equation: In the classical heat transfer, by applying heat balance over a small control

Figure 2. Stages in homogenization of 1/3rd wavy fibre composite.

Table 1. Comparison of micro- vs. macro-approaches in terms of maximum number of equations (nodes) to be solved at $L = 300$ units and different volume fractions.

V_f	Micro (nodes)	Macro (nodes)
0.10	39595	819
0.20	55265	952
0.30	71461	1104
0.40	87719	1129
0.50	100925	1183
0.55	106675	1209
0.60	116369	1310

volume, 3D heat conduction equation obtained is as follows:

$$\left[\frac{\partial}{\partial x} k_x \frac{\partial T}{\partial x} + \frac{\partial}{\partial y} k_y \frac{\partial T}{\partial y} + \frac{\partial}{\partial z} k_z \frac{\partial T}{\partial z} \right] dV + q_g dV = C \frac{\partial T}{\partial \tau} dV, \quad (1)$$

where q_g is the internal heat generation per unit volume (W m^{-3}), k the thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$) and c the specific heat ($\text{J kg}^{-1} \text{K}^{-1}$).

For steady-state conduction and without internal heat generation, equation (1) reduces to

$$\left[\frac{\partial}{\partial x} k_x \frac{\partial T}{\partial x} + \frac{\partial}{\partial y} k_y \frac{\partial T}{\partial y} + \frac{\partial}{\partial z} k_z \frac{\partial T}{\partial z} \right] dV = 0. \quad (2)$$

On integrating the above equation and applying appropriate boundary conditions, temperature distribution and heat flow rate are obtained.

2.2b Finite element formulation for heat flow: The classical heat conduction analysis is quite difficult for complex domains; this can be easily solved by the finite element method with the integration of CAE software. The basis for finite element method (FEM) is a piecewise polynomial approximation for temperature field within each element:

$$T(x, y, z) = \sum_{i=0}^{\text{nodes}} N_i(x, y, z) T_i, \quad (3)$$

where N_i are shape functions and T_i are nodal temperatures.

Shape functions are specified by discretizing the domain with different types of elements. To evaluate T_i , the most popular method, for heat transfer problems, the Galerkin's approach is used as given below:

$$\int \varphi \left[k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2} + k_z \frac{\partial^2 T}{\partial z^2} \right] dV = 0, \quad (4)$$

$$\varphi = [N_i] \{ \Psi_i \},$$

where Ψ_i is the virtual nodal temperature vector.

Since there are i nodes, equation (4) creates a set of ' i ' number of ordinary differential equations which are integrated to form a set of algebraic equations:

$$[K][T] = [R], \quad (5)$$

where $[K]$ is global conductivity matrix and $[R]$ is an effective global load vector.

After implementation of the boundary conditions, above algebraic equations are solved for unknown $[T]$ and reaction load at the boundaries.

In the well-established methodology by earlier researchers [3,14,15], the prescribed temperature boundary conditions T_1 and T_2 are applied on the faces perpendicular to heat flow direction and other faces are insulated, which permit the heat to flow in temperature gradient direction, there by analysis reduces to 1D problem. Further, reaction heat load is obtained by FE solution and effective thermal conductivity of the composite is found by Fourier's law of heat conduction, i.e.,

$$Q = -k_z A \frac{dT}{dz}, \quad (6)$$

where k_z is the thermal conductivity of composite in heat flow direction and A the area perpendicular to heat flow direction.

Similarly, thermal conductivity in the other two directions can be found by applying similar boundary conditions on the appropriate faces.

In the present study, commercial finite element software ANSYS 15 is used. Geometry and discretized model of 1/3rd wavy-span composite is shown in figure 3. A 3-D higher order tetrahedron element having 10 nodes with a single degree of freedom (temperature) at each node, named

Figure 3. Geometry and discretized model for 1/3rd wavy-span composite.

SOLID87 is used for the discretization of the individual constituents of the composite. The finite element mesh is properly refined and converged results are verified with Hasselman–Johnson and Farmer–Covert models for validation purpose and the results are presented in ‘Results and discussion’ section.

