Accurate quasi static capacitance for abrupt homojunction under forward and reverse polarization

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Abstract. In this work, we present a new approach to derive the capacitance–voltage characteristic for an abrupt homojunction with uniform doping (N_A in p -region and N_D in n -region) under forward and reverse polarization. Under thermal equilibrium conditions, we show that it is possible to obtain analytically the exact capacitance–voltage characteristic without any simulation for the symmetric case ($N_A = N_D$). We also propose a model of the total capacitance under forward and reverse polarization for the asymmetric case ($N_A \neq N_D$).

Keywords. Homojunction; capacitance–voltage; analytical model; mobile carriers; physical junction; metallurgical junction.

1. Introduction

The p – n junction is the basic structure in a number of semiconductor devices based on electronic and opto-electronic components like diodes, bipolar-junction transistors (BJT), solar cells and detectors. The understanding of the electrical properties and the behaviour of this structure is thus essential for the design and optimization of all these semiconductor devices (Mazhari and Mahajan 2005).

The capacitance–voltage characteristic of a homojunction has been first investigated by Shockley (1949). In this model, it is assumed that in the depletion region there are no mobile carriers (full depletion approximation). For further details, the physics of the p – n junctions are explained by Sze (1981).

Furthermore, other models have been developed taking into account the presence of mobile carriers in the depletion region. For instance, some authors (Van Halen 1988; Van Halen and Habib 1993) have proposed a numerical study that leads to a specific model for the case of any applied voltage for a specified doping profile.

Haggag and Hess (2000) have built a general model for a symmetric homojunction that has been confirmed by numerical simulation, considering some limit of the transition region. This case has been already studied formally in (Van Den Biesen 1985), but for a more general limit of the transition region. In addition, many works have been performed in order to study numerically the C – V characteristic of a homojunction taking into account the presence of free carriers in the depletion region (Laux and Hess 1999).

In this work, our aim is to contribute to the study of this characteristic for any abrupt homojunction with a uniform doping and for any polarization in presence of free carriers

(holes and electrons) in the depletion region. Hereunder, we will present a systematic study of all ranges of the applied voltage.

In our approach, we derive an analytical expression of this characteristic for any symmetric abrupt homojunction with uniform doping under reverse and forward polarization. In addition, we present a new model of this characteristic for the asymmetric case, by means of an introduction of an adapted diffusion potential (called effective diffusion potential) in the expression of C – V characteristic of symmetrical homojunction. Our approach seems to be very satisfactory in the sense that our results are in good agreement with experimental results and with various simulation works.

This work is organized as follows. In §2, we will describe briefly the Shockley's model. In §3, we will present our approach in detail. We will first give the explicit expression of C – V characteristic of symmetric homojunction, and then we deduce a new model for the asymmetric case. Finally, we will report the conclusion and perspectives.

2. Shockley's model

The classical treatment of a p – n junction transition capacitance assumes that a depletion region exists around the metallurgical junction (Schottky 1939, 1942; Shockley 1949; Sze 1981).

The capacitance–voltage characteristic of an abrupt homojunction and its normalized version are given respectively by (Sze 1981):

$$C(V_{\text{app}}) = \frac{\epsilon}{L_{Dn,p}} \sqrt{\frac{N_{A,D}}{N_A + N_D}} (2x)^{-1/2}, \quad (1)$$

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and

$$\frac{C(V_{\text{app}})}{C_0} = \sqrt{\frac{V_d}{u_T}} x^{-\frac{1}{2}}, \quad (2)$$

where

$$x = \frac{V_d - V_{\text{app}}}{u_T}, \quad (3)$$

and

$$L_{D_{n,p}} = \sqrt{\frac{\varepsilon u_T}{q N_{D,A}}}, \quad (4)$$

is the Debye extrinsic length in p -type and n -type of this structure, ε the permittivity of the semiconductor material, q the magnitude of electric charge, u_T the thermal voltage, V_d the built-in potential (diffusion potential), V_{app} an external applied voltage and C_0 the capacitance at zero bias; N_A and N_D are the acceptor and donor concentrations in the p and n regions, respectively (all donor and acceptor impurities are ionized). In this context, the built-in potential takes the form, (Pota 2004):

$$V_d = u_T \ln \left(\frac{N_A N_D}{n_i^2} \right). \quad (5)$$

A major problem within this model is the singularity which occurs when $V_{\text{app}} = V_d$ characterizing an infinite capacitance (first anomaly). This is essentially due to the fact that it is assumed that there are no free carriers in the depletion region. The neglect of mobile carriers gives rise to significant errors in the calculation of electric field, potential distributions and depletion layer thicknesses (Kennedy 1975; Mohammadi and Selvakumar 1996). In the Shockley's model, the capacitance has not been defined for $V_{\text{app}} > V_d$ (second anomaly).

