Bull. Mater. Sci., Vol. 14, No. 4, August 1991, pp. 977–981. © Printed in India.

Some properties of vortices in layered superconducting structures

A F VOLKOV
Institute of Radioengineering and Electronics, Academy of Sciences of the USSR, GSP-3, Moscow 103907, USSR.

Abstract. Some properties of vortices in layered superconductors and superconducting structures with the Josephson interaction between S-layers are studied theoretically. The Josephson vortex lattice is formed in the system if the magnetic field H_e is parallel to the layers. Long-wave oscillations of this lattice have a sound-like spectrum; the velocity of "magnetic sound" propagating across the layers is small and diminishes with increase of H_e. Using an analogy with electron-hole plasma in semiconductors, we investigate also the behaviour of the layered structure in the field H_e perpendicular to the layers at temperatures close to the Kosterlitz–Thouless transition temperature T_{KT}. The spatial dependence of $H(r)$ and the impedance of the system are determined at $T > T_{\mathrm{KT}}$.

Keywords. Josephson vortice lattice; layered superconductors; Kosterlitz–Thouless transition.

1. Introduction

An increase of interest in the study of layered superconductors is related mainly to the fact that some of the high T_c superconductors are layered compounds. Besides, $\{S,N\}$ or $\{S,I\}$ structures may appear in superconductors with a regular array of the twinning planes if the order parameter is suppressed at these planes (Blazey *et al* 1988) (here, $\{S,N\}$ and $\{S,I\}$ denote a periodical system of superconducting (S), normal (N), and insulating (I) layers). The $\{S,N\}$ or $\{S,I\}$ systems can also be made artificially (Ruggeiro *et al* 1982).

Here, we study the $\{S,I\}$ system, but some of the results are valid also for the $\{S,N\}$ system. The $\{S,I\}$ system with the Josephson interaction between S-layers in a field H_e parallel to the layers is considered in the next section. We will analyse some properties of a lattice formed by the Josephson vortices (J-vortices). In §3, we study some properties of the 2-d vortices in the field perpendicular to the layers at temperatures close to the Kosterlitz–Thouless transition temperature T_{KT}.

2. Josephson vortex lattice

Consider the $\{S,I\}$ system in an applied magnetic field H_e parallel to the layers. If H_e exceeds a threshold value H_{J1}, the Josephson vortices penetrate into the system. At $H_e \gg H_{J1}$ a dense lattice of the J-vortices (J-lattice) is formed in the system. The deformation of the J-lattice is described by an equation (Volkov 1989a, b)

$$\lambda_J^2 \sum_m \vartheta_m'' \exp[-a|n-m|] = -l_H^{-2}[\sin(\vartheta_{n+1}-\vartheta_n)+\sin(\vartheta_{n-1}-\vartheta_n)] + \tau\dot{\vartheta}_n + \omega_0^{-2}\ddot{\vartheta}_n, \tag{1}$$

where $\vartheta_n = \phi_n - \phi_n^{(0)}$ is a deviation of the phase difference ϕ_n at the nth I-layer from an equilibrium value $\phi_n^{(0)}, \theta'' = \partial^2\theta/\partial x^2$, the x-axis is lying in the plane of layers, λ_J is the Josephson penetration depth, $l_H = 2(\lambda_J/\lambda)\mathbf{H}[\sinh(a\rho(\mathbf{H}))/\rho(\mathbf{H})]^{1/2}, \rho(\mathbf{H}) = (1+\mathbf{H}^2)^{1/2}$, $\dot{\theta} = \partial\theta/\partial t$. A constant $\mathbf{H}$ is related to a magnetic induction B_J caused by the J-lattice

$$H_0\mathbf{H} = B_J a = 2\tanh(a/2)H_e(1-f), f = (1/2)[2\lambda_J H_e \tanh(a/2)/\lambda H_0]^{-4} \ll 1, \tag{2}$$