2.3 Boundary conditions

Temperature difference (dT) of 100°C is maintained on two isothermal surfaces perpendicular to heat flow direction. All other surfaces are subjected to an insulation boundary condition.

2.4 Material properties

For the validation of the models developed in the present analysis, the following properties of fibres and matrix are considered from Christo [1]:

Polyimide matrix with thermal conductivity, $K_m = 0.19 \text{ W m}^{-1} \text{ K}^{-1}$.

(T-300) Carbon fibre with thermal conductivity, $K_f = 8.365218 \text{ W m}^{-1} \text{ K}^{-1}$.

For all the other cases, matrix thermal conductivity (K_m) is $1 \text{ W m}^{-1} \text{ K}^{-1}$ and fiber thermal conductivities (K_f) ranges from 0.1 to $1\text{E}5 \text{ W m}^{-1} \text{ K}^{-1}$.

3. Results and discussion

The analytical solution for 1-D heat conduction in homogeneous slabs is readily available in the form of Fourier’s

Figure 4. Variation in transverse thermal conductivity with respect to V_f of straight fibre composite.

heat conduction equation. However, the analytical solution for 1-D heat conduction is quite complex for heterogeneous materials such as fibre-reinforced composite materials. Thus, the numerical finite element models have been developed to suit the different cases considered in this study. The models are first tested for mesh-independent solution by imposing earlier stipulated boundary conditions, then with the heat flux obtained from ANSYS software, effective thermal conductivity of the composite is found by equation (6).

A finite element model developed using ANSYS software is first validated with the Hasselman and Johnson model [9] (H-J) and Farmer and Covert model [10] (F-C). Figure 4 portrays the comparison of results between Hasselman–Johnson model, Farmer–Covert model and present FEM model. It reveals that the results predicted by the finite element model for a perfectly aligned fibre case exactly matches for complete range of volume fraction with F-C model and

up to nearly 50% V_f with H–J and deviation thereon from H–J model is due to assumptions made in the model and quite coherent with the higher order F–C model.

Effective longitudinal thermal conductivity for straight fibre composite can be easily found by simple rule of mixtures, as heat flows through fibre and matrix in low resistance parallel path (direction 1), whereas for wavy fibre composite, heat flow through the fibre takes place in wavy path, encountering a series of resistances from fibre and matrix at the curved vicinities. This reduces heat flow in longitudinal direction and the same is contributed to the through thickness transverse direction 3.

From figure 5, it is observed that with the increase in wave offset %, in longitudinal direction 1, K_{of1}/K_{st1} decreases nonlinearly and this variation is high at $V_f = 0.1$ compared to other higher V_f . At lower V_f , due to lower size of the fibre, heat flow through the composite encounters high series

resistance as compared to higher V_f . At the higher V_f , major portion of fibre is straight or close to straight that causes for the reduction in resistance to heat flow. In direction 2, K_{of2}/K_{st2} variation is negligible at all V_f which is near unity. In through thickness transverse direction, K_{of3}/K_{st3} variation is negligible at lower V_f and it gets aggravated with an increase in V_f . For offset % beyond 75% variation is very high; this is due to low resistance, nearly parallel path created by fibre waviness.

Similarly from figure 6, for different fibre–matrix thermal conductivity mismatch ratios, with the increase in wave offset %, in longitudinal direction 1, K_{of1}/K_{st1} decreases nonlinearly. In direction 2, K_{of2}/K_{st2} variation is negligible at all σ , which is near unity. In through thickness transverse direction, variation in K_{of3}/K_{st3} is insignificant for matrix dominant and for fibre dominant cases, it increases gradually up to 75% offset, beyond this value, variation is rapid. In the first

Figure 5. Variation in K_{of1}/K_{st1} with respect to offset %, at $\sigma = 100$ and wave span = max., for different volume fraction.

Figure 6. Variation in K_{of1}/K_{st1} with respect to offset %, at $V_f = 0.6$ and wave span = max., for different thermal conductivity mismatch ratios.

two directions, variation in K_{of}/K_{st} is not affected by K_f/K_m . In third direction, this ratio is observed to be increasing with K_f/K_m .