3. Our approach

3.1 Equilibrium case

The equations governing the potential (or the electric field) distribution, the electron and the hole densities are given in the case of one space dimension.

The population densities of electrons and holes are governed by the Fermi–Dirac statistical law (Sze 1981). For non-degenerate semiconductors, these densities obey Boltzmann statistics approximation (Ghandi 1968), i.e.

$$n_e(x) = n_i e^{\frac{E_F - E_i}{kT}} = n_i e^{\frac{V_i - V_F}{u_T}} \propto e^{+\frac{V^e(x)}{u_T}}, \quad (6)$$

and

$$p_e(x) = n_i e^{\frac{E_i - E_F}{kT}} = n_i e^{\frac{V_F - V_i}{u_T}} \propto e^{-\frac{V^e(x)}{u_T}}, \quad (7)$$

where n_i represents the intrinsic density, $E_i = qV_i(x)$ the intrinsic energy level, $E_{Fn} = E_{Fp} = qV_i(x)$ the unique

quasi-level Fermi energy (Slotboom 1977) and $V^e(x) = V_i(x) - V_F$ the local potential.

Making use of law of mass action (Sze 1981; Mohammad Asadul Haque and Jalalur Rahman 2009; Jomaa and Macaskill 2010)

$$n_e(x) \times p_e(x) = n_i^2 \quad \forall -\infty < x < +\infty. \quad (8)$$

Noting that the charge density $\rho(x)$ vanishes at the infinities (Adirovich *et al* 1958)

$$\rho(\pm\infty) = 0, \quad (9)$$

and that the potential difference between the ohmic contacts is defining the barrier potential at the equilibrium V_b^e (or equivalently, the diffusion potential V_d)

$$V^e(+\infty) - V^e(-\infty) = V_b^e \cong V_d, \quad (10)$$

the expressions of the free carrier densities reduce to the following expressions:

$$n_e(x) = N_D^e e^{\frac{V^e(x) - V_b^e}{u_T}}, \quad (11)$$

and

$$p_e(x) = N_A^e e^{-\frac{V^e(x)}{u_T}}, \quad (12)$$

where the quantities $N_{A,D}^e$ are given by:

$$N_{A,D}^e = \frac{N_{A,D} + \sqrt{N_{A,D}^2 + 4n_i^2}}{2}. \quad (13)$$

In this case, the total potential V_b^e across this p – n structure becomes:

$$V_b^e = u_T \ln \left(\frac{N_A^e N_D^e}{n_i^2} \right) \cong V_d. \quad (14)$$

Now, we have to determine the electric field:

$$E^e(x) = -\frac{dV^e(x)}{dx}, \quad (15)$$

where the electrostatic potential satisfies the Poisson's equation:

$$\frac{dV^e(x)}{dx} = -\frac{q}{\varepsilon} [p_e(x) - n_e(x) + (-1)^\kappa N_\kappa], \quad (16)$$

where $N_\kappa = \kappa N_A + (1 - \kappa)N_D$ and where $\kappa = 0$ and 1 correspond respectively to the n -type region and the p -type region. This leads to (Nuyts and Overstraeten 1971):

$$\begin{aligned} E^e(x) dE^e(x) &= \frac{q}{\varepsilon} [p_e(x) - n_e(x) + (-1)^\kappa N_\kappa] dV^e(x). \end{aligned} \quad (17)$$

