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New effects in high T_c superconductors

P BHATTACHARYYA
Theoretical Physics Group, Tata Institute of Fundamental Research, Homi Bhabha Road, Colaba, Bombay 400005, India

Abstract. We show that a proper interpretation of the critical field measurements can distinguish spin fluctuation mechanism from others in the context of the pairing hypothesis. We can predict the existence of a new mode in the mixed state of a superconductor.

Keywords. Spin fluctuation mechanism; critical field; midgap states.

1. Introduction

Within the context of the pairing mechanism claiming to explain the transition temperatures of the high T_c materials a number of mechanisms (Bedell and Pines 1988; Emery 1987) have been postulated. We are interested, primarily, in pairing mechanisms induced by spin fluctuations for the reason that a proper interpretation of critical field measurements can, we show, distinguish it unambiguously from the others. This mechanism is consistent with the form inferred from normal state properties (Varma *et al* 1989).

The idea is that if the effective interaction is dominantly mediated by spin fluctuations, the spectral density for the quanta of spin fluctuations is modified by the presence of a magnetic field. This causes the transition temperature $T_c(H)$ to be different from what it would be in the absence of any spin fluctuation interaction. Since this is the same as $H_{c2}(T)$, a proper calculation of the critical field in the strong coupling formalism will reveal the essential differences. This is reminiscent of the spin fluctuation stabilization (Anderson and Brinkman 1975) of the A-phase of the superfluid He^3.

2. Formalism

Electron-phonon interactions can be included in calculations of the upper critical field valid for the entire regime of temperature (Werthamer and McMillan 1967; Eilenberger and Ambegaokar 1967). Here we describe the modifications necessitated by the magnetic field dependence of the effective interaction and the layered structure characteristic of high T_c compounds extending the formulation (Schossmann and Schachinger 1986) which was most useful for the situation. Since we wish to highlight the differences arising from the effect of spin fluctuation-mediated interaction, we will only be interested in the situation where H is along the C-axis. The off-diagonal propagator is the solution of (Schossmann and Schachinger 1986)

$$F_i(rr', zz', u) = G_i(rr', zz', u)\Sigma_{o,i}(r'z', u)\, G_i^*(r'r, z'z, u),$$

where, in the local approximation $\Sigma_0(rr', u) = \Sigma_0(ru)\delta(r - r')$. Consequently,

$$F_i(rr', zz', \zeta_l) = T\Sigma_n \lambda(\zeta_l - \zeta_n)\hat{O}F_i(rz, \zeta_n) + (t_+\pi)\hat{O}F_i(rz, \zeta_l), \tag{1}$$

and the one-particle propagator

$$G_i(rr', zz', u) = (k_F/2\pi v_F)(I_{z-z'}(p)/|r-r'|)\delta_{zz'} \times \exp[i(k_F \mp eH/mv_F)\,\mathrm{sgn}(\tilde{\zeta}_l) + t_+ \pi\,\mathrm{sgn}(\tilde{\zeta}_l)] \quad (2)$$

with the integral operator

$$\hat{O}f = (k_F/2\pi v_F)^2 \sum_{k_z,k_z'} \int (\mathrm{d}z'\mathrm{d}^2s'/|s-s'|^2)\exp(-iJ|s-s'|/v_F) \times \exp i(k_z - k_z')(z-z')\exp[-(2/v_F)\tilde{\zeta}_l|s-s'| - 2ie\int_s^{s'} \mathrm{d}y\cdot A(y)]f(s'z', \zeta_l). \quad (3)$$

This is the generalization to a strong coupling theory. The rest of the calculation proceeds in the usual manner. The form of $\hat{O}$ allows the separation of variables

$$f(sz, \zeta_l) = \Delta(\zeta_l)f(sz) \quad (4)$$

and upper critical field determined by the leading eigenvalue

$$\chi(\tilde{\zeta}_l) = (2/\sqrt{\alpha})\int_0^\infty \mathrm{d}q\exp(-q^2)\tan^{-1}(q\sqrt{\alpha}/|\tilde{\zeta}_l|), \quad (5)$$

where $\alpha = eH_{c_2}v_F^2$. Consider the effect of including the spin fluctuation interaction. In the presence of a magnetic field, the effective interaction is of the form

$$\lambda(H) = (1 + \alpha/\alpha_c)\lambda_0.$$

Note that it is not possible to distinguish between spin singlet and $S = 1, S_z = 0$ pairing (Anderson and Brinkman 1975).

