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Some key properties of low-dimensional electron gas in semiconductors

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Abstract. Conditions for the formation of low-dimensional electron gas in semiconductors and different structures supporting it are discussed. Some of the new devices in which carriers have low-dimensional motion are introduced. The properties of electrons needed to be studied for the optimisation of the device performance are mentioned. In particular, the charge control and mobility of electrons in high electron mobility transistors, gain and loss processes in quantum well lasers, and excitonic line width in multiple quantum wells are discussed.

Keywords. Low-dimensional electron gas; semiconductors; engineered structures; electron and optical devices.

1. Introduction

In the presence of a one-dimensional potential well of dimension comparable to the de Broglie wavelength of electrons across the width (z-direction), the motion of the electrons becomes quantised. The electrons are however free to move along the other two directions (x and y) and a two-dimensional electron gas (2DEG) is formed (Ando *et al* 1982; Nag and Basu 1982; Esaki 1985; Basu 1986). The electrons are then confined in different sub-bands and their dispersion relation is given by

$$E(\mathbf{k}) = E_i + \frac{\hbar^2 k^2}{2m^*}, \quad (\mathbf{k} = k_x, k_y), \tag{1}$$

where E_i is the energy of the ith sub-band and m^* the (isotropic) effective mass. The E–$\mathbf{k}$ relationship is shown in figure 1.

Similarly if the narrow well extends over two directions one gets a one-dimensional electron gas (1DEG) (obtained in a quantum wire system). When the well is sufficiently narrow in all the three directions, a zero DEG (0DEG; in a quantum dot or box system) results.

The idea of obtaining quantisation in the surface inversion layer of a semiconductor was put forward by Schrieffer (1957). However, it was possible to realise 2DEG, only after planar technology became mature. The first observation of the 2D effect was made by Fowler *et al* (1966) in silicon metal oxide semiconductor field effect transistor (MOSFET). The 2DEG in GaAs and other III–V and II–VI compounds and alloys was found in superlattices (SL), quantum wells (QW) and heterojunctions (HJ), a few years later (Esaki 1985; Basu 1986). Recently 1DEG and 0DEG have also been realised.

The low-dimensional electron gas exhibits many novel physical phenomena like quantum and fractional quantum Hall effects, localisation and many body effects, mobility enhancement, excitonic effects persisting even at room temperature and resonant tunnelling phenomena. Several new devices have emerged which are likely to be used in future electronic and optoelectronic systems. The high electron mobility transistors (HEMT) show higher speed and lower power consumption than existing metal semiconductor field effect transistor (MESFET) (Morkoc 1985).

The QW lasers offer lower threshold current density and slower rise of the threshold current density with increase in temperature (Tsang 1985). The saturation of excitonic absorption with low intensity optical signals at room temperature are exploited to produce optical switches, modulators and detectors (Chemla *et al* 1984). The new resonant tunnelling diodes and transistors offer promise of very high frequency amplifiers and oscillators (Capasso *et al* 1988).

The operation of all the above mentioned devices is controlled by the motion of confined carriers. For better understanding of the operation and also for optimisation of the device performance, electronic properties must be studied thoroughly. In the present paper, we shall mention some of the key device parameters needing optimisation and point out some of the key electronic properties that merit detailed investigation for this purpose.

The organisation of the paper is as follows. The different structures used for the realisation of 2DEG are described in § 2. The charge control and mobility behaviour of 2DEG and 1DEG in HEMT are discussed in § 3. Some features of QW lasers, and the hot-electron behaviour in these devices are pointed out in § 4. The excitonic behaviour in multiple QW and some applications of the study of excitonic effects are mentioned in § 5. Section 6 gives the summary and conclusions.

2. Different structures supporting 2DEG

The band diagram of an HJ made of two perfectly lattice-matched semiconductors, GaAs and $Al_xGa_{1-x}As$, is shown in figure 2a. The AlGaAs layer is modulation-

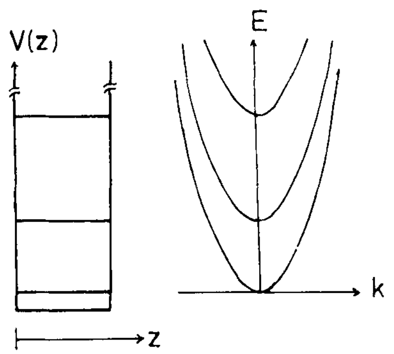

Figure 1. Quantisation of an electron in a narrow potential well, and formation of sub-bands. The *E–k* relation is shown.