By integration and using the following specific boundary conditions, as follows:

$$E^e(\pm\infty) = 0, \quad (18)$$

$$n_e(-\infty) = N_D^e e^{-\frac{V_b^e}{u_T}}, p_e(-\infty) = N_A^e, \quad (19)$$

$$p_e(+\infty) = N_A^e e^{-\frac{V_b^e}{u_T}}, n_e(+\infty) = N_D^e, \quad (20)$$

and

$$V^e(-\infty) = 0, V^e(+\infty) = V_b^e, \quad (21)$$

we find the following expressions for the electric field in function of the potential in the two regions (n and p), respectively:

$$E_\kappa^e(x) = -\frac{u_T}{L_{D_\kappa}} \sqrt{y_1^e - 4 \frac{n_i}{N_\kappa} \sinh\left(\frac{y_1^e}{2}\right) \sinh(y_2^e)}, \quad (22)$$

where

$$y_1^e = \frac{(1-\kappa)V_b^e - (-1)^\kappa V^e(x)}{u_T}, \quad (23)$$

and

$$y_2^e = \frac{y_1^e}{2} - \text{arc sinh}\left(\frac{N_\kappa}{2n_i}\right), \quad (24)$$

with

$$L_{D_\kappa} = \sqrt{\frac{\varepsilon u_T}{q N_\kappa}}. \quad (25)$$

We note that $\sinh$ corresponds to the sinus-hyperbolic function and arc sinh its inverse function.

Using the continuity of the electric field at the metallurgical junction point, i.e. $x = 0$, we can deduce the following expressions of the potential and the electric field at this point (the $n-p$ interface):

$$V^e(0) = V_0^e = \frac{N_A}{N_A + N_D} V_b^e + u_T \frac{N_A^e - N_D^e}{N_A + N_D} \left(1 - \frac{n_i^2}{N_A^e N_D^e}\right), \quad (26)$$

and

$$E^e(0) = E_0^e = -\frac{u_T}{L_{D_\kappa}} \sqrt{\frac{V_b^e - V_0^e}{u_T} - 4 \frac{n_i}{N_\kappa} \sinh\left(\frac{V_b^e - V_0^e}{2u_T}\right) \sinh\left(\frac{V_0^e}{2u_T} + \ln\left(\sqrt{\frac{N_D^e}{N_A^e}}\right)\right)}. \quad (27)$$

In this case, the electric field at the metallurgical junction point reduces to the following compact form for $V_b^e \geq 12 u_T$:

$$E^e(0) = -\frac{u_T}{L_{D_\kappa}} \times \sqrt{\frac{N_{(1-\lambda)}}{N_A + N_D} \left(\frac{V_b^e}{u_T} - 2\right) + e^{-\frac{N_{(1-\lambda)}}{N_A + N_D} \left(\frac{V_b^e}{u_T} - 2\right)} - 1}, \quad (28)$$

where $N_\lambda = \lambda N_A + (1-\lambda)N_D$, $N_{(1-\lambda)} = (1-\lambda)N_A + \lambda N_D$ and with $\lambda = 0$ corresponding to the case where $N_A \geq N_D$ and $\lambda = 1$ to the converse case.

At physical junction point, i.e. $x = x_{pj}^e$, we have (Murray 2009; Thide 2004):

$$n_e(x_{pj}^e) = p_e(x_{pj}^e). \quad (29)$$

This permits us to obtain the potential at the physical junction:

$$V_{pj}^e = V(x_{pj}^e) = \frac{V_b^e}{2} + u_T \ln\left(\sqrt{\frac{N_A^e}{N_D^e}}\right) = u_T \ln\left(\frac{N_A^e}{n_i}\right). \quad (30)$$

Then, the electric field at this point in the two cases $\lambda = 0$ and $\lambda = 1$, respectively is given by:

$$E_{pj}^e = E(x_{pj}^e) = -\frac{u_T}{L_{D_\lambda}} \sqrt{2 \ln\left(\frac{N_\lambda^e}{n_i}\right) - 2 \frac{N_\lambda^e - n_i}{N_\lambda^e + n_i}}, \quad (31)$$

where

$$L_{D_\lambda} = \sqrt{\frac{\varepsilon u_T}{q N_\lambda}}, \quad (32)$$

$$N_\lambda^e = \lambda N_A^e + (1-\lambda) N_D^e. \quad (33)$$

The total charge Q_λ^e per unit area is given by (Nuyts and Overstraeten 1971):

$$Q_\lambda^e = -\varepsilon E_{pj}^e + (-1)^\lambda q N_\lambda x_{pj}^e, \quad (34)$$

where x_{pj}^e is deduced from the expressions of the electric field and Faraday's equation (Murray 2009; Thide 2004):

$$x_{pj}^e = -\int_{V_0^e}^{V_{pj}^e} \frac{dV^e(x)}{E_\lambda^e(x)}, \quad (35)$$

where $E_\lambda^e(x)$ is the electric field in the weakly doped region, whose expression is given by (22), where κ is replaced by λ .