From figure 7, it is observed that with the increase in wave-span %, in longitudinal direction 1, K_{sp1}/K_{st1} decreases non-linearly and this variation is high at $V_f = 0.1$ compared to other higher V_f . In direction 2, K_{sp2}/K_{st2} variation is negligible at all V_f , which is near unity. In through thickness transverse direction, K_{sp3}/K_{st3} variation is negligible at lower V_f and it gets aggravated with an increase in V_f .

Similarly from figure 8, for different fibre–matrix thermal conductivity mismatch ratios, with the increase in wave-span %, in longitudinal direction 1, K_{sp1}/K_{st1} decreases non-linearly. In direction 2, K_{sp2}/K_{st2} variation is negligible at all σ , which is near unity. In through thickness transverse direction, variation in K_{sp3}/K_{st3} is insignificant for matrix dominant and for fibre dominant cases, it increases gradually. In the first two directions, variation in K_{sp}/K_{st} is not affected by

K_f/K_m . In third direction, this ratio is observed to be increasing with K_f/K_m .

From figures 9–11, it is observed that similar to earlier researchers [16], with increase in V_f , longitudinal effective thermal conductivity of composite K_{sp1} varies linearly at all σ . For better pictorial view, plots for lower and higher σ are omitted and % error between macro- and micro-approaches lies between 0.04 to 0.38 as shown in table 2. In plane effective transverse thermal conductivity of composite K_{sp2} , varies non-linearly with the variation in V_f , gradual rise is observed in fibre thermal conductivity dominant cases and gradual fall for matrix dominant cases, also effective thermal conductivity in direction 2, nearly reached the saturation stage for thermal conductivity mismatch ratios beyond 100 at their corresponding volume fraction. The percentage error between macro- and micro-mechanical approaches lies between 0.02 and 0.19 (table 2).

Figure 7. Variation in K_{sp1}/K_{st1} with respect to wave span %, at $\sigma = 100$ and wave offset = max., for different volume fractions.

Figure 8. Variation in K_{sp1}/K_{st1} with respect to wave span %, at $V_f = 0.6$ and wave offset = max., for different thermal conductivity mismatch ratios.

For wavy-span case, through thickness transverse thermal conductivity, K_{sp3} increases with increase in wavy-span. The percentage error between micro- and macro-mechanical

Figure 9. Variation in K_{sp1} with respect to V_f for 1/3rd wavy composite for different σ values.

approaches for K_{sp3} lies between 0.04 and 4.49 as mentioned in table 2.

It is observed that, % error at V_f 0.6 is high, also gets aggravated with the increase in thermal conductivity mismatch ratio. Reason for this high % error at V_f 0.6, can be understood with figure 12. In micro-mechanics approach, V_f from 0.10 to 0.55, resistance to heat flow is high in direction 3, as heat first flows through fibre portion A1 and then through matrix enveloping A1, whereas at V_f 0.6, resistance to heat flow is low i.e., heat flows simultaneously through high thermal conductivity fibre portion A2 and low thermal conductivity matrix portion A3. Due to this high thermal conductivity, fibre contributes more heat as compared to low thermal conductivity matrix when it was in series, but this is not same with the macro approach, as heat flow is uniform in direction 3.

An attempt is made to reduce the % error by considering V_f 0.6 with slight modification in the approach. In the modified

Figure 10. Variation in K_2 with respect to V_f for 1/3rd wavy composite.

Figure 11. Variation in K_3 with respect to V_f for 1/3rd wavy composite.

Table 2. % Error between macro- and micro-mechanics approach results for 1/3rd wavy span composite.

