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Deformation behaviour of materials—A phenomenological approach

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1. Introduction

Deformation of bulk materials under different modes of loading is characterized by a variety of tests. Uniaxial tensile test is the simplest one where the relation between stress and strain is determined in both the linear and nonlinear regions. Dislocation theory has been developed to explain the nonlinear or plastic deformation behaviour of materials but it is used only to interpret properties of well-characterized single crystals. The complexities increase while explaining the results of polycrystals or commercial materials and there is a big gap between the data required by a design engineer and the predictions of dislocation theory. So far, it has only helped to establish guidelines to identify important parameters for material development. A first principle quantitative description of the mechanical properties of real materials is a very distant objective. However, a phenomenological approach appears to be more well-suited to correlate the dislocation mechanisms with the observed stress-strain behaviour in a material.

There are two elementary processes involved in plastic deformation of materials: (a) movement of dislocations and (b) generation of dislocations. Both these processes are heterogeneous. In addition, the matrix in which the dislocation moves also lacks homogeneity due to distribution of defects.

To describe various deformation behaviours the starting point is a mechanical transport equation proposed by Orowan as given below

$$\dot{\varepsilon} = b\rho V, \tag{1}$$

where $\dot{\varepsilon}$ is the shear strain rate, V is the mean velocity of dislocation lines with Burger's vector b and ρ is the dislocation density. The heterogeneity is averaged out by assuming a strain-dependent density of dislocations moving with a stress-dependent average velocity.

2. Kinetic and evolution equations

These equations describe the dynamical mechanical response of a material with a given structure as specified by certain state or structure parameters. There are phenomenological theories with a single state parameter like the one proposed by Kocks *et al* (Estrin and Meking 1984) where the kinetic equation is expressed as

$$\sigma = \sigma\ (S;\ \dot{\varepsilon},\ T), \tag{2}$$

where S is the structure parameter, σ is the stress and T is the temperature. The change of structure parameter S with progressive strain is governed by the evolution equation

$$dS/d\varepsilon = f(S; \dot{\varepsilon}, T), \tag{3}$$

The structure parameter changes from an initial value to a saturation value S_S which is a function of $\dot{\varepsilon}$ and T. The corresponding value of stress σ_S can be determined from (2) as

$$\sigma_S = \sigma_S(\dot{\varepsilon}, T), \tag{4}$$

and the deformation behaviour will evolve from a transient to a steady state. Substantial internal drag stresses act on moving dislocations and under suitable conditions it may balance the applied stress. In the model of Kock *et al* the average dislocation density ρ has been taken as the structure parameter S. The evolution of structure can also be determined in terms of the mechanical strength of an obstacle $\hat{\sigma}$ given by

$$\hat{\sigma} = \alpha\, Gb\rho^{\frac{1}{2}}, \tag{5}$$

where α is a numerical factor and G is the shear modulus. The flow stress σ is related to the strain rate by the kinetic equation given in the form of a power law as,

$$\sigma/\hat{\sigma} = (\dot{\varepsilon}/\hat{\dot{\varepsilon}})^{1/m}, \tag{6}$$

where $m = (\partial \ln \dot{\varepsilon}/\partial \ln \sigma)|_{\hat{\sigma}, T} \gg 1$ and $\hat{\dot{\varepsilon}}$ is a normalization parameter. The evolution equation in the model of Kock *et al* is written as

$$d\rho/d\varepsilon = k_1\, \rho^{\frac{1}{2}} - k_2\rho. \tag{7}$$

The first term in (7) is the production term i.e. the dislocations which become immobile after travelling through a distance equivalent to spacing between dislocations. It is also called a thermal hardening term. The second term is the loss term corresponding to thermally activated dynamic recovery. If the system is full of impenetrable sinks the production term in (7) will be constant and proportional to the inverse of mean distances between them i.e. $\rho^{-\frac{1}{2}}$. k_1 is the constant which can be evaluated from the slope of stress-strain curve in stage II or athermal hardening. k_2 is the constant for dynamic recovery and is a function of strain rate and temperature.