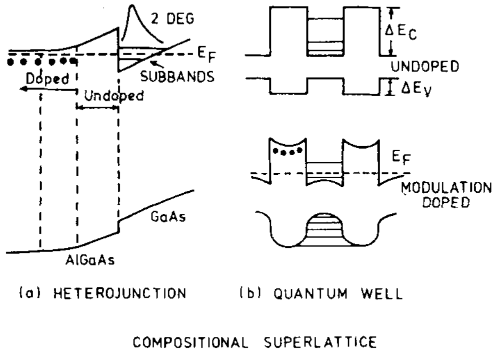

Figure 2. Band diagrams in (**a**) a heterojunction and (**b**) a quantum well.

doped, such that an undoped spacer layer of AlGaAs exists between unintentionally doped GaAs and donor-doped AlGaAs. The electrons given up by donor atoms come to the GaAs side in which they are confined. A QW made by sandwiching a GaAs layer between two AlGaAs layers and the different sub-bands are shown in figure 2b. Note that the difference in band gap $\triangle E_g$ between the two materials appears as steps $\triangle E_c$ and $\triangle E_v$ in the conduction and valence bands respectively at the heterointerface. $\triangle E_c$ corresponds to the difference of electron affinities of two materials and $\triangle E_g = \triangle E_c + \triangle E_v$.

A periodic repetition of QW makes a multiple QW (MQW: wavefunction in each QW is independent) or a superlattice (SL; wavefunctions of the QW overlap). HJ and QW are the modules of a kind of SL known as compositional SL. There exists another kind: the doping SL (figure 3). It may be grown by growing alternate layers of *p*- and *n*-types of a semiconductor (NIPI) (figure 3a) or by employing alternate monolayers of donors and acceptors (figure 3b). The latter is known as a δ-doped SL in which a V-shaped potential well confines the motion of electrons and holes.

In all the above structures, the narrow potential well is one-dimensional in nature and the resulting electron gas is two-dimensional. Structures supporting 1DEG will be mentioned in § 3.

The density-of-states (DOS) function for electrons in bulk and low-dimensional systems are plotted in figure 4.

3. HEMT: charge control and mobility

The structure of an HEMT is shown in figure 5. An undoped AlGaAs layer is grown between the GaAs substrate and *n*-type AlGaAs layer. The n^+ GaAs layer is grown to facilitate contact-making. The source and drain regions are grown by

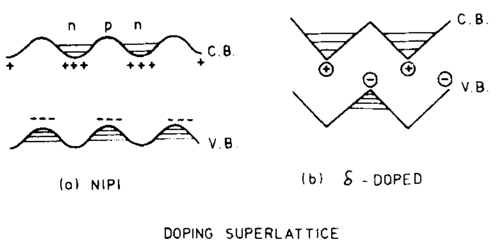

Figure 3. Band diagrams in (**a**) an NIPI and (**b**) a δ-doped structure.

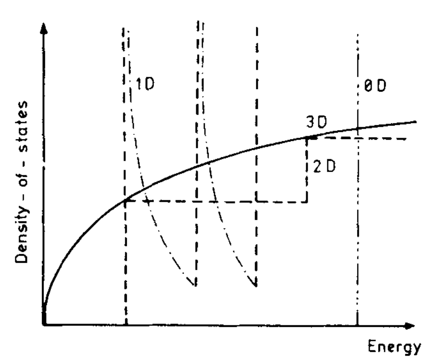

Figure 4. Density-of-states function for 3D, 2D, 1D and 0DEG.

alloying and metallisation. After etching away the cap n^+ GaAs layer and a part of AlCaAs, gate metallisation is performed (Morkoc 1985). As shown in the figure the 2DEG is formed at the interface and is pinched off near the drain.

