

## Interlayer coupling in peroxitonic model of superconductivity

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**Abstract.** The unusual ESR spectra of  $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$  is ascribed to a  $\text{Cu}^{2+}$  ion, whose one ligand in  $\text{CuO}_2$  plane is involved in a peroxiton  $\text{O}^- - \text{Cu}^+ - \text{O}^-$  bond. The analysis shows that the presence of a ligand in  $\text{O}^-$  substantially increases the energies of  $d$ -hole orbitals. Since this increase for  $d_{x^2-y^2}$  orbital is much larger than that for  $d_{3z^2-r^2}$  orbital, their separation is drastically reduced and there occurs a significant orthorhombic mixing between the two. This charge transfer from a planar  $d_{x^2-y^2}$  orbital to a  $d_{3z^2-r^2}$  orbital, which lies mainly perpendicular to the plane, provides a mechanism for interlayer coupling. The effect of this coupling on superconducting transition temperature is discussed.

**Keywords.** Interlayer coupling; peroxitonic model.

### 1. Introduction

Nearly isotropic ESR spectrum of  $\text{YBaCu}_3\text{O}_{7-x}$  observed by Meheran *et al* (1988) was recently explained (Rai 1989) by ascribing it to a  $\text{Cu}^{2+}$  ion, whose one ligand in  $\text{CuO}_2$  plane is involved in a  $\text{O}^- - \text{Cu}^+ - \text{O}^-$  peroxiton bond with an oxygen and copper of neighbouring  $\text{CuO}_2$  unit (figure 1). The analysis shows that the presence of a ligand in  $\text{O}^-$  form, instead of  $\text{O}^{2-}$ , drastically reduces the separation between  $d_{x^2-y^2}$  and  $d_{3z^2-r^2}$  orbitals and causes a significant orthorhombic mixing between the two. Thus there occurs an oxygen hole-assisted charge transfer from a planar  $d_{x^2-y^2}$  orbital to a  $d_{3z^2-r^2}$  orbital, which lies mainly in a direction perpendicular to the plane. A simultaneous charge transfer of this type at the nearest copper sites on two adjacent  $\text{CuO}_2$  planes will affect an out-of-phase displacement of bridge  $\text{O}_4$  oxygen atoms in the direction perpendicular to the planes (figure 1). This provides a mechanism for interlayer coupling. Our aim in the present paper is to investigate the effect of this coupling on superconducting transition temperature.

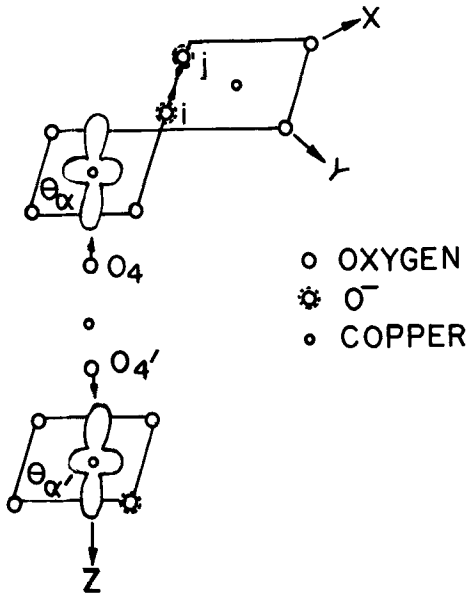
### 2. Theory

The appropriate hamiltonian can be written as

$$H = H_0 + H', \quad (1)$$

$$H_0 = \sum_{k\sigma} [(t_k^e + E_\epsilon)n_{k\sigma}^e + t_k^\pi \pi_{k\sigma}^+ \pi_{k\sigma}] + \sum_{\langle ia \rangle \langle ij \rangle} \left[ u \sum_{\substack{(rr') \\ = (\epsilon\theta)}} n_{a\sigma}^r n_{a'\sigma'}^{r'} \right. \\ \left. + U_\pi n_{i\uparrow}^\pi n_{i\downarrow}^\pi - J \pi_{i\uparrow}^+ \pi_{j\downarrow}^+ \pi_{j\uparrow} \pi_{i\downarrow} + h(\epsilon_{a\sigma}^+ \pi_{i\sigma} + \pi_{i\sigma}^+ \epsilon_{a\sigma}) + Q_{ij} n_{i\sigma}^\pi n_{j-\sigma}^\pi \right], \quad (2)$$