The change in dislocation density predicted by (7) will cause a change of mechanical strength of the obstacle or hardness parameter as given by (5).

$$\begin{aligned} d\hat{\sigma}/d\varepsilon &= \tfrac{1}{2}(\alpha Gb)\, \rho^{-\frac{1}{2}}\, (d\rho/d\varepsilon) \\ &= \theta_0\, [1 - (\hat{\sigma}/\hat{\sigma}_s)], \end{aligned} \tag{8}$$

where $\theta_0 = \alpha\, Gb\, k_1/2$, $\hat{\sigma}_s = \alpha\, Gb\, k_1/k_2$, when there will be no increase of ρ; further the mechanical strength of the obstacle will saturate, i.e. $\hat{\sigma} \to \hat{\sigma}_s$ and $d\hat{\sigma}/d\varepsilon \to 0$.

The evolution equation proposed by Kock *et al* may be changed depending on the existing condition of the material. If the mean free path of dislocations is determined by distances between grain boundaries or precipitates d, then (7) will have to be modified as

$$d\rho/d\varepsilon = k - k_2\rho, \tag{9}$$

where k is proportional to the inverse of mean free path and k_2 has the same meaning as in (7).

The single parameter kinetic equation (2) can be used to find an analytical description of stress-strain behaviour. At constant temperature σ is a function of only the strain rate $\dot{\varepsilon}$ and the structure parameter which is identified as the hardness parameter in kinetic equation (6). So one can write,

$$\mathrm{d}\sigma = (\partial\sigma/\partial \ln \dot{\varepsilon})_s \ln \dot{\varepsilon} + (\partial\sigma/\partial s)_{\dot{\varepsilon}}\, \mathrm{d}s$$

$$= \frac{\sigma}{m}\mathrm{d}\ln\dot{\varepsilon} + \frac{\sigma}{\hat{\sigma}}\cdot \mathrm{d}\hat{\sigma} \tag{10}$$

and the rate of variation of stress with strain is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\varepsilon} = \frac{\sigma}{m}\frac{\mathrm{d}\ln\dot{\varepsilon}}{\mathrm{d}\varepsilon} + \frac{\sigma}{\hat{\sigma}}\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\varepsilon}. \tag{11}$$

In the uniaxial tensile test $\dot{\varepsilon}$ is maintained constant and the first term in (11) will vanish. Substituting for $\mathrm{d}\hat{\sigma}/\mathrm{d}\varepsilon$ from (8) and using (6) one can write,

$$\theta = \mathrm{d}\sigma/\mathrm{d}\varepsilon = \theta_0\,(\dot{\varepsilon}/\hat{\dot{\varepsilon}})^{1/m}\,[1-(\sigma/\hat{\sigma}_s)]. \tag{12}$$

Thus, the stress-strain behaviour will show θ vs σ as a straight line. If one presumes the other model for evolution given by (9), θ vs σ^2 will be a straight line. Although the monotonic stress strain behaviour could be explained by the one-parameter model with a reasonable success the transient behaviours could not be explained within this framework. The two-parameter models have been introduced and a typical model proposed by Estrin and Kubin (1986, EK hereafter) has been taken up here for discussion.

In the EK model the two structure parameters are the mobile dislocation density ρ_m and the forest dislocation density ρ_s. The kinetic equation is the same Arrhenius type as given below.

$$\dot{\varepsilon} = A\rho_m \exp|(\sigma - \sigma_f)/S|, \tag{13}$$

where σ is the applied stress σ_f is the back stress due to forest dislocations. S is the strain rate sensitivity of flow stress. The strain rate $\dot{\varepsilon}$ and σ_f are given by,

$$\dot{\varepsilon} = \rho_m b V \quad \sigma_f = \alpha G b \rho_f^{\frac{1}{2}}, \tag{14}$$

where G and b are respectively the shear modulus and Burger's vector of the material and V is the average velocity of the dislocation. Taking the logarithm of (13) and differentiating with respect to time one gets,

$$\dot{\sigma} = S(\ddot{\varepsilon}/\dot{\varepsilon}) + h\dot{\varepsilon}. \tag{15}$$

where the rate of strain hardening, $h = h^+ + h^-$ with,

$$h^+ = Gb\left.\frac{\partial \rho_f^{\frac{1}{2}}}{\partial\varepsilon}\right|_{\dot{\varepsilon}},$$

$$h^- = -S\left.\frac{\partial \ln \rho_m}{\partial\varepsilon}\right|_{\dot{\varepsilon}}.$$

The term h^+ is due to increase in the density of forest dislocation whereas h^- is the softening due to release of mobile dislocation.