We note that x_{pj}^e depends only on the fact if N_A is strictly superior or inferior of N_D .

3.2 Non-equilibrium case

Now, in the non-equilibrium case, we can follow the same steps. Hereunder, the only difference by quasi-Fermi levels

for electrons and holes is due to the externally applied voltage, V_{app} (Sze 1981),

$$E_{F_n} - E_{F_p} = qV_{\text{app}}. \quad (36)$$

This implies that the law of mass action will now be rewritten (Adirovich *et al* 1958; Ghandi 1968; Sze 1981) as,

$$n_{\text{ne}}(x) \times p_{\text{ne}}(x) = n_i^2 e^{\frac{V_{\text{app}}}{u_T}}. \quad (37)$$

In order to study the capacitance–voltage characteristic for any homojunction with an uniform doping under any polarization in the presence of mobile carriers in the depletion region, we set:

$$N_i = n_i e^{\frac{V_{\text{app}}}{2u_T}}, \quad (38)$$

as a new ‘effective intrinsic concentration’ for each fixed value of the applied voltage, V_{app} .

In this framework, our mobile carrier densities read as,

$$n_{\text{ne}}(x) = N_D^{\text{ne}} e^{+\frac{V_{\text{ne}}(x) - V_b^{\text{ne}}}{u_T}}, \quad (39)$$

and

$$p_{\text{ne}}(x) = N_A^{\text{ne}} e^{-\frac{V_{\text{ne}}(x)}{u_T}}, \quad (40)$$

where

$$N_{A,D}^{\text{ne}} = \frac{N_{A,D} + \sqrt{N_{A,D}^2 + 4N_i^2}}{2}. \quad (41)$$

Now, the barrier potential becomes:

$$V_b^{\text{ne}} = u_T \ln \left(\frac{N_A^{\text{ne}} N_D^{\text{ne}}}{N_i^2} \right). \quad (42)$$

So, following this reason, we obtain exactly the same expressions as in the case of equilibrium, taking care to change $N_{A,D}^e$ by $N_{A,D}^{\text{ne}}$, V_b^e by V_b^{ne} and n_i by N_i .

Thus the electric fields in κ -region get the following form:

$$E_{\kappa}^{\text{ne}}(x) = -\frac{u_T}{L_{D_{\kappa}}} \sqrt{y_1^{\text{ne}} - 4 \frac{N_i}{N_{\kappa}} \sinh \left(\frac{y_1^{\text{ne}}}{2} \right) \sinh(y_2^{\text{ne}})}, \quad (43)$$

where

$$y_1^{\text{ne}} = \frac{(1 - \kappa) V_b^{\text{ne}} - (-1)^{\kappa} V_{\text{ne}}(x)}{u_T}, \quad (44)$$

and

$$y_2^{\text{ne}} = \frac{y_1^{\text{ne}}}{2} - \text{arc sinh} \left(\frac{N_{\kappa}}{2N_i} \right). \quad (45)$$

Hence, at the physical junction point, we have:

$$V_{\text{pj}}^{\text{ne}} = V(x_{\text{pj}}^{\text{ne}}) = \frac{V_b^{\text{ne}}}{2} + u_T \ln \left(\sqrt{\frac{N_A^{\text{ne}}}{N_D^{\text{ne}}}} \right) = u_T \ln \left(\frac{N_A^{\text{ne}}}{N_i} \right), \quad (46)$$

$$E_{\text{pj}}^{\text{ne}} = E(x_{\text{pj}}^{\text{ne}}) = -\frac{u_T}{L_{D_{\lambda}}} \sqrt{2 \ln \left(\frac{N_{\lambda}^{\text{ne}}}{N_i} \right) - 2 \frac{N_{\lambda}^{\text{ne}} - N_i}{N_{\lambda}^{\text{ne}} + N_i}}, \quad (47)$$

and

$$x_{\text{pj}}^{\text{ne}} = - \int_{V_0^{\text{ne}}}^{V_{\text{pj}}^{\text{ne}}} \frac{dV_{\text{ne}}(x)}{E_{\lambda}^{\text{ne}}(x)} = \int_{V_0^{\text{ne}}}^{V_{\text{pj}}^{\text{ne}}} f[V_{\text{app}}, V_{\text{ne}}(x)] dV_{\text{ne}}(x). \quad (48)$$