An algebraic solution for $H_{c2}(T)$ is possible when the parameter $\alpha \ll 1$ like the situation in Nb, for instance. This is possible in the 1 square well approximation so that

$$(T_c/T_{c0})(1 + (\alpha/\alpha_0)\beta_c^2(1 + \alpha/\alpha_c))(\omega_c/T_c)^{-\alpha/\alpha_c} = 1. \quad (6)$$

Generalization to many-band pairing and the effects of the Coulomb term μ^* can be incorporated by $\omega_c \to \omega_c/e$. Equation (6) is the main result of these calculations.

Let us make a few comments regarding the interaction responsible for superconductivity. We have sufficient evidence that it is a good approximation to replace the frequency-dependent coupling constant by a square well (Dynes 1974; Bhattacharyya and Jha 1978) with

$$\lambda_0 = (N(0)/2k_F^2)\sum_{\mathrm{layers}}\int_0^{2qF} \mathrm{d}qq\phi_s(q, z = la, \omega = 0) \propto [1 - \ln((1+\delta^2)/\delta^2)], \quad (7)$$

where $aq_F = \delta^2 r_s, r_s$ being a measure of the mean density in units of the Bohr radius. This is a prediction of the dependence of T_{c0} on parameters like the mean density and the interlayer spacing.

Let us make a few pertinent remarks regarding the solution. Note that the initial slope is

$$T_c(\alpha) = 1 - \alpha/\alpha_0$$

and does not depend upon the nature of the pairing mechanism. The low temperature critical field α_l determines the parameter α_c since it is a solution of

$$1 = (\alpha_l/\alpha_0)(1 + \alpha_0/\alpha_c).$$

Validity of the calculations can be checked by noting that, in the absence of the spin fluctuation term, these results for $T_c(\alpha)$ are completely consistent with the numerical results of Schossmann and Schachinger.

At intermediate temperatures, it is evident that $T_c(\alpha)$ will curve upwards though the extent will depend upon ω_c. This is in conformity with experiments (Orlando *et al* 1987). The effect of elastic scattering can be achieved by replacing α by $\tilde{\alpha} = \alpha - 3\alpha_0 t_+ \zeta(2)/4$. This results in the upward curvature being decreased and the effect of spin fluctuation interaction reduced. This is reasonable.

3. Mid gap state

Next we consider the effects of the interaction of ultrasound. We predict the existence of a new mode which exists in the mixed state of superconductors because of the coupling of the sound mode with the amplitude mode of the order parameter in the gauged Hamiltonian. In the Nambu notation, the symmetry (Littlewood and Varma 1982)

$$\begin{aligned} &\Psi(r) \to \exp(i\alpha_0(r)\tau_0)\Psi(r) \\ &\nabla \to \nabla + i\nabla\alpha_0(r)\tau_0 \end{aligned} \tag{8}$$

couples to amplitude fluctuations which the gauged Hamiltonian respects.

Because the vortex lattice induces a translational symmetry, pairing is between different branches of the excitation spectrum. In order to calculate the energy of the mode, we consider the distribution in local equilibrium. Galilean invariance in the two-fluid model requires a density response of the form

$$\begin{aligned} n^{\mathrm{l.e.}}(k\omega) = {} & u_{k\_}^2 f(E_{k\_,n}(\mu_n))\delta(\omega - E_{k\_,n}(\mu_s + v_s^2/2)) \\ & + v_{k\_}^2 f^+(E_{k\_,n}(\mu_n))\delta(\omega + E_{k\_,n}(\mu_s + v_s^2/2)). \end{aligned} \tag{9}$$

where $E_{k,n}^2 = \xi_{kn}^2 + \Delta^2, \xi_{kn}(\mu) = f((k + nK)^2) - \mu$ and $k_- = k - mv_s$. This means that the Cooper pairs are in equilibrium with the chemical potential $\mu_s + v_s^2/2$ while the excitations are in equilibrium with respect to a chemical potential μ_n. Relaxation to equilibrium takes place by the process of quasiparticle relaxation leading to branch relaxation. Recalling the similarity with spin relaxation in superfluid He^3, we see that local equilibrium is established when

$$\mu^{\mathrm{l.e.}} = \mu_s + v_s^2/2 = \mu_0, \tag{10}$$

where the chemical potential μ_0 is that in constant equilibrium. There is a mode corresponding to this energy. This is the mid gap state and is a direct consequence of the coexistence of diagonal and off-diagonal LRO.

4. Conclusions

We have shown that measurements of the critical field can distinguish unambiguously between the spin fluctuation interaction and the rest. Evidently the technological advantages of designing materials with controllable critical fields make it necessary to evolve a consensus on whether the upward curvature is an intrinsic effect or not. We have also proved the existence of a new mode in the mixed state of superconductors.

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