where $H_0 = \Phi_0/(2\pi\lambda^2)$, a is the thickness of the S-layer. The J-lattice deformations induced, for example, by the Abrikosov vortices penetrating at high enough H_e into the S-layers of a finite thickness lead to a mutual attraction of the Abrikosov vortices and to a change of the sample magnetization (Volkov 1989c). Equation (1) describes, in particular, weakly damped (at low temperatures) oscillations of the J-lattice (Volkov 1989a). In the long-wave limit these oscillations have a sound-like spectrum. The velocity of "magnetic sound" propagating along the layers equals $v = \lambda_J\omega_0/\tanh(a/2\lambda)$, where ω_0 is the Josephson plasma frequency. The velocity of sound propagating across the layers, $v_\perp = \omega_0 a/l_H$, is rather low and diminish when H_e increases. The last mode may be excited, for instance, by an ac current flowing along the layers with frequency $\omega < v/L_x$. Then, the microwave absorption as a function of H_e must have sharp peaks at low temperatures when the condition $\omega = 2\pi v_\perp n/L_z$ is fulfilled (Volkov 1989b).

At $H < 1$ the distance between adjacent peaks is nearly constant and equals $\delta H_e/H_0 \cong (\pi/2)(\omega_0/\omega)(a/\lambda)(\sinh(a/\lambda))^{1/2}\,[(\lambda_J L_z/\lambda^2)\tanh(a/2\lambda)]^{-1}$. The peaks observed in microwave absorption in YBCO single crystals (Blazey *et al* 1988; Bugai *et al* 1988) seem to be related to these resonances.

3. Dynamics of the two-dimensional vortices

Experiments performed recently on single crystals of high T_c superconductors, revealed that Kosterlitz–Thouless transition takes place at a temperature T_{KT} slightly lower than the BCS transition temperature T_{c0} (Dubson *et al* 1988; Stamp *et al* 1988; Martin *et al* 1989; Artemenko *et al* 1989). It was assumed that at $T \geqslant T_{KT}$ free vortices appear in Cu–O layers as a result of breaking of bound vortex-antivortex pairs. The last ones are formed in a 2d system due to thermal excitations (see, for example, Minnhagen 1987). In the presence of transport current the motion of free vortices caused by the current leads to the energy dissipation and to the appearance of voltage proportional at $T \geqslant T_{KT}$ to the current: $V = R_v I$. It is interesting to study the screening effects near T_{KT}. In this section, using phenomenological approach, we investigate the penetration of an ac and dc field H_e into layered superconducting structures at temperatures $T_{KT} < T < T_{c0}$ when free vortices play the main role.

We use the well-known analogy between 2-d vortices and 2-d Coulomb gas. Consider a layered superconductor (or a $\{S, I\}$ system) and neglect the weak Josephson interaction between S-layers. The mean field approximation will be used provided that there are many free vortices over the screening length λ_*. The continuity equation for a real density of free vortices N_n and antivortices N_p has the form

$$\partial N_{n,p}/\partial t = -\partial_\perp j_{n,p} + G - \beta N_n N_p. \tag{3}$$

Here $j_{n,p} = v_{n,p}N_{n,p}$ is the vortex (antivortex) flow density, a term G determines the vortex generation rate and depends on $N_{n,p}$ and on the current density j; however, at $T > T_{KT}$ this dependence is not essential and will be neglected. The last term in (3) describes the recombination rate caused by mutual vortex-antivortex capture. The Lorentz force $(\mathbf{j}^*\mathbf{e}_z)\Phi_0/c$ acting on vortices leads to their viscous motion; hence, for $\mathbf{j}_{n,p}$ we get

$$\mathbf{j}_{n,p} = \pm \alpha N_{n,p}(e_z{}^*\mathrm{curl}\,\mathbf{H}) - D\partial_\perp N_{n,p}, \tag{4}$$

