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Upper critical field of a *D*-wave superconductor stabilized by antiferromagnetic fluctuations

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Abstract. We have derived microscopic equations for the upper critical magnetic field of a *D*-wave superconductor stabilized by antiferromagnetic spin fluctuations. We present numerical results for the reduced field $h_{c2}(t)$ as a function of reduced temperature t when the magnetic field is along the z axis and also zero-order results when it is along the x or y axis. Two different *D*-wave models are considered. The angular dependence of H_{c2} for T near T_c is given as a function of polar angle θ in the z–x or z–y plane.

Keywords. Upper critical field; *D*-wave superconductor; anisotropic superconductivity.

1. Introduction

Several authors have studied the possibility of *D*-wave superconductivity stabilized through antiferromagnetic spin fluctuations (Miyake 1987; Miyake *et al* 1984, 1986; Millis *et al* 1988; Williams and Carbotte 1989). The model could apply to the heavy fermion superconductors (Steglich 1985; Rauchschwalbe *et al* 1987) and possibly to other materials. Scharnberg and Klemm (1980, 1981, 1985) calculated the upper critical magnetic field $H_{c2}(T)$ for anisotropic superconductors with particular emphasis on the *P*-wave case which implies triplet spin pairing. Here we consider *D*-wave pairing with singlet spin.

2. Formalism

The interelectron interaction for scattering from $\mathbf{K}$ to $\mathbf{K}'$ on the Fermi surface is given in terms of the electron spin interaction I and the magnetic susceptibility $\chi(\mathbf{q}, \omega_n)$ with ω_n the n'th Matsubara frequency. Using a separable model, we write for this interaction

$$I^2\chi(\mathbf{K}_F - \mathbf{K}'_F; \omega_n - \omega_m) \equiv I^2\chi_0(\mathbf{K}_F - K'_F)\Phi(\omega_n - \omega_m), \tag{1}$$

with χ_0 the static susceptibility. The spectral representation for Φ is

$$\Phi(\omega_n - \omega_m) = \frac{2}{I^2 N(0)\pi}\int_0^\infty \mathrm{d}\omega \frac{\omega A_{\mathrm{SF}}(\omega)}{\omega^2 + (\omega_n - \omega_m)^2} \tag{2}$$

with $N(0)$ the electron density of states at the Fermi surface and $A_{\mathrm{SF}}(\omega)$ a spectral density for spin fluctuations.

A simple model for χ_0 is

$$I^2\chi_0(\mathbf{K}'_F - \mathbf{K}_F) = J_0 - J_1\eta_i(\hat{\mathbf{K}})\eta_i(\hat{\mathbf{K}}') \tag{3}$$

where

$$\eta_1(\hat{\mathbf{K}}) = \frac{\sqrt{15}}{2}(\hat{k}_x^2 = \hat{k}_y^2) \quad \text{and} \quad \eta_2(\hat{\mathbf{K}}) = \frac{\sqrt{5}}{2}(\hat{K}_x^2 + \hat{K}_y^2 - 2\hat{K}_z^2). \tag{4}$$

The ratio of J_1 to J_0 is denoted by g and the equation for the space (**R**) momentum (**K**) and energy (ω_n)-dependent gap $\phi(\mathbf{R}, \mathbf{K}, \omega_n)$ which we assume to be of the form $\phi(R)\Delta(\omega_n)\eta_i(\hat{\mathbf{K}})$ is

$$\phi(\mathbf{R})\Delta(\omega_n) = \frac{gT}{2v_F}\sum_m \int \frac{\mathrm{d}^3K}{K^2}\eta_i^2(\hat{\mathbf{K}})\exp[-(2|\omega_m|K)/v_F]$$

$$\exp[-i\,\mathrm{sgn}(\omega_m)\mathbf{K}\pi(R)] \times \lambda(\omega_n - \omega_m)\phi(\mathbf{R})\Delta(\omega_m). \tag{5}$$

Here T is the temperature and v_F the Fermi velocity. The operator $\pi(R) = 1/i\nabla_{\mathbf{R}} + 2eA(\mathbf{R})$ where e is the electron charge and $\mathbf{A}$ is the vector potential. Finally

$$\lambda(\omega_n - \omega_m) = \int_0^\infty \mathrm{d}\omega \frac{2\omega A_{\mathrm{SF}}(\omega)}{\omega^2 + (\omega_n - \omega_m)^2}. \tag{6}$$

To solve (5), it is necessary, in general, to expand $\phi(\mathbf{R})$ in a complete set of functions. We can use the generalized Abrikosov solutions $\phi_N(\mathbf{R})$ (Scharnberg and Klemm 1980, 1981, 1985) and write

$$\phi(\mathbf{R}) = \sum_{N=0}^{\infty} b_N \phi_N(\mathbf{R}).$$

3. Results

For the magnetic field H along the z-axis the lowest Abrikosov solution ϕ_0 is the complete solution for the $\eta_2(\hat{\mathbf{K}})$ model. Results for the reduced upper critical field $h_{c2}(t)$ as a function of reduced temperature are shown as the short dashed line in the upper