Using the continuity of the electric field at metallurgical junction point, i.e. $x = 0$, we can deduce the following expressions of the potential and the electric field at this point:

$$V^{\text{ne}}(0) = V_0^{\text{ne}} = \frac{N_A}{N_A + N_D} V_b^{\text{ne}} + u_T \frac{N_A^{\text{ne}} - N_D^{\text{ne}}}{N_A + N_D} \left(1 - \frac{N_i^2}{N_A^{\text{ne}} N_D^{\text{ne}}} \right), \quad (49)$$

and

$$E^{\text{ne}}(0) = E_0^{\text{ne}} = -\frac{u_T}{L_{D_{\kappa}}} \left[\frac{V_b^{\text{ne}} - V_0^{\text{ne}}}{u_T} - 4 \frac{N_i}{N_{\kappa}} \sinh \left(\frac{V_b^{\text{ne}} - V_0^{\text{ne}}}{2u_T} \right) \times \sinh \left[\frac{V_0^{\text{ne}}}{2u_T} + \ln \left(\sqrt{\frac{N_D^{\text{ne}}}{N_A^{\text{ne}}}} \right) \right] \right]. \quad (50)$$

The total charge Q_{λ}^{ne} per unit area, for the case $N_A \geq N_D$, will be written as:

$$Q_{\lambda}^{\text{ne}} = -\varepsilon E_{\text{pj}}^{\text{ne}} + qN_D x_{\text{pj}}^{\text{ne}}. \quad (51)$$

In general, the total capacitance C per unit area is defined as follows (Jomaa and Macaskill 2010):

$$C(V_{\text{app}}) = -\frac{dQ_{\lambda}^{\text{ne}}}{dV_{\text{app}}} = \varepsilon \frac{dE_{\text{pj}}^{\text{ne}}}{dV_{\text{app}}} - qN_D \frac{dx_{\text{pj}}^{\text{ne}}}{dV_{\text{app}}}. \quad (52)$$

This equation gives us the capacitance–voltage characteristic of an abrupt homojunction with uniform doping under any polarization in the presence of free carriers in the depletion region.

Using the following relation (Smirnov 1975),

$$\begin{aligned} & \frac{d}{dy} \left(\int_{z_1(y)}^{z_2(y)} f[y, z(y)] dz \right) \\ &= f[y, z_2(y)] \frac{dz_2(y)}{dy} - f[y, z_1(y)] \frac{dz_1(y)}{dy} \\ &+ \int_{z_1(y)}^{z_2(y)} \frac{\partial f[y, z(y)]}{\partial y} dy, \end{aligned} \quad (53)$$

for evaluating the last term in the previous relation, i.e.

$$\frac{dx_{pj}^{ne}}{dV_{app}} = \frac{d}{dV_{app}} \left(\int_{V_0^{ne}}^{V_{pj}^{ne}} f[V_{app}, V^{ne}(x)] dV^{ne}(x) \right), \quad (54)$$

we obtain

$$C = C_1 + C_2 + C_3 + C_4 = C_1 + C_{234}, \quad (55)$$

where

$$C_1 = \varepsilon \frac{dE_{pj}^{ne}}{dV_{app}}, \quad (56)$$

$$C_2 = qN_D f[V_{jp}, V_{app}] \frac{dV_{pj}^{ne}}{dV_{app}} = -\frac{qN_A N_D}{2E_{pj}^{ne} \sqrt{N_A^2 + 4N_i^2}}, \quad (57)$$

$$C_3 = -qN_D f[V_0, V_{app}] \frac{dV_0^{ne}}{dV_{app}} = q \frac{N_D}{N_A + N_D} \times \frac{(a + b + c + d)}{4E_0^{ne} N_i^2 \sqrt{(N_A^2 + 4N_i^2)(N_D^2 + 4N_i^2)}}, \quad (58)$$

$$C_4 = qN_D \int_{V_0^{ne}}^{V_{pj}^{ne}} \frac{\partial f[V_{app}, V^{ne}(x)]}{\partial y} dV^{ne}(x), \quad (59)$$