The transconductance of the field effect transistor (FET) is an important parameter that needs optimisation. It depends on the 2D electron density (n_{2D}) and on the mobility of 2DEG. Therefore, for obtaining the values of g_m or for modelling the device, a knowledge of the variation of n_{2D} with gate voltage V_g is necessary. The band diagram for a positive gate voltage applied to the Schottky gate, is as shown in figure 6. The potentials are usually obtained by solving the Poisson equation and matching the fields and potentials at the heterointerface. Since the electrons are confined in the present case, quantum mechanical considerations should be brought in. If $\chi_i(z)$ is the wavefunction of electrons in the ith sub-band at a distance z from the interface, where $V(z)$ is the potential, then $\chi_i(z)$ and $V(z)$ must be obtained from the Schrödinger and Poisson equations expressed as

$$\frac{\hbar^2 d^2\chi_i(z)}{2m^* \mathrm{d}z^2} - [E_i - V(z)]\chi_i(z) = 0, \tag{2a}$$

$$\frac{\mathrm{d}^2 V(z)}{\mathrm{d}z^2} = \frac{e}{\varepsilon}\left\{\sum_i N_i |\chi_i(z)|^2 + N_{\mathrm{depl}}\right\}. \tag{2b}$$

Here, N_i is the carrier density in the ith sub-band having energy E_i, ε the permittivity of GaAs and N_{depl} the total depletion layer charge. The above two equations should be solved self-consistently by numerical iteration techniques. For HEMT modelling, however, an approximate method has been employed (Raychaudhury *et al* 1984; Morkoc 1985). The potential in GaAs is taken to be triangular in shape [$V(z) = eF_s z$; F_s is the surface potential]. Equation (2a) then transforms into the Airy equations, $\chi_i(z)$ become Airy functions and the sub-band energies E_i can then be related to the roots of the Airy functions. Usually at 300 K, two sub-bands are assumed to be occupied. Under these approximations, one is required to solve the following two equations (Raychaudhury *et al* 1984), taking the applied gate voltage to be zero (figure 7):

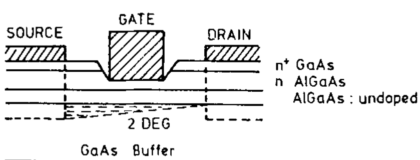

Figure 5. Cross-sectional view of an HEMT.

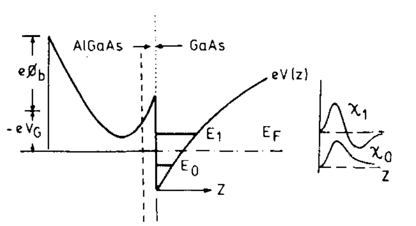

Figure 6. Band diagram in an HEMT for a positive gate voltage (V_g).

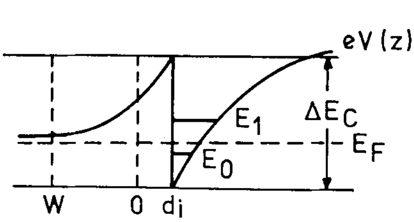

Figure 7. Band diagram in an HEMT under no gate bias.

$$n_{2D} = \frac{m^* kT}{\pi \hbar^2} \ln \left\{ \sum_i \left[1 + \exp \frac{eV(d_i^+) - E_i}{kT} \right] \right\}, \tag{3a}$$

$$n_{2D} = \frac{2\varepsilon N_d}{e} \left[-V(d_i^-) + V(-\infty) + \delta + N_d d_i^2 \right] \Bigg\}^{1/2} - N_d d_i. \tag{3b}$$

A plot of n_{2D} versus the spacer thickness d_i in an In-GaAs/In-AlAs heterojunction is given in figure 8. The calculated values (Raychaudhury *et al* 1984) are in qualitative agreement with the experimental data if $\triangle E_c = 0{\cdot}32$ eV is chosen in accordance with the affinity rule. If, however, $\triangle E_c = 0{\cdot}55$ eV is chosen in conformity with recent data (People *et al* 1983), then agreement is quite satisfactory. In recent years, some doubts have been expressed about the validity of electron affinity rule (Kroemer 1985), and the present calculations lend support to the alternate suggestions.

