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Properties of combined phonon-high-energy boson mechanism

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Abstract. We consider the properties of a superconductor with a joint mechanism. One involves, otherwise unspecified, high-energy excitations and is modelled with a BCS pairing potential. The other is of low energy, presumably involving phonons. When the second contribution is significant, the resulting properties can differ greatly from BCS, and for most but not all properties, in a direction opposite to that expected for conventional strong coupling systems.

Keywords. High energy excitation plus phonon mechanism; thermodynamics; upper critical field; joint mechanism.

1. Introduction

We have calculated the superconducting properties of a superconductor with a joint mechanism (Marsiglio and Carbotte 1987a; Marsiglio *et al* 1987a; Marsiglio and Carbotte 1988; Carbotte 1988; Allender *et al* 1973). One is a high energy excitation with a characteristic frequency ω_c that is much larger than T_c. The other is a low-energy mechanism that is presumably phonon in nature. The high-energy mechanism is modelled by a BCS constant pairing interaction $N(0)V$, where $N(0)$ is the electronic density of states at the Fermi surface. This can be treated within the usual Eliashberg theory by introducing an effective attractive Coulomb pseudopotential combining the repulsive μ^* with the attractive $N(0)V$: $\mu^*_{\text{eff}} = -N(0)V + \mu^*$. If $N(0)V$ is large enough, the repulsive μ^* is overcome and μ^*_{eff} is negative. If this is the case, we can characterize this part of the interaction by introducing a critical temperature $T_{c0} = 1{\cdot}13\,\omega_c \exp(1/\mu_{\text{eff}})$, where the subscript zero indicates that this critical temperature applies to the high-energy mechanism by itself. The low-energy interaction is treated exactly within Eliashberg theory (Daams and Carbotte 1981; Rainer and Bergmann 1974; Allen and Mitrovic 1982) by including it through an $\alpha^2F(\omega)$ (McMillan and Rowell 1982) spectral function. For simplicity, we have chosen an $\alpha^2F(\omega)$ of the form $\alpha^2F(\omega) = A\delta(\omega - \omega_E)$, where ω_E is the Einstein frequency and A is a parameter that is adjusted to get different values of T_c.

2. Formalism

The equations for the Matsubara gaps $\tilde{\Delta}(i\omega_n)$ and renormalized frequencies $\tilde{\omega}(i\omega_n)$ with $\omega_n \equiv \pi T(2n-1)\, n = 0, \pm 1, \pm 2, \ldots$ are (Daams and Carbotte 1981; Rainer and Bergmann 1974; Allen and Mitrovic 1982)

$$\tilde{\Delta}(i\omega_n) = \pi T \sum_m (\lambda(m-n) - \mu^*_{\text{eff}}) \frac{\tilde{\Delta}(i\omega_m)}{\tilde{\Delta}^2(i\omega_m) + \tilde{\omega}^2(i\omega_m)}, \tag{1}$$

$$\tilde{\omega}(i\omega_n) = \omega_n + \pi T \sum_m \lambda(m-n) \frac{\tilde{\omega}(i\omega_m)}{\tilde{\Delta}^2(i\omega_m) + \tilde{\omega}^2(i\omega_m)}. \tag{2}$$

The free-energy difference between normal and superconducting state is given by

$$\Delta F = 2\pi N(0) T \sum_{n>0} \left[2\left([\tilde{\Delta}^2(i\omega_n) + \tilde{\omega}^2(i\omega_n)]^{1/2} - \tilde{\omega}(i\omega_n) - \frac{1}{2}\frac{\tilde{\Delta}^2(i\omega_n)}{[\tilde{\Delta}^2(i\omega_n) + \tilde{\omega}^2(i\omega_n)]^{1/2}} \right) - [\tilde{\omega}(i\omega_n) - \tilde{\omega}^0(i\omega_n)] \left(\frac{\tilde{\omega}^2(i\omega_n)}{[\tilde{\Delta}^2(i\omega_n) + \tilde{\omega}^2(i\omega_n)]^{1/2}} - 1 \right) \right]. \tag{3}$$

where $\tilde{\omega}^0(i\omega_n)$ is given by equation (2) with the gap on the right hand side ($\tilde{\Delta}(i\omega_n)$) set equal to zero. In the above equations, T is the temperature and

$$\lambda(m-n) \equiv \int \frac{2\omega\alpha^2 F(\omega)\mathrm{d}\omega}{\omega^2 + (\omega_n - \omega_m)^2}. \tag{4}$$