The evolution equations will now be coupled differential equations involving both

ρ_f and ρ_m as given below

$$\frac{d\rho_m}{d\varepsilon}=\frac{1}{\rho_m bV}\frac{d\rho_m}{dt}=\frac{C_1}{b^2}(\rho_f/\rho_m)-C_2\rho_m-\frac{C_3}{b}\rho_f^{\frac{1}{2}}. \tag{16}$$

The first term indicates the production of mobile dislocation due to segments of dislocation pinned by forest dislocations and acting as the Frank-Read source. The second term indicates the process of annihilation by mobile dislocations and the third term represents immobilization of mobile dislocations. C_1, C_2 and C_3 are constants. A similar equation can be written for the density of forest dislocations ρ_f as,

$$\frac{d\rho_f}{d\varepsilon}=\frac{1}{\rho_m bV}\cdot\frac{d\rho_f}{dt}=C_2\rho_m+(C_3/b)\,\rho_f^{\frac{1}{2}}-C_4\rho_f. \tag{17}$$

Here the first term considers the process of creation of immobile, small loops or dislocation debris by jog dragging and the last term indicates the process of annihilation of forest dislocation by climb or cross slip.

A solution of these coupled equations (16) and (17) along with the kinetic equation have been used to predict the monotonic stress-strain behaviour as successfully as the one-parameter models. But the unique feature of the two-parameter model lies in its ability to explain the transient stress-strain behaviour as discussed below.

3. Transient deformation behaviour

At a given point of deformation a local imperfection is introduced in the specimen in terms of deviation from the uniform values of σ, ε, $\dot{\varepsilon}$, ρ_f and ρ_m. These deviations $\delta\sigma$, $\delta\varepsilon$, $\delta\dot{\varepsilon}$, $\delta\rho_f$ and $\delta\rho_m$ will obey the constitutive equation and are therefore inter-related. One can write,

$$\delta\rho_f=(d\rho_f/d\varepsilon)\,\delta\varepsilon;\ \delta\rho_m=(d\rho_m/d\varepsilon)\,\delta\varepsilon \tag{18}$$

and linearize (15) to

$$S(\delta\dot{\varepsilon}/\dot{\varepsilon})=\delta\sigma-h\,\delta\varepsilon. \tag{19}$$

Since for an incompressible material $\delta\sigma=\pm\sigma\delta\varepsilon$ where the plus and minus signs are respectively stand for tension and compression, (19) reduces to

$$\delta\dot{\varepsilon}/\dot{\varepsilon}=-[(h\mp\sigma)/S]\,\delta\varepsilon. \tag{20}$$

Now, a stability analysis is carried out in terms of the growth or suppression of the inhomogeneity in strain, $\delta\varepsilon$, with the application of further strain ε. It is assumed that

$$\delta\varepsilon=(\delta\varepsilon)_0\exp[\lambda\varepsilon], \tag{21}$$

or $$\delta\dot{\varepsilon}=d(\delta\varepsilon)/dt=\lambda\dot{\varepsilon}\,\delta\varepsilon.$$

Using (20) and (21) together one gets,

$$\delta\dot{\varepsilon}/\dot{\varepsilon}=\lambda\,\delta\varepsilon=-[(h\mp\sigma)/S]\,\delta\varepsilon$$

or $$\lambda=-(h\mp\sigma)/S. \tag{22}$$

If one plots both the rate of strain hardening and σ as a function of strain, ε, as given

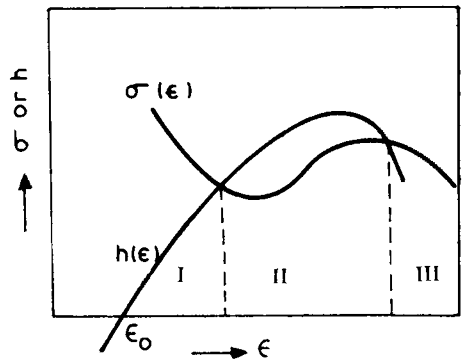

Figure 1. Variation of strain hardening rate, h, and σ with strain, ε.

in figure 1, it is evident that a bifurcation from uniform to non-uniform deformation is associated with the strain where λ vanishes.

In compression the bifurcation corresponds to $h+\sigma=0$ i.e. $h(\varepsilon_1^B)<0$ or $\varepsilon_1^B<\varepsilon_0$. At $\varepsilon>\varepsilon_0$, $h+\sigma$ is greater than zero and λ is negative and $\delta\varepsilon$