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On statistical behaviour of stress drops in Portevin–Le Chatelier effect

A CHATTERJEE*, P MUKHERJEE, N GAYATHRI, P BARAT, ARNAB BARAT[†] and A SARKAR[††]

Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Kolkata 700 064, India
[†]Indian Statistical Institute, 203 B.T. Road, Kolkata 700 105, India
[††]Mechanical Metallurgy Section, Bhabha Atomic Research Centre, Mumbai 400 085, India

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Abstract. **The Portevin–Le Chatelier (PLC) effect is a kind of plastic instability observed in many dilute alloys when deformed at certain ranges of strain rate and temperature. In this paper we present a comprehensive statistical analysis of the observed experimental data obtained during PLC effect and establish that the occurrence probability of the stress drops in the dynamical process responsible for PLC effect is Poisson in nature.**

Keywords. **Plastic deformation; Portevin–Le Chatelier effect; aluminum alloy; steel; Poisson distribution.**

1. Introduction

Many interstitial and substitutional alloys exhibit repeated stress drops followed by periods of reloading during tensile deformation in certain ranges of strain rate and temperature (Mazot 1979; Balik and Lukac 1993). This repeated yielding of these alloys is referred to as the Portevin–Le Chatelier (PLC) effect and has been extensively studied over the decades (Van den Beukel 1980; Kubin and Estrin 1990; Rizzi and Hahner 2004). It is a striking example of the collective behaviour of dislocations leading to complex spatiotemporal patterns. Due to the complexity of the problem, the methods of nonlinear dynamics and statistical analysis have been applied to understand the underlying dynamics of the PLC effect (Ananthakrishna *et al* 1999; Kugiumtzis *et al* 2004). These studies have provided a considerable understanding of the mechanism of PLC effect. The general consensus explains the origin of the PLC effect as a dynamical interaction of the mobile dislocations and the diffusing solute atoms, which is denoted as the dynamic strain aging (DSA) (Van den Beukel 1980; Kubin and Estrin 1990; Rizzi and Hahner 2004). Mobile dislocations which are the carrier of the plastic strain move jerkily between the obstacles provided by the other dislocations. Solute atoms diffuse in the stress field generated by the mobile dislocations and pin them further while they are arrested at the obstacles. This DSA leads to negative strain rate sensitivity of the flow stress for certain ranges of applied strain rate and temperature when the mobile dislocations and the solute atoms have comparable mobility. Bands of localized deformation are then formed, in association with stress serrations and close investigations of the PLC effect revealed the occurrence of different types of stress serrations. These serrations are well characterized in polycrystals, where they exhibit three main types of behaviour of the bands: static, hopping and propagating, which are traditionally labeled as types C, B and A, respectively. Type C bands appear almost at random in the sample without propagating, type B bands exhibit an oscillatory or intermittent propagation and type A bands propagate continuously. Recent analyses suggest that distinct dynamic features could be associated with each of these band types (Bharathi *et al* 2001, 2002). At low strain rates static (type C) bands are associated with weak spatial interactions, consistent with randomness in their spatial distribution. In contrast, at high strain rates, strong spatial correlations are associated with type A propagating bands, leading to self-organized criticality regime. At medium strain rates, partially relaxed spatial interactions lead to marginal spatial coupling linked to type B hopping bands. In this case, a chaotic regime was demonstrated (Kubin *et al* 2002).

The occurrence of the stress drops during the PLC effect is an outcome of complex nonlinear threshold dynamics in the material. This dynamics is a combined effect of different temporal and spatial processes taking place in a highly heterogeneous media over a wide range of temporal and spatial scales. Despite this complexity, one can consider the PLC stress drops as a point process in space and time, by neglecting the spatial scale of the bands and the temporal scale of the duration of each stress drop. Hence, one can study statistical properties of this process and test the methods that may explain the observed load drops. In this paper, we present a comprehensive analysis of the statistical nature of the occurrence probability of the stress drops during PLC effect in Al–2·5%Mg alloy and low carbon steel.

*Author for correspondence (arnomitra@vecc.gov.in)

2. Experimental

Al–Mg alloys containing a nominal percentage of Mg and low carbon steel exhibit PLC effect at room temperature for a wide range of strain rates (Chihab *et al* 1987; Pink and Kumar 1995). We have carried out tensile tests at room temperature on flat Al–2·5%Mg alloy and cylindrical low carbon steel samples at different strain rates (10^{-5} s^{-1} to 10^{-3} s^{-1}). In this range of strain rate, we could observe three types of stress serrations. The details of the experimental procedures can be found elsewhere (Barat *et al* 2005).

3. Method of analysis and results

Figure 1 shows the observed PLC effect in Al–2·5%Mg alloy and low carbon steel samples. In our experiments, we have recorded the load at an interval of 0·05 s. True stress was calculated from the load values using instantaneous cross-sectional area of the sample. For our analysis, we have performed a symbolic representation of the experimental stress time series data. When there was a stress drop, it was designated by one and its absence by zero. Hence, the resulting experimental data were binary in nature consisting of a sequence of 0s and 1s.

During the tensile deformation of the samples, the occurrences of stress drops in the stress-time curve apparently looked to be random and rare. These rare occurrences of the stress drops lead us to believe that their distribution is Poisson in nature. In order to validate our assumptions, we have adopted the following statistical analysis.

Any distribution or process is characterized by certain specific parameters which ultimately govern the distribution or the process. In a Poisson process, the distribution can be represented by the form

$$f(x) = \frac{\lambda(t)^x e^{-\lambda(t)}}{x!}, \tag{1}$$

where λ is a function of time and x the number of occurrences. The parameter $\lambda(t)$ of the Poisson distribution is usually estimated by mean of the data, which are the minimum variance unbiased estimator and the maximum likelihood estimator (Fisher 1971). Table 1 shows estimated mean of the experimental data in binary form at some arbitrarily chosen strain rate values obtained for Al–2·5%Mg alloy and low carbon steel. The small values of the mean signify that the occurrences of the stress drops are rare. To analyse the experimental data statistically, we have subdivided the entire data set obtained from a particular strain rate experiment, into subgroups of 4 to 8 time-points. The occurrences of the load drops for this modified data set should follow a Poisson distribution with a new parameter, $n_1\lambda$ (n_1 being the number of time-points in a single subgroup). We formulate the empirical Poisson distribution from this new parameter and compute the expected frequency of occurrence by (1). From the modified data set, we also tabulate a frequency distribution and compute the relative frequency. This relative frequency will be close to the expected frequency if the data really follow Poisson distribution. To confirm this, we carried out the χ^2 and Kullback–Leibler (KL) tests (Rao 1973). The results of the χ^2 test and the corresponding p-values (Sellke *et al* 2001) are shown in table 2. It is seen that the p-values for

Table 1. Values of test statistic (z) and corresponding p-values for some arbitrary chosen strain rates.

Al–2·5%Mg alloy		Low carbon steel	
Strain rate (s^{-1})	Mean	Strain rate (s^{-1})	Mean
$8{\cdot}06 \times 10^{-5}$	0·0684	$6{\cdot}30 \times 10^{-5}$	0·0756
$3{\cdot}90 \times 10^{-4}$	0·0962	$3{\cdot}84 \times 10^{-4}$	0·1421
$6{\cdot}25 \times 10^{-4}$	0·1250	$8{\cdot}28 \times 10^{-4}$	0·1607
$1{\cdot}20 \times 10^{-3}$	0·2093	$1{\cdot}34 \times 10^{-3}$	0·1675
$1{\cdot}94 \times 10^{-3}$	0·2491	$2{\cdot}85 \times 10^{-3}$	0·2115

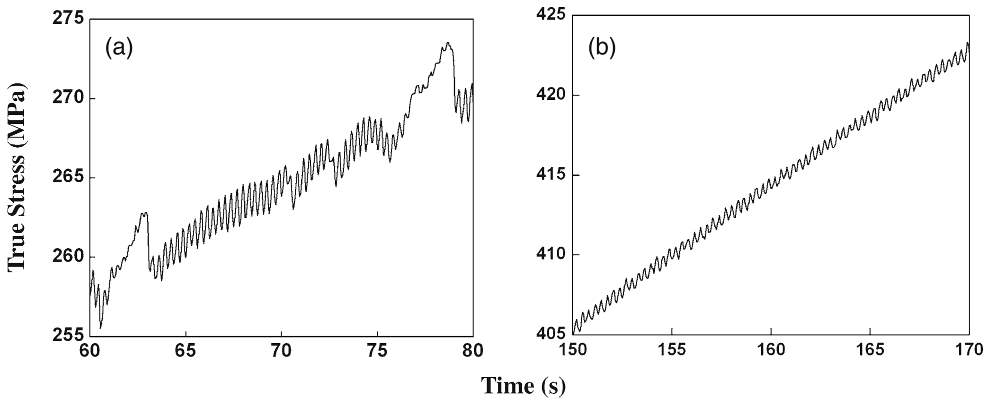

Figure 1. Typical segment of stress-time curves for **(a)** Al–2·5%Mg alloy deformed at a strain rate of $6{\cdot}25 \times 10^{-4}$ s^{-1} and **(b)** low carbon steel deformed at a strain rate of $6{\cdot}30 \times 10^{-5}$ s^{-1}. PLC serrations are prominent in true stress time curves.

Table 2. Results of χ^2 test and corresponding p-values of data sets with subgroups of 4 time-points.

Al–2·5%Mg alloy			Low carbon steel		
Strain rate (s^{-1})	Value of χ^2-test statistic	p-value	Strain rate (s^{-1})	Value of χ^2-test statistic	p-value
$8{\cdot}06 \times 10^{-5}$	0·0202	0·9992	$6{\cdot}30 \times 10^{-5}$	0·0001	0·9999
$3{\cdot}90 \times 10^{-4}$	0·0415	0·9978	$3{\cdot}84 \times 10^{-4}$	0·0152	0·9995
$6{\cdot}25 \times 10^{-4}$	0·0504	0·9970	$8{\cdot}28 \times 10^{-4}$	0·0749	0·9947
$1{\cdot}20 \times 10^{-3}$	0·0899	0·9930	$1{\cdot}34 \times 10^{-3}$	0·0317	0·9985
$1{\cdot}94 \times 10^{-3}$	0·1326	0·9877	$2{\cdot}85 \times 10^{-3}$	0·0577	0·9964

Table 3. Values of test statistic (z) and corresponding p-values for some arbitrary chosen strain rates.

	Al–2·5%Mg alloy				Low carbon steel			
	Strain rate (s^{-1})				Strain rate (s^{-1})			
	$8{\cdot}06 \times 10^{-5}$		$1{\cdot}94 \times 10^{-3}$		$3{\cdot}84 \times 10^{-4}$		$2{\cdot}85 \times 10^{-3}$	
Data sets	z	p	z	p	z	p	z	p
1–2	340·0444	0·0	41·0903	0·0	379·8972	0·0	158·4237	0·0
2–3	489·3531	0·0	15·6070	0·0	121·3015	0·0	98·5782	0·0
3–4	145·6694	0·0	5·0375	0·0	54·2555	0·0	5·2577	0·0
4–5	897·7044	0·0	17·0666	0·0	15·2467	0·0	64·3697	0·0

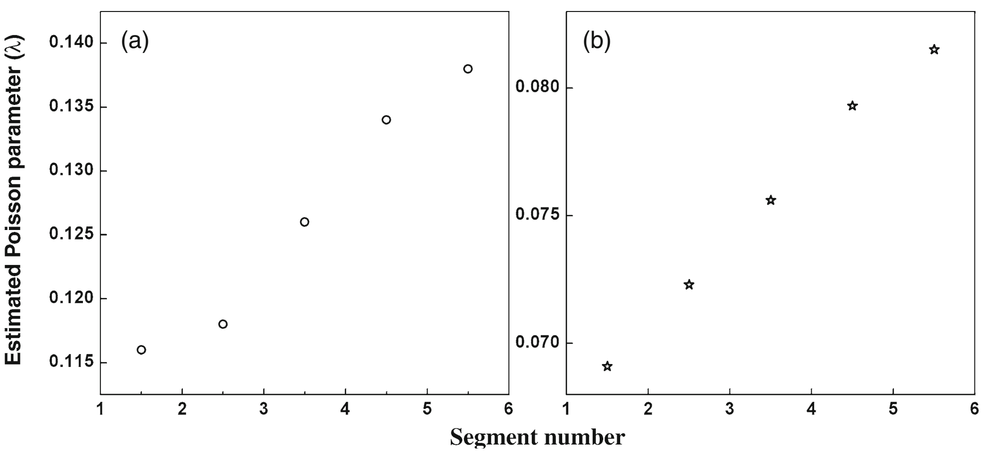

Figure 2. Typical plot showing variation of estimated Poisson parameter, λ, with time (segment number) for **(a)** Al–2·5%Mg alloy deformed at strain rate, $6{\cdot}25 \times 10^{-4}$ s^{-1} and **(b)** low carbon steel deformed at strain rate, $6{\cdot}30 \times 10^{-5}$ s^{-1}. λ increases with time indicating that λ is not constant in a deformation test.

PLC data obtained from all strain rates are >0·95. KL distances are also the measures of the distance between empirical and experimental distribution. The small values of KL distances indicate that the two distributions are alike. For the analysed data, the KL distance are found to vary from 0·01 to 0·08. Hence, from the results of χ^2 and KL tests, it can be claimed with full confidence that the experimental data actually follow Poisson distribution.

To prove through the confirmatory test that λ is not a constant but varies with time during the test, we divide each data set, obtained from a particular strain rate experiment, into 4–6 segments of equal time-points, m and estimate the value of λ for each segment. Choosing two contiguous segments, a two-sided hypothesis test (Clark 1963) was performed at 95% confidence level to test the null hypothesis (H_0: $\lambda_1 = \lambda_2$) against the alternate hypothesis (H_A: $\lambda_1 \neq \lambda_2$), where λ_1 and λ_2 are the values of λ for the two chosen contiguous segments X and Y, respectively.

Under H_0, $X - Y$ will have the mean parameter $\lambda_1 - \lambda_2$ (which is zero) and the variance $\sigma_1^2/m + \sigma_2^2/m$ (σ_1 and σ_2

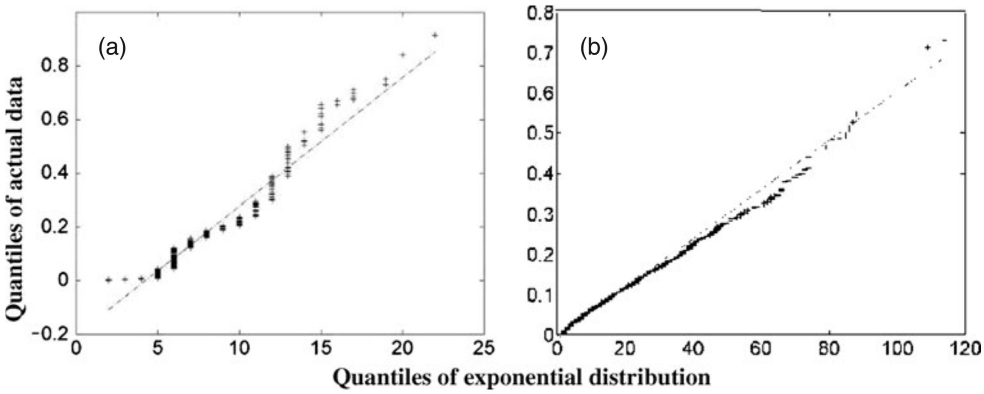

Figure 3. Quantile quantile plot for **(a)** Al–2·5%Mg alloy deformed at a strain rate, $6{\cdot}25 \times 10^{-4}$ s^{-1} and **(b)** low carbon steel deformed at a strain rate, $6{\cdot}30 \times 10^{-5}$ s^{-1}. Quantiles of waiting time distribution are plotted against quantiles of specific exponential distribution. QQ plots are close to 45° line.

are the standard deviation of the first and second segments, respectively). By the law of large numbers (Grimmett and Stirzaker 1992) or the central limit theorem (Feller 1971)

$$\left[\{(X-Y)-0\}/\left\{\left(\sigma_1^2/m+\sigma_2^2/m\right)\right\}^{1/2}\right]=z,$$

will follow a normal distribution with mean and variance equal to 0 and 1, respectively. The values of our test statistic z, and the corresponding p-values for both the samples are listed in table 3. The p-values are $<0{\cdot}025$ or $>0{\cdot}975$ i.e. cut-off value $>1{\cdot}96$ or $<-1{\cdot}96$. Thus we reject the Null hypothesis and say that the process is not stationary. From the estimated values of λ in each of the four or six segments, we see that there is a generic increasing trend of λ as a function of time. Figure 2 shows a typical plot of the variation of λ with segment number for Al–2·5%Mg alloy and low carbon steel where the values of λ are 0·1250 for the Al–2·5%Mg alloy deformed at a strain rate of $6{\cdot}25 \times 10^{-4}$ s^{-1} and 0·0756 for the low carbon steel deformed at a strain rate of $6{\cdot}30 \times 10^{-5}$ s^{-1}.

If the time dependence of λ is linear, the waiting time (time difference between the successive load drops) should follow an exponential distribution (Feller 1971). In this regard, a quantile quantile plot (QQ-plot) (Nair and Freeny 1994) of the waiting time distribution against the specific exponential distribution was drawn. Figure 3 shows the typical QQ-plots for Al–2·5%Mg alloy and low carbon steel. For most of the cases, the QQ-plots are close to the 45° line indicating that the waiting time follows an exponential distribution. This proves that λ varies linearly with time for the two types of samples.

4. Discussion

Poisson process represents a set of events which are mutually independent. In our present work, we could establish that the occurrence probability of the stress drops during the PLC effect is Poisson in nature. During its propagation through the material, the movement of deformation bands is hindered by the presence of several obstacles and successively they are pinned by the solute atoms. The pinning of a deformation band is governed by several factors like distribution of the obstacles, local solute distribution and the mobility of the deformation font etc. The band movement will be initiated further provided the pinning barrier is overcome by increased stress level and the thermal activation associated with the respective pinning site. The system response to each unpinning of deformation band is manifested as a stress drop in macroscopic scale. Hence, the instant at which a stress drop will occur will be governed by all these factors. But there is no knowledge *a priori* to the system when the deformation band will encounter the obstacle or if the thermal activation will be high enough to unpin the band. Moreover, the thermal fluctuations are completely random in nature. Hence, occurrence of each individual stress drop is an independent event and it is not influenced by the occurrence of the earlier ones.

5. Conclusions

In conclusion, we have carried out uniaxial tensile tests on Al–2·5%Mg alloy and low carbon steel at a wide range of strain rates where PLC effect is observed. The experimental stress time series data are analysed using different statistical approaches. Analysis revealed that the occurrences of stress drops during the PLC effect follow Poisson distribution. The mean of the Poisson distribution increases linearly with time or strain. The time interval between two consecutive stress drops is found to obey an exponential distribution. All these observations indicate that the stress drop during the PLC effect is an outcome of a Poisson process.

References

Ananthakrishna G, Noronha S J, Fressengeas C and Kubin L P 1999 *Phys. Rev.* **E60** 5455

Balik J and Lukac P 1993 *Acta Metal. Mater.* **41** 1447

Barat P, Sarkar A, Mukherjee P and Bandyopadhyay S K 2005 *Phys. Rev. Lett.* **94** 055502

Bharathi M S, Lebyodkin M, Ananthakrishna G, Fressengeas C and Kubin L P 2001 *Phys. Rev. Lett.* **87** 165508

Bharathi M S, Lebyodkin M, Ananthakrishna G, Fressengeas C and Kubin L P 2002 *Acta Mater.* **50** 2813

Chihab K, Estrin Y, Kubin L P and Vergnol J 1987 *Scr. Metall.* **21** 203

Clark C A 1963 *Rev. Educ. Res.* **33** 455

Feller W 1971 *An introduction to probability theory and its applications* (New York: John Wiley) **Vol. II**

Fisher R A 1971 *Statistical methods and scientific inferences* (New York: Hafner) 3rd ed.

Grimmett G R and Stirzaker D R 1992 *Probability and random processes* (Oxford: Clarendon Press) 2nd ed.

Kubin L P and Estrin Y 1990 *Acta Metall. Mater.* **38** 397

Kubin L P, Fressengeas C and Ananthakrishna G 2002 *Dislocations in solids* (eds) F R N Nabarro and M S Duesbery (Amsterdam: Elsevier Science) **11** p. 101

Kugiumtzis D, Kehagias A, Aifantis E C and Neuhauser H 2004 *Phys. Rev.* **E70** 036110

Mazot P 1979 *Acta Metall.* **21** 943

Nair Vijayan N and Freeny Anne E 1994 *Statistical methods for physical science* (eds.) Celotta Robert and Lucatorto Thomas (San Diego: Academic Press) **28** p. 192

Pink E and Kumar S 1995 *Mater. Sci. Eng.* **A201** 58

Rao C R 1973 *Linear statistical inference and its applications* (New York: John Wiley)

Rizzi E and Hahner P 2004 *Int. J. Plast.* **20** 121

Sellke T, Bayarri M J and Berger J O 2001 *The American Statistician* **55** 62

Van den Beukel A 1980 *Acta Metall.* **28** 965