$$H' = \sum_{\substack{\langle ia \rangle \langle aa' \rangle \\ \sigma}} \left[ E_\theta \theta_{a\sigma}^+ \theta_{a\sigma} + (V_\epsilon n_{a\sigma}^e + V_\theta n_{a\sigma}^e) n_{i\sigma}^\pi + d_i n_{i\sigma}^\pi (\epsilon_{a\sigma}^+ \theta_{a\sigma} + \theta_{a\sigma}^+ \epsilon_{a\sigma}) \right. \\ \left. + \left\{ \hbar w b_{aa'}^+ b_{aa'} + \lambda_\perp \left( \frac{\hbar}{2\mu_\perp w_\perp} \right)^{1/2} (b_{aa'}^+ + b_{aa'}) \right\} n_{a\sigma}^\theta n_{a'\sigma'}^\theta \right]. \quad (3)$$



**Figure 1.** Occupancy of  $d_{3z^2-r^2}$  orbital affected by a  $O^-$  ligand and an out-of-phase displacement of bridge  $O_4$  and  $O_{4'}$  oxygen atoms due to simultaneous occupancy of  $d_{3z^2-r^2}$  orbitals on sites  $\alpha$  and  $\alpha'$ , the nearest neighbours on two adjacent  $CuO_2$  planes.

$H_0$  is essentially the hamiltonian proposed by Chakraverty *et al* (1987). Here  $E$  refers to the hybridized  $d_{x^2-y^2} - p_\sigma$  antibonding orbital,  $\theta$  to  $s_{3z^2-r^2}$  and  $\pi$  to oxygen  $p_\pi$ . The operators  $\varepsilon^+(\varepsilon)$  etc are the corresponding band or site hole creation (annihilation) operators.  $Q_{ij}$  involves the  $O_i - O_j$  bond fluctuation in a  $CuO_2$  plane due to the presence or absence of a hole on each of these two oxygen atoms.

The quantities  $V_\varepsilon$  and  $V_\theta$  in  $H'$  are respectively the increase in Coulomb energies of  $\varepsilon$  and  $\theta$  hole orbitals due to the presence of a hole on a nearest neighbour oxygen site in  $CuO_2$  plane,  $d_i = d$  or  $-d$  depending upon whether the site  $i$  is on  $x$  or  $y$  axis (figure 1) and  $d$  is an orthorhombic parameter (Rai 1989). The notation  $\langle \alpha\alpha' \rangle_\perp$  indicates that  $\alpha$  and  $\alpha'$  are the nearest copper sites on two adjacent  $CuO_2$  planes. The presence or absence of a hole on each of the two  $\theta_\alpha$  and  $\theta_{\alpha'}$  orbitals is responsible for the out-of-phase oscillation of the two bridge  $O_4$  oxygen atoms (figure 1) and  $b_{\alpha\alpha'}^+$  ( $b_{\alpha\alpha'}$ ) are the phonon creation (destruction) operators corresponding to this oscillation.