and with

$$C_{234} = C_2 + C_3 + C_4, \quad (60)$$

$$a = -(N_A - N_D) [N_A^2 N_D^2 + 2(N_A^2 + N_D^2) N_i^2], \quad (61)$$

$$b = [N_A(N_A - N_D) N_D^2 + 2(N_A^2 - N_A N_D - N_D^2) N_i^2 - 16N_i^4] \sqrt{N_A^2 + 4N_i^2}, \quad (62)$$

$$c = [N_D(N_A - N_D) N_A^2 + 2N_D^2 N_i^2 + 16N_i^4] \times \sqrt{N_A^2 + 4N_i^2}, \quad (63)$$

$$d = N_A N_D (N_A - N_D) \sqrt{(N_A^2 + 4N_i^2)(N_D^2 + 4N_i^2)}. \quad (64)$$

The quantities C_1 , C_2 and C_3 are easily computed, and the term C_4 can be evaluated using numerical integration (Press *et al* 1988). The case $N_D \geq N_A$ is similar.

For instance, for a symmetric p - n junction, the term C_{234} vanishes and the capacitance is given only by the term C_1 .

For a dissymmetric case, the term C_{234} will be added to the first symmetric one, C_1 .

Hence, in the symmetric case, i.e. $N_A = N_D$, the exact capacitance takes the following expression:

$$C(V_{app}) = \frac{\varepsilon}{L_{Dn}} \times \frac{\frac{\sqrt{N_D^2 + 4N_i^2} - 2N_i}{2N_i}}{\sqrt{2\ln\left(\frac{N_D + \sqrt{N_D^2 + 4N_i^2}}{2N_i}\right) - 2\left(\frac{\sqrt{N_D^2 + 4N_i^2} - 2N_i}{N_D}\right)}}. \quad (65)$$

To deduce its expression in the equilibrium case, it is sufficient to evaluate it at $V_{app} = 0$:

$$C_0 = \frac{\varepsilon}{L_{Dn}} \times \frac{\frac{\sqrt{N_D^2 + 4N_i^2} - 2N_i}{2N_i}}{\sqrt{2\ln\left(\frac{N_D + \sqrt{N_D^2 + 4N_i^2}}{2N_i}\right) - \left(\frac{\sqrt{N_D^2 + 4N_i^2} - 2N_i}{N_D}\right)}} \cong \frac{\varepsilon}{L_{Dn} \sqrt{\frac{V_d}{u_T} - 2}}. \quad (66)$$

The expression of the normalized capacitance for the symmetric case can be rewritten as:

$$\frac{C(V_{app})}{C_0} = \frac{\sqrt{1 + 4e^{-x}} - 2e^{-\frac{x}{2}}}{\sqrt{x + 2\ln\left(\frac{1 + \sqrt{1 + 4e^{-x}}}{2}\right) - 2\left(\sqrt{1 + 4e^{-x}} - 2e^{-\frac{x}{2}}\right)}} \times \sqrt{\frac{V_d}{u_T} - 2}, \quad (67)$$

where x is defined by (3).

Finally, the expression of the total capacitance for a homojunction with any uniform doping and under any polarization is given by:

$$C = C_1 + C_{234}. \quad (68)$$

Since it is practically impossible to solve analytically the term C_4 and because this term is important comparatively to the other terms entering in the expression of C_{234} , we can just be able to give an approximated expression of the total capacitance for the asymmetric case. In order to solve this problem, we propose the following new model for the total capacitance:

$$C(V_{app}) = C_0 \frac{\sqrt{1 + 4e^{-x_{eff}}} - 2e^{-\frac{x_{eff}}{2}}}{\sqrt{x_{eff} + 2\ln\left(\frac{1 + \sqrt{1 + 4e^{-x_{eff}}}}{2}\right) - 2\left(\sqrt{1 + 4e^{-x_{eff}}} - 2e^{-\frac{x_{eff}}{2}}\right)}} \times \sqrt{\frac{V_{d,eff}}{u_T} - 2}, \quad (69)$$

where

$$x_{eff} = \frac{V_{d,eff} - V_{app}}{u_T}. \quad (70)$$

This new model requires two input parameters, viz. C_0 and $V_{d,eff}$.

The C - V characteristic presents two asymptotic behaviours.