The upper critical field $H_{c2}(T)$ follows in the clean limit from the equation (Schossmann and Schachinger 1986; Marsiglio *et al* 1987b)

$$\tilde{\Delta}(i\omega_n) = \pi T \sum_m (\lambda(n-m) - \mu^*_{\rm eff})\tilde{\Delta}(i\omega_m)\chi(i\omega_m), \tag{5}$$

where

$$\tilde{\omega}(i\omega_n) = \omega_n + \pi T \sum_m \lambda(n-m)\,\mathrm{sgn}\,(\omega_m) \tag{6}$$

and

$$\chi(\tilde{\omega}(i\omega_n)) = \frac{2}{\sqrt{\alpha}} \int_0^\infty \mathrm{d}q\, e^{-q^2} \tan^{-1}\left\{ \frac{\sqrt{\alpha}q}{|\tilde{\omega}(i\omega_n)|} \right\} \tag{7}$$

with

$$\alpha = \tfrac{1}{2} e H_{c2}(T) v_F^2, \tag{8}$$

where e is the electron charge and v_F the Fermi velocity.

3. Results

In the uppermost frame of figure 1, we show results for the total isotope effect $\beta = -\mathrm{d}\ln T_c/\mathrm{d}\ln M$, where M is the atomic mass. We have plotted β as a function of T_c/ω_E for various values of T_{c0}/ω_E. Note that each curve starts at the specified value of T_{c0}/ω_E and these starting points correspond to cases where the superconductivity is due exclusively to the high-energy mechanism. We then turn on the phonon contribution so that $T_c > T_{c0}$. These curves are all universal and apply for any T_c and phonon energy ω_E. We see from the figure that β rises rapidly from zero as T_c increases above its initial value T_{c0} and so a modest phonon contribution to T_c can lead to an isotope effect which differs considerably from zero. For larger values of T_{c0}/ω_E, however, β climbs less rapidly.

In the second and third frames, we show results for the normalized specific heat jump at T_c, $\Delta C/\gamma T_c$, and the strong coupling correction to the upper critical field at $T = 0$,