Using a unitary transformation

$$\bar{H} = u^+ H u, \tag{4}$$

with

$$u = \prod_{\langle ai \rangle \sigma \sigma'} \exp [\Phi_i (\varepsilon_{\alpha\sigma}^+ \theta_{\alpha\sigma} - \theta_{\alpha\sigma}^+ \varepsilon_{\alpha\sigma}) n_{i\sigma}^\pi] \tag{5}$$

and

$$\tan 2\Phi_i = 2d_i / [E_\theta + V_\theta - E_\varepsilon - V_\varepsilon]. \tag{6}$$

Neglecting the interactions between three or more sites in the same  $CuO_2$  plane we

notice that, excepting the terms with  $h$  and  $\lambda_{\perp}$  as coefficients, the mixing between  $\varepsilon$  and  $\theta$  orbitals is eliminated. The  $h$  and  $\lambda_{\perp}$  dependent terms are modified as

$$\sum_{\langle xi \rangle \sigma} h [1 + (1 - \cos \Phi_i) n_{\alpha\sigma}^{\theta}] (\varepsilon_{\alpha\sigma}^+ \theta_{\alpha\sigma} + \theta_{\alpha\sigma}^+ \varepsilon_{\alpha\sigma}), \quad (7)$$

and

$$\begin{aligned} \sum_{\substack{\langle xx \rangle_{\perp} \\ \sigma\sigma'\sigma''}} \lambda_{\perp} \left( \frac{h}{2\mu_{\perp} w_{\perp}} \right)^{1/2} (b_{\sigma\sigma'}^+ + b_{\sigma\sigma'}) n_{\alpha'\sigma'}^{\theta} \{ n_{\alpha\sigma}^{\varepsilon} n_{i\sigma'}^{\pi} \sin^2 \Phi_i \\ + n_{\alpha\sigma}^{\theta} \cos^2 \Phi_i - \frac{1}{2} \sin 2\Phi_i (\varepsilon_{\alpha\sigma}^+ \theta_{\alpha\sigma} + \theta_{\alpha\sigma}^+ \varepsilon_{\alpha\sigma}) n_{i\sigma'}^{\pi} \}. \end{aligned} \quad (8)$$

Neglecting the last term in the curly bracket in (8), dropping all terms in  $\bar{H}$  which involve only  $\theta$ -dependence, and using the Lange–Firsov transformation operator

$$S = \exp \left[ - \sum_{\substack{\langle \alpha\alpha' \rangle \\ \sigma \sigma' \sigma''}} \lambda_{\perp} \left( \frac{h}{2\mu_{\perp} w_{\perp}} \right)^{1/2} \frac{1}{hw_{\perp}} (b_{\alpha\alpha'}^+ - b_{\alpha\alpha'}) n_{\alpha'\sigma'}^{\theta} n_{i\sigma''}^{\pi} n_{\alpha\sigma}^{\varepsilon} \sin^2 \Phi_i \right] \quad (9)$$

it is seen that the zero phonon expectation value of  $(S^+ \bar{H} S)$  regains the form of the original hamiltonian (Chakraverty *et al* 1987) with renormalized functions  $E_{\varepsilon}$  and  $h$  given by

$$E'_{\varepsilon} = E_{\varepsilon} + \frac{\delta}{2} \{ E_{\theta} + V_{\theta} + V_{\varepsilon} - E_{\varepsilon} - [4d^2 + (E_{\theta} + V_{\theta} - E_{\varepsilon} - V_{\varepsilon})^2]^{1/2} \}, \quad (10)$$

$$h' = h [1 + a_1 \delta + a_2 \delta^2 + a_3 \delta^3], \quad (11)$$

$$\delta = (1 - 2x) = \langle n_{i\sigma}^{\pi} \rangle = \frac{1}{2 \sin^2 \Phi} \langle n_{\alpha\sigma}^{\theta} \rangle, \quad (12)$$

$$\begin{aligned} a_1 &= 2 \sin^2 \Phi (1 - \cos \Phi) \\ a_2 &= 2q_{-} \sin^2 \Phi (1 - 4 \sin^2 \Phi - 2 \sin^2 \Phi \cos \Phi), \\ a_3 &= 4q_{+} \sin^4 \Phi (4 - 2 \cos^2 \Phi) + 4q_{-} \sin^4 \Phi (1 - \cos \Phi), \end{aligned} \quad (13)$$

$$q_{-} = \exp \left( - \frac{2 \sin^4 \Phi}{2h\mu_{\perp} w_{\perp} 3} \right) - 1, \quad (14)$$

$$q_{+} = \exp \left( \frac{\lambda_{\perp}^2 \sin^4 \Phi}{2h\mu_{\perp} w_{\perp} 3} \right) - 1.$$