The other parameter of interest is the mobility of 2DEG. Here, both the defect and phonon scatterings are to be considered, and the expressions for various relaxation times are to be obtained from the Boltzmann equation. It is to be remembered that, due to modulation doping, the 2DEG in GaAs is spatially separated from the donors in the doped AlGaAs layer (figure 2a). The consequent reduction of impurity scattering leads to an enhancement of mobility especially at low temperatures. The usual momentum conservation rule for phonon scattering is also modified due to lower dimensionality. It is also found that the screening effect is quite important in limiting the values of the mobility. In addition to the usual scattering mechanisms encountered in bulk semiconductors, a new kind of defect scattering known as interface roughness (or surface roughness) scattering (Ando *et al* 1982; Ando 1982) becomes dominant at low temperature for higher values of carrier density. The interface between two materials cannot be perfectly smooth and hence hills and valleys appear. This interface roughness gives rise to a potential variation which is responsible for the scattering of the electrons. Another point to be considered in the study of mobility behaviour is that many sub-bands are usually occupied and intersub-band scattering caused by defects or phonons are to be included.

For higher values of drain voltage, the current voltage characteristics show saturation behaviour. The 2DEG then enters into the hot electron regime, and the study of hot 2DEG merits special attention. The additional points to be considered here are the intersub-band scattering and the transfer of electrons from the GaAs to the AlGaAs layer in real space (Hess 1988).

It is usually found that the high field drift velocity of carriers is not affected at all by impurity scattering. An increase in the value of transconductance in the saturation region may therefore be accomplished by using larger values of n_{2D}. A very large value of g_m has been obtained by using δ-doping of very large

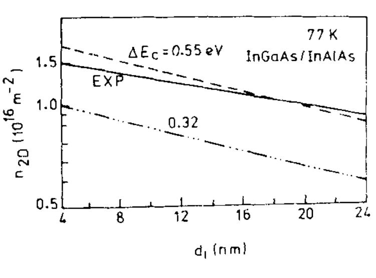

Figure 8. Variation of 2D carrier density versus spacer width (d_i).

concentrations ($\approx 10^{18}$ cm^{-3}) (Schubert *et al* 1986). Such FET structures are termed δ-FET.

The mobility enhancement in HEMT or in HJ is due to the spatial separation between 2DEG and impurities. It is to be noted that in a 1D system the electrons from a state k_x can be scattered to a state $\pm k_x'$ only, where x is the direction of free motion. The drastic reduction in the number of final states in comparison to 2D is expected to reduce the scattering rates and to give a substantial rise in the values of mobility (Sakaki 1980). The 1DEG has been realised in narrow channel MOSFET or HEMT and such transistors still show the I–V characteristics typical of FET. So far, studies on 1D systems are confined mainly to the investigation about sub-band structures and localisation effects. There has not been any practical evidence of high mobility behaviour. A calculation of mobility limited by alloy scattering as presented in figure 9 indicates, however, that high values of mobility may not be achieved for lower concentrations and smaller widths of the well (Basu and Sarkar 1986). A similar conclusion is also drawn in connection with interface roughness scattering (Basu *et al* 1987).

4. QW laser

The energy band diagram of an MQW laser is shown in figure 10a. The structure differs from the usual double heterostructure (DH) laser in that the GaAs layer sandwiched between two cladding AlGaAs layers in a DH laser is replaced by an

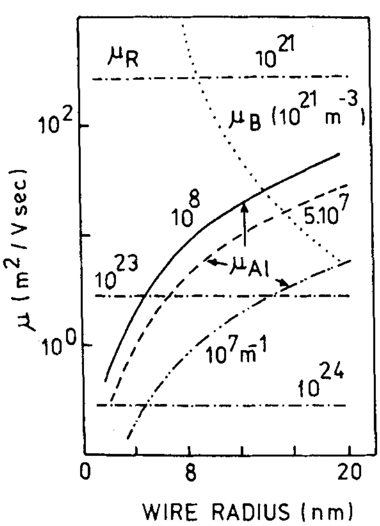

Figure 9. Alloy scattering limited mobility (μ_{AL}) in a 1D system for different 1DEG densities. μ_R and μ_B denote respectively the remote impurity and background impurity scattering limited mobility.