For an applied voltage going up to $V_d - 7u_T$, the capacitance can be written as:

$$C(V_{app}) = -\frac{dQ_{\lambda}^{ne}}{dV_{app}} = \varepsilon \frac{dE_{pj}^{ne}}{dV_{app}} - qN_D \frac{dx_{pj}^{ne}}{dV_{app}}.$$

In figure 1, we give several examples of symmetric abrupt homojunction Si/Si ($\varepsilon \approx 100$ pF/m) at room temperature.

In figures 2 and 3, we treat the case of dissymmetric p - n junction (Si/Si), with $N_D = 10^{21} \text{ m}^{-3}$, $N_D = 10^{22} \text{ m}^{-3}$, respectively. In this case, we note that for important asymmetries the maximum stability at the point is defined by:

$$V_{app,max} \approx 2u_T \ln \left(\frac{N_{\lambda}}{n_i} \right) - 2.29u_T, \quad (71)$$

and

$$C_{max} \approx 0.308 \frac{\varepsilon}{L_{D_{\lambda}}}. \quad (72)$$

In figure 4, we show the contribution of a purely symmetric part C_1 of the capacitance and one of its purely dissymmetric part C_{234} . It appears that in general case, the two terms play the role of two capacitances in parallel, since $C = C_1 + C_{234}$.

We try to present some modeling of the peaks corresponding to different values of C_{max} of various symmetric abrupt homojunction. It is seen to obey the following laws:

$$V_{app,max} \cong 2u_T \ln \left(\frac{N_D}{n_i} \right) - 3.44854275690 u_T, \quad (73)$$

$$C_{max} = C(V_{app,max}) \cong 0.24332401386 \frac{\varepsilon}{L_{D_n}}, \quad (74)$$

and

$$N_i(V_{app,max}) \cong 0.17830291980 N_D. \quad (75)$$

Figure 1. Normalized capacitance-voltage characteristic for symmetric p - n junction (Si/Si) at room temperature.

The analytical methods have the advantage of promoting insight into device behaviour, guiding the interpretation of numerical simulations (Barybin and Santos 2007). Equations (73)–(75) can be used as performance reference for any future numerical simulations for symmetric homojunction.

In general, the peak of $C(V_{app})$ corresponds to an applied voltage:

$$V_{app,max} \approx 2u_T \ln \left(\frac{N_{\lambda}}{n_i} \right) - (2.9 \pm 0.6) u_T. \quad (76)$$

Figure 5 shows maximum C_{max} for different asymmetries. Setting:

$$\alpha = \frac{C_{max} L_{D_{\lambda}}}{\varepsilon}, \quad \beta = \frac{N_{\lambda}}{N_{(1-\lambda)}}, \quad (77)$$

we find that α varies from the value 0.243 in the limit case of non-asymmetry (i.e. symmetrical case) to the stabilized value of 0.308 for higher asymmetries as discussed before.

Figure 2. Capacitance-voltage characteristic of dissymmetric p - n junction ($N_D = 10^{15} \text{ cm}^{-3}$).

Figure 3. Capacitance-voltage characteristic of dissymmetric p - n junction ($N_D = 10^{16} \text{ cm}^{-3}$).

Figure 4. Decomposition of capacitance (dissymmetric p - n junction) into a purely symmetric part C_1 and a purely dissymmetric part C_{234} .

Figure 5. Variation of normalized maximum $C_{\max}$ for different asymmetries.

Figure 6. Comparison between our model and numerical calculations for dissymmetric p - n junction (Si/Si, $T = 300^\circ\text{K}$, $N_D = 10^{16} \text{ cm}^{-3}$ and $N_A = 10^{18} \text{ cm}^{-3}$).

In figure 6, we compare our model (see (69)) with numerical results.

4. Conclusions

Finally, we conclude that our approach appears to be well appropriate to study the C - V characteristic of any abrupt homojunction with uniform doping and under any polarization in comparison with the experimental results (Serra *et al* 1993) and other works (Schmidt *et al* 2007; Kavasoglu *et al* 2008). In fact, our results are not only in good agreement with the other simulation works (Haggag and Hess 2000), but they give an exact expression of the total capacitance in function of the applied voltage in the symmetric case. In the asymmetric situation, our model seems to be accurate since the error at the maximum does not exceed 10% for higher asymmetries. We plan to treat in our future work the characteristic of heterojunctions following the same approach.

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