where $\partial_\perp = (\partial_x, \partial_y, 0)$, the "mobility" α is related to D by a relationship $(\alpha/D) = \Phi_0 a/(4\pi T)$. At last, we need the equation for H. It can be obtained from an expression for the current density j averaged over an area containing many vortices: $\mathbf{j} = -(4\pi/c\tilde{\lambda}^2)\mathbf{A}_c + \sigma_q\mathbf{E}$, where $\mathbf{A}_c = \langle \mathbf{A} - \mathbf{A}_v \rangle$, $\mathbf{A}_v$ is a part of vector potential describing the vortices, σ_q is the quasiparticle conductivity. The length $\tilde{\lambda}$ differs from the usual London penetration λ depth because at T_{KT} the condensate density may alter. Up to now, there is no full theory to Kosterlitz–Thouless transition in layered superconductors (see, however, Artemenko and Kruglov 1990), and we consider $\tilde{\lambda}$ as a phenomenological length vanishing at T_{c0}. Using the expression for $\mathbf{j}$, we get for the $\mathbf{H}$-component normal to the layers

$$\tilde{\lambda}^2\partial_\perp^2 H - [1 + (4\pi\tilde{\lambda}^2\sigma_q/c^2)\partial/\partial t]H = \Phi_0 N_-, \tag{5}$$

where $N_- = N_n - N_p$. Equations (3)–(5) should be supplemented by boundary conditions. One of them is the condition for H

$$[H(r_b) - H_b] = 0. \tag{6}$$

Another one is the condition for $N_{n,p}$ and $j_{n,p}$

$$[j_{n,p}(r_b) \pm s(N_b - N_{n,p})(r_b)] = 0. \tag{7}$$

Here H_b and N_b are the magnetic field and the vortex density at the boundary, s depends on the recombination rate at the boundary and on velocity with which vortices escape the sample. We will consider them as phenomenological parameters.

Equations (3)–(5) together with the conditions (6)–(7) describe a layered superconductor near T_{KT}. In equilibrium and in the absence of an external field H_e, the distribution of the vortex density in space will be uniform if $N_b = N_0 \equiv (G/\beta)^{1/2}$. In the 2-d case G depends on T exponentially: $N_n = N_p = \xi_{GL}^{-2}\exp[-C(T/T_{KT} - 1)^{-1/2}]$ (Minnhagen 1987). If $N_b \neq N_0$ the vortex density varies in space over the characteristic scale $l_r = (D\tau_r)^{1/2}$, $\tau_r = (2\beta N_0)^{-1}$ is the recombination time. Consider now the penetration of a constant field H_e into a layered superconductor having the form of an infinite slab $|x| < L$.

In this case, the boundary condition for H is $H(\pm L) = H_e$. Suppose that H_e is small and $N_b = N_0$. Linearizing equations (3)–(5) in a stationary state and performing necessary calculations, we get for deviations $\delta N_n \equiv N_n - N_0 = -\delta N_p = (b/2)(H - H_e)/\Phi_0$ and for H.

$$H(x) = (H_e/(1 + b))[b + \cosh(x/\lambda_*)/\cosh(L/\lambda_*)], \tag{8}$$

where $\lambda_* = \tilde{\lambda}(1 + b)^{-1/2}$ is a renormalized screening length, $b = 2\alpha N_0\Phi_0/D$ describes a contribution of free vortices to the screening. One can see that in the case of a thick slab $(L \gg \lambda_*)$, $H(0)/H_e = b/(1 + b)$, i.e. this ratio is close to unity because for typical values of parameters $b > 1$ (for example, for BSCCO). It means that a vanishingly weak field H_e will penetrate into a sample at temperatures $T_{KT} < T < T_{c0}$. In the case of alternating current $I_{tr}(t)$ and $H_e = 0$, the boundary condition for H is

$H(\pm L) = \pm H_I(t)$, where $H_I(t) = (2\pi/c) I_{tr}(t) \equiv \tilde{H}\, \mathrm{Im}\exp(i\omega t)$ (I_{tr} is the transport current per unit length). We solve again (3)–(5) assuming that the current is small, $N_b = N_0$, and frequencies are not too high: $\omega[(b\lambda_*^2/D) + 4\pi\tilde{\lambda}^2\sigma_q/c^2] \ll (1+b)$. Then, for the field we have

$$H(x,t) = \tilde{H}(1+b)^{-1}\ \mathrm{Im}\ \{\exp(i\omega t)[(s_1 + c_1(1+b)\varkappa l_s)\sinh(x/\lambda_*)/\sinh(L/\lambda_*) + b\ \sinh(\varkappa x)]\ [s_1 + \varkappa l_s c_1]^{-1}\} \tag{9}$$