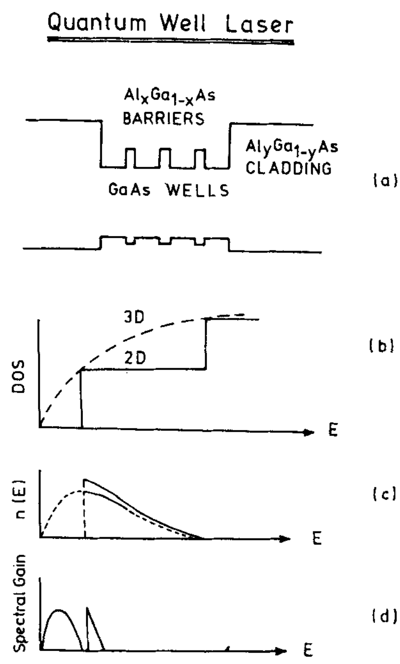

Figure 10. **(a)** Band diagram of a QW laser, **(b)** DOS function for bulk and 2D, **(c)** electron distribution as a function of energy, and **(d)** gain spectra.

MQW (note that x in the barrier layers in MQW is different from y in the cladding layers). The DOS function for bulk and 2D electrons are again shown in figure 10b. The spectral distribution of gain is shown in figure 10d and the distribution of electrons in energy for the same amount of peak gain in DH and MQW lasers are depicted in figure 10c. It appears that the number of electrons needed to achieve the same values of peak gain is less in MQW than in DH lasers. Therefore, for the same amount of loss, a lesser number of injected carriers or lesser current density is needed in MQW. The threshold current density, J_{th}, is therefore smaller in the MQW laser. Moreover, the temperature dependence of J_{th} is weaker in MQW. Both these advantages are derived out of the altered DOS function in 2D.

It is to be noted that the electrons injected into GaAs MQW from the left AlGaAs layer (figure 10a) are hot with respect to the actual distribution. The DOS function can be exploited to advantage, if these hot electrons give up energy and come to the bottom of the sub-band. The energy relaxation of hot electrons by LO phonon emission and the mean free path associated with electron–phonon coupling are, therefore, topics for investigation.

The power loss of hot electrons in the MQW system has been studied as a function of heating field and is given in figure 11. A calculation has been made about power loss including the quasi-2D nature of the electron gas, modified momentum conservation rule for 2D systems, and degenerate distribution of electrons; the results do not agree with experimental data. Better agreement is achieved when the complete wave vector-, frequency- and electron temperature-dependence of the screening parameter (q_s) is included in the calculation (Basu and Kundu 1986). The various calculated curves are included in figure 11 for comparison. Some workers believe that the electrons emit enough LO phonons to make the phonon distribution hot, and this hot phonon effect can account for the observed lower rate of cooling of electrons (Shah 1986). The matter is still being debated in the literature.

It is well-known that, for gain to occur in a semiconductor laser, the frequency ν of radiation must satisfy the condition $E_g < h\nu < \triangle F$, where E_g is the band gap and $\triangle F$ the difference between quasi-fermi levels for electrons and holes. A comparison between DOS functions for 3D and 2D electrons point out that more electrons are accommodated in 2D systems than in 3D systems in the energy range where gain occurs. The DOS of 1D systems is more favourable in this respect, and

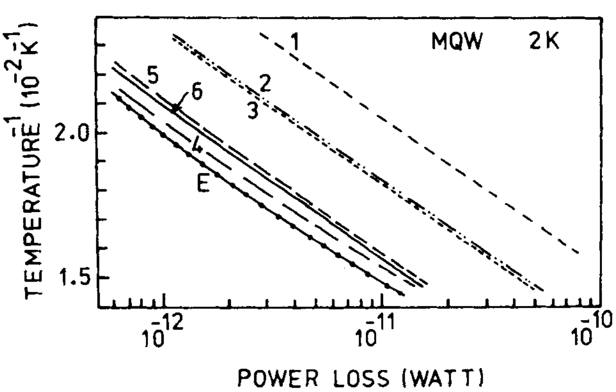

Figure 11. Power loss vs reciprocal electron temperature in a GaAs–AlGaAs MQW. E: experimental curve; 1: pure 2D; 2,3: phonon overlap function included; 4: $q_s(0, 0, 0)$; 5: $q_s(q, \omega_0, T_e)$ 6: $q_s(q, \omega_0, T_e)$, damping included.

correspondingly, J_{th} will be lower (Temkin *et al* 1987). Work on gain and loss mechanisms in QW wires (or 1D systems) is therefore pursued to reduce threshold current density.