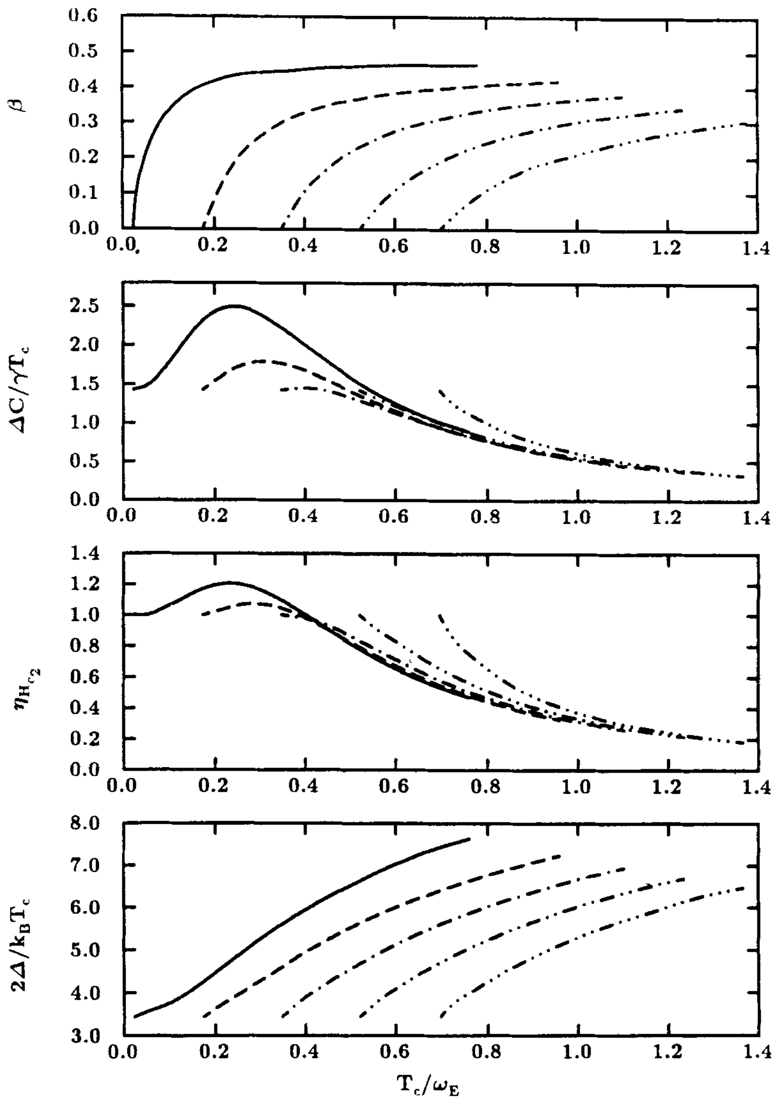

Figure 1. Isotope effect β, the normalized specific heat jump $\Delta C(T_c)/\gamma T_c$, the correction factor $\eta_{H_{c2}}$ for the upper critical field H_{c2} and the gap to critical temperature ratio $2\Delta/k_B T_c$ as a function of T_c/ω_E for various values of T_{c0}/ω_E, namely, 0·022 (solid), 0·173 (long dash), 0·348 (dash dot), 0·521 (dash double dot), and 0·696 (dash triple dot). Here T_{c0} is the critical temperature d:e to the high-energy mechanism alone, and T_c includes the low-energy contribution.

$\eta_{H_{c2}}$, respectively. The latter quantity is defined by the equation $H_{c2}(0) = \eta_{H_{c2}} H_{c2}^{\mathrm{BCS}}(0)$. For both quantities, the curves for small values of T_{c0}/ω_E begin at their BCS values, 1·43 and 1·0, respectively. They then increase as T_c/ω_E is increased, eventually reaching maxima before turning over and falling well below their BCS values. This behaviour is similar to that exhibited by these quantities for the pure strong coupling case without any contributions from a high-energy mechanism. (Marsiglio *et al* 1987c; Marsiglio and Carbotte 1987b). For the larger values of T_{c0}/ω_E, the maxima disappear and the drop below BCS occurs immediately. This behaviour is characteristic of a joint mechanism.

In the bottom frame, we plot the ration $2\Delta/k_B T_c$ vs T_c/ω_E, where Δ is the zero-temperature energy gap. Unlike $\Delta C/\gamma T_c$ and $\eta_{H_{c2}}$, this quantity always remains larger than the BCS value $2\Delta/k_B T_c = 3{\cdot}54$. There is no qualitative change in behaviour of the curves going from a small T_{c0}/ω_E to a large one and so one cannot really see any signs of a joint mechanism by looking at $2\Delta/k_B T_c$ by itself.

4. Conclusion

In conclusion, for a joint high and low-energy mechanism, the low energy contribution can significantly offset some superconducting properties in a characteristic way which is distinct from the usual strong coupling regime.

References

Allender D, Bray J and Bardeen J 1973 *Phys. Rev.* **B7** 1020
Allen P B and Mitrovic B 1982 in *Solid state physics* (eds) H Ehrenreich, P Seitz and D Turnbull (New York: Academic Press) p. 4
Carbotte J P 1988 in *CAP-NSERC Summer Institute on Theoretical Physics* (eds) F C Khanna, H Umezawa, G Kunstatten and H C Lee (Singapore: World Scientific) p. 28
Daams J M and Carbotte J P 1981 *J. Low Temp. Phys.* **43** 263
Marsiglio F and Carbotte J P 1987a *Phys. Rev.* **B36** 3633
Marsiglio F and Carbotte J P 1987b *Phys. Rev.* **B36** 3937
Marsiglio F and Carbotte J P 1988 *Rev. Solid State Sci.* **1** 423
Marsiglio F, Akis R and Carbotte J P 1987a *Solid State Commun.* **64** 905
Marsiglio F, Schossmann M, Schachinger E and Carbotte J P 1987b *Phys. Rev.* **B35** 3226
Marsiglio F, Akis R and Carbotte J P 1987c *Phys. Rev.* **B36** 5245
McMillan W L and Rowell J M 1969 in *Superconductivity* (ed.) R D Parks (New York: Marcel Dekker) Vol. 1 p. 561
Rainer D and Bergmann G 1974 *J. Low Temp. Phys.* **14** 501
Schossmann M and Schachinger E 1986 *Phys. Rev.* **B33** 6123