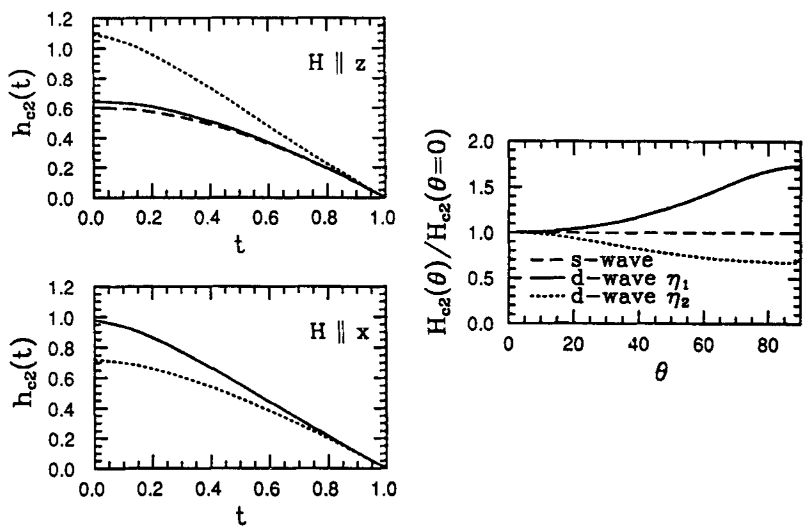

Figure 1. Reduced critical field $h_{c2}(t)$ as a function of reduced temperature t. The top left frame is for the magnetic field H parallel to the z axis. The short-dashed curve applies to the η_2 model and represents a complete solution. The solid and long dashed curve apply to the η_1 model with $N = 0$ (long dashed) and $N = 0, 4$ (solid) solutions respectively. The lower left frame holds for H parallel to the x or y axis and gives approximate results only with $N = 0$. The right frame gives $H_{c2}(\theta)/H_{c2}(\theta = 0)$ near T_c as a function of angle θ in the z–x or z–y plane with $\theta = 0$ to the z axis.

frame left of figure 1. Note the upward curvature near $t = 1$. The $t = 0$ value is much higher than for an S-wave superconductor ($h_{c2}(0) = 0{\cdot}73$) and the p-wave polar state ($h_{c2}(0) = 0{\cdot}85$). The case applies for $g = 1$ and $T_c/\omega_{SF} = 0{\cdot}05$ with ω_{SF} the spin fluctuation frequency in the spectral density $A_{SF}(\omega)$ which is taken to be a delta function. The corresponding value of $\lambda(0) = 0{\cdot}56$. To get a complete solution for the $\eta_1(\hat{\mathbf{K}})$ model, one needs $N = 0, 4, 8, 12\ldots$ etc. Results with $N = 0$ only are given as the long dashed line in the top left frame of figure 1. We see that the temperature dependence of $h_{c2}(t)$ is now completely different from that found in the $\eta_2(\hat{\mathbf{K}})$ model and is closer to the familiar S-wave dependence. Adding on the $N = 4$ function in the expansion for $\phi(\mathbf{R})$ gives the solid curve which is everywhere higher than the long dashed curve. The curves differ most at $t = 0$, although the differences are not very large, indicating that an expansion in N retaining only the two lowest terms is probably fairly accurate.

In the lower left frame of figure 1, we show results for $h_{c2}(t)$ vs t when H is parallel to the x or y axis. In this case, ϕ_N's with $N = 0, 2, 4, 6, 8\ldots$, all contribute to $h_{c2}(t)$. We present results only for $N = 0$. In contrast to the previous case ($H \| z$), the solid curve for $\eta_1(\hat{\mathbf{K}})$ is now higher than the short dashed curve for the $\eta_2(\hat{\mathbf{K}})$ model. Again, we note that $h_{c2}(0)$ is quite different from the S-wave case (Helfand and Werthamer 1966; Werthamer *et al* 1966).

Near $T = T_c$, we can work out an explicit and simplified equation for H_{c2} as a function of angle θ in the z–x or z–y plane. In this case, the Abrikosov solution with $N = 0$ is valid and H_{c2} is given by the equation

$$\Delta(\omega_n) = g\pi T \sum_m \lambda(n-m)\Delta(\omega_m) \times \left[\frac{1}{|\omega_m|} - \frac{3}{7}\frac{\alpha^*}{|\omega_m|^3}\left[\tfrac{1}{3}\sin^2\theta + \cos^2\theta\right]^{1/2}\right] \quad \text{for } \eta_1 \tag{7}$$

and a similar expression for η_2 with $\frac{3}{7} \to \frac{3}{21}$ and $\frac{1}{3} \to \frac{11}{5}$. Here $\alpha^* = eH_{c2}v_F^2/2$. Results for $H_{c2}(\theta)/H_{c2}(\theta = 0)$ are shown in the right frame of figure 1. For an S-wave superconductor (long dashed curve), there is no θ dependence, and the curve is flat at value 1. For the $\eta_1(\hat{\mathbf{K}})$ (solid curve) model, the field increases with θ; while for model $\eta_2(\hat{\mathbf{K}})$ (short-dashed curve), it decreases. This result is in accord with the results given in the two upper frames.

The deduced shapes of $h_{c2}(t)$ for $H \| x$ and T near T_c are slightly higher than the ones in the lower left frame of figure 1, indicating that the full results for $h_{c2}(t)$ will also be higher than the approximate results presented.

4. Conclusion

In conclusion, the upper critical field of a D-wave superconductor has been calculated for $H \| z$ and $\| x$ or y axis. The dependence on angle is very strong and the dependence on reduced temperature can be very different from the conventional S-wave case.

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