V_f	$K_f/K_m = 0.1$			$K_f/K_m = 10$			$K_f/K_m = 100$			$K_f/K_m = 1000$			$K_f/K_m = \alpha$		
	K_{sp1} % error	K_{sp2} % error	K_{sp3} % error	K_{sp1} % error	K_{sp2} % error	K_{sp3} % error	K_{sp1} % error	K_{sp2} % error	K_{sp3} % error	K_{sp1} % error	K_{sp2} % error	K_{sp3} % error	K_{sp1} % error	K_{sp2} % error	K_{sp3} % error
0.10	0.08	0.08	0.06	0.14	0.06	0.08	0.36	0.08	0.11	0.38	0.02	0.23	0.37	0.06	0.28
0.20	0.16	0.11	0.07	0.20	0.02	0.15	0.30	0.05	0.09	0.32	0.09	0.21	0.31	0.01	0.30
0.30	0.25	0.70	0.63	0.18	0.04	0.23	0.25	0.02	0.28	0.27	0.09	0.28	0.26	0.05	0.42
0.40	0.17	0.37	0.48	0.16	0.02	0.35	0.22	0.10	0.45	0.23	0.06	0.46	0.22	0.13	0.53
0.50	0.08	1.00	0.37	0.13	0.08	0.33	0.20	0.15	1.09	0.21	0.05	1.04	0.20	0.01	1.03
0.55	0.23	0.19	0.42	0.04	0.23	0.90	0.12	0.10	1.63	0.12	0.12	1.75	0.11	0.11	1.79
0.60	0.15	0.30	0.04	0.11	0.01	0.97	0.18	0.02	3.13	0.18	0.14	4.44	0.17	0.12	4.49

Figure 12. Superimposed composites of 0.55 and 0.60 volume fraction.

Table 3. % Error between micro- and (modified) macro-mechanics approach results for K_{sp3} .

K_f/K_m	Modification	K_{sp3}							
		Macro	5%	10%	20%	50%	60%	70%	80%
0.1	Case i	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
	Case ii	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
	Case iii	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
10	Case i	0.97	0.86	0.76	0.56	0.27	0.26	0.27	0.11
	Case ii	0.97	0.86	0.76	0.88	0.43	0.26	0.27	0.11
	Case iii	0.97	0.86	0.76	0.69	0.43	0.43	0.46	0.31
100	Case i	3.13	2.93	2.75	2.31	1.40	1.16	0.95	0.77
	Case ii	3.13	2.93	2.75	2.83	1.40	1.16	0.95	0.77
	Case iii	3.13	3.01	2.75	2.39	1.51	1.28	1.07	0.90

approach, length of segment B (figure 2) is extended, instead of cutting, the wavy portion ends abruptly and keeping the overall length of composite same as original. In this approach, three cases are considered by extending length of segment B: (case-i) Extension on steep side of curve, (case-ii) equal extension on both sides and (case-iii) extension on opposite end of steep side. Error % results between

original micro- and (modified) macro-approach for K_{sp3} are shown in table 3 and % error values of thermal conductivity of composite in direction 1 (K_{sp1}) and direction 2 (K_{sp2}) are omitted in the table, as they are negligible. The % error values of table 3 infer that increase in % length, decreases the % error between micro- and modified macro-mechanics approaches.

4. Conclusions

In the present work, the effect of fibre waviness, crest offset, wavy-span on the thermal conductivities of a composite and also homogenization of wavy composite by using finite element method is studied for V_f ranging from 0.1 to 0.6 and thermal conductivity mismatch ratios from 0.1 to α . It is found that due to internal anisotropy, wavy fibre composite compared with straight fibre composite has lower K_{sp1} and higher K_{sp3} at their corresponding volume fractions. As the crest offset and wavy-span increases, K_{sp1} decreases and K_{sp3} increases. It is evident from the results that macro-mechanics approach yields minimal errors with minimum effort. Same can be extended to composite with other kinds of dissimilarities which is quite difficult with pure micro-mechanical models.

Nomenclature

a = Height of the composite in direction 3
 b = Width of the composite in direction 2
 L = Length of composite in direction 1
 L_o = Wavy profile's crest deviation in transverse direction to the fibre axis from the surface of the fibre
 R = Radius of the fibre
 $\sigma = K_f/K_m$ = Ratio of fibre to matrix thermal conductivities
 K_{ofi}/K_{sti} = Ratio of effective thermal conductivities of composite with wave-crest offset to straight fibre composite
 K_{spi}/K_{sti} = Ratio of effective thermal conductivities of composite with waviness over part of composite length to straight fibre composite, where $i = 1, 2, 3$.

Subscripts

1 = Longitudinal direction

2 = In-plane transverse direction
 3 = Through thickness transverse direction
 of = Wave offset
 sp = Wavy-span
 st = Straight fibre.

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