5. Excitons and optical devices

The 2D nature of bound electron-hole pairs or excitons in QW structure has been studied by a large number of workers. The QW structures have recently become quite interesting since they exhibit exceptional optical properties. Very clear excitonic peaks are observed in the absorption spectra in QW even at room temperature while, in nearly all bulk semiconductors, excitonic peaks are barely visible above 77K (Chemla *et al* 1984; Chemla 1987). This property of QW opens up new opportunities for practical applications of excitonic effects in optical switches, modulators or detectors. Recently, it has been observed that the excitonic peaks shift as an electric field is applied perpendicular to the QW layers. This new physical mechanism forms the basis for new optical devices like optical modulators or voltage tunable detectors (Chemla 1987).

It is necessary that the excitonic peaks remain distinct and do not merge with the continuum, in order that the new optical devices function properly. The broadening mechanisms for the peaks or the linewidth (LW) of excitons is therefore an important topic for investigation. The main contribution to the LW at room temperature has been identified as that from polar optic phonons; but acoustic phonons and impurities are also thought to be important at lower temperatures (Chemla *et al* 1984). The thickness fluctuation of the well gives rise to an inhomogeneous broadening of the peak and the excitonic LW at low temperatures gives a measure of the smoothness of the heterointerface (Weisbuch *et al* 1981).

Recently, very good quality In–GaAs QW have been grown by chemical beam epitaxy (CBE) and gas source or solid source molecular beam epitaxy (GSMBE or SSMBE). We believe that the excitonic LW in these high quality QW is determined by alloy-disorder (without composition fluctuation) and therefore is homogeneous in nature. We assume that excitons are scattered by alloy-disorder potentials arising under virtual crystal approximation, calculate the scattering time τ and relate it to the LW $\triangle E$ by $\triangle E = \hbar/\tau$ (Basu and Sarkar 1988).

The calculated results are given in figure 12 and are compared with available experimental data. It appears that the agreement between our calculation and the data is quite satisfactory.

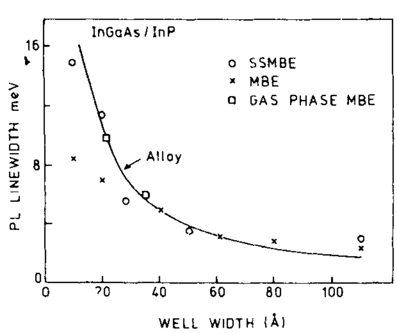

Figure 12. Photoluminescence linewidth (in meV) vs well width.

6. Summary and conclusions

In this paper various structures using III–V compound and alloy semiconductors, employed for the study of low-dimensional electron gas, are described and the importance of the study of 2D and 1D systems is mentioned. Several new electronic and opto-electronic devices have been fabricated, the operation of which is governed by the properties of low-dimensional electron gas in the devices. Some of the remarkable properties of electrons in these devices are discussed. It is found that the variation of carrier density with gate bias in a HEMT may be described by considering constant surface field and population in two sub-bands. The mobility of 2DEG is limited by the usual scattering mechanisms; however, interface roughness scattering degrades the mobility. The values of mobility in 1D systems are in some cases drastically reduced due to alloy and interface roughness scattering, contrary to the original expectation. In QW lasers, hot electrons are found to relax energy at a slower rate, because of the influence of screening. The excitonic peaks in MQW, surviving even at room temperature, are broadened due to several mechanisms. Alloy-disorder broadening in ternary QW appears to be the dominant mechanism in high quality samples.

The above mentioned properties of carriers in low-dimensional systems have been discussed in the present paper. A thorough investigation of the properties is useful for the optimisation of the performance of HEMT, QW lasers and optical elements using MQW.

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