Here $s_1 = \sinh(\varkappa L)$, $c_1 = \cosh(\varkappa L)$, $\varkappa^2 = i\omega/(D(1+b))$, $l_s = D/s$. Equation (9) determines the spatial distribution of the field H. From (9) we can find the impedance of the system $Z(\omega)$. In limiting cases one has for real part of Z: (a) $\omega \ll \omega_1 \equiv 2D(1+b)/L^2$, $\mathrm{Re}Z \cong R_v$; (b) $\omega \gg \omega_1, \mathrm{Re}Z \cong R_v\omega(L+l_s)\ (1+2\varkappa_0 l_s)\ \{2D(1+b)\varkappa_0[(1+\varkappa_0 l_s)^2 + (\varkappa_0 l_s)^2]\}^{-1}$. Here $\varkappa_0 = |\varkappa|/\sqrt{2}$, $R_v = 4\pi b\ D/(L+l_s)c^2$ is the dc resistance caused by viscous flux flow. Hence, the dispersion of ReZ appears at frequencies ω_1 and $\omega_2 = 2D(1+b)/l_s^2$. With the increase of ω ReZ increases and approaches to a constant $R_v\ (1 + L/l_s)$ at $\omega \gg \omega_2$. If one takes into account the contribution of bound pairs to the dissipation, additional dispersion may arise at frequencies satisfying the condition $r_\omega \equiv (D/\omega)^{1/2} = \xi_+(T_\omega)$ (Ambegaokar *et al* 1978).

5. Conclusion

In this report we have analysed some properties of vortices in layered superconductors. The J-vortex lattice appears in the system if an applied magnetic field is parallel to the layers. The most interesting phenomenon which can be observed in the J-lattice is the sound-like oscillations of this lattice. This "magnetic sound" can be excited by an ac external field $H_I(t)$ and can propagate through the system with weak damping if we deal with the $\{S, I\}$ system at low temperatures.

Vortices with a magnetic moment perpendicular to the layers have the most interesting properties near the Kosterlitz–Thouless transition temperature. The phenomenological theory presented above allows one to determine, for example, characteristics of the 2-d vortices from the spectral measurements of impedance.

Acknowledgement

The author is grateful to Dr S N Artemenko for useful discussions.

References

Ambegaokar V *et al* 1978 *Phys. Rev. Lett.* **40** 783
Artemenko S N and Kruglov A N 1990 *Phys. Lett. A* (in press)
Artemenko S N *et al* 1989 *Pis'ma Zh. Eksp. Teor. Fiz.* **49** 352
Artemenko S N *et al* 1989 *Phys. Lett.* **A138** 428
Blazey K M *et al* 1988 *Physica* **C153–155** 56
Blazey K M *et al* 1988 *Europhys. Lett.* **6** 457
Bugai A A *et al* 1988 *Pis'ma Zh. Eksp. Teor. Fiz.* **48** 209
Dubson M A *et al* 1988 *Phys. Rev. Lett.* **60** 1061
Martin S *et al* 1989 *Phys. Rev. Lett.* **62** 677

Minnhagen P 1987 *Rev. Mod. Phys.* **59** 1001
Ruggeiro S T, Barbee T M and Beasley M R 1982 *Phys. Rev.* **26** 4894
Stamp P *et al* 1988 *Phys. Rev.* **B38** 2847
Volkov A F 1989 *Phys. Lett.* **A138** 213
Volkov A F 1989 *Pis'ma v Zh. Eksp. Teor. Fiz.* **50** 127 (in Russian)
Volkov A F 1989 *Sverkhprovodimost* 2.n.12.17 (in Russian)