In writing (12) we have noted that the expectation value of a hole at any oxygen site in a  $\text{CuO}_2$  plane is  $\frac{1}{2}(1 - 2x)$ . The expectation value of  $n_{\alpha\sigma}^{\theta}$  is non-zero only if one ligand of  $\text{Cu}^{2+}$  ion at  $\alpha$  is in  $\text{O}^-$  form so that there is an orthorhombic mixing between  $|\varepsilon_{\alpha\sigma}\rangle$  and  $|\theta_{\alpha\sigma}\rangle$  orbitals resulting in a combined orbital of the form  $(\cos \Phi |\varepsilon_{\alpha\sigma}\rangle \pm \sin \Phi |\theta_{\alpha\sigma}\rangle)$  (Rai 1989). Since there are two oxygen sites per copper in a  $\text{CuO}_2$  unit  $\langle n_{\alpha\sigma}^{\theta} \rangle = \sin^2 \Phi (1 - 2x)$ .

Noting that the superconducting transition temperature in a peroxitonic model is proportional to  $h^2$  a relation between  $T_c$  and the doping parameter  $\delta$  can readily be written as

$$T_c = T_0 [1 + a_1 \delta + a_2 \delta^2 + a_3 \delta^3]^2. \quad (15)$$

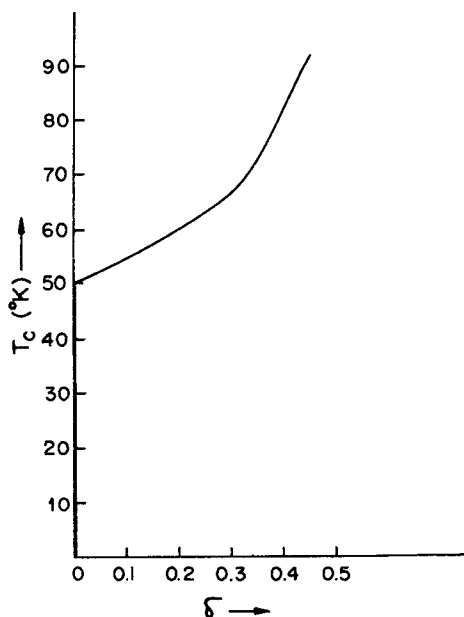


Figure 2. Variation of superconducting transition temperature  $T_c$  versus doping parameter  $\delta = 0.5 - x$  for  $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ .

### 3. Discussion

Figure 2 shows a plot of  $T_c$  versus  $\delta$  for the value of  $T_0 = 50^\circ\text{K}$ ,  $(\lambda_\perp / (2h\mu W_\perp^3))^{1/2} = 1.208$  and  $\Phi = 55.2^\circ$ . These parameter values are chosen to simulate the experimental  $T_c$  versus  $\delta$  curve (Goucharov *et al* 1988) as best as possible. Even though the curve near  $\delta \approx 0.2$  does not resemble the experimental  $60^\circ\text{K}$  plateau the slope in this region is much slower than that above  $\delta = 0.3$ . Above  $\delta = 0.3$  the present plot resembles the experimental curve (figure 2). The present value of  $\Phi = 55.2^\circ$  is close to the value of  $\Phi = 49^\circ$  obtained after a phase correction of the value required to explain the ESR result (Rai 1989). The present calculation clearly brings out the importance of oscillation of  $\text{O}_4$  bridge oxygen atoms in interlayer coupling. The importance of these oscillations is also emphasized in experimental infrared and Raman spectroscopic results (Goncharov *et al* 1988; Yonggang *et al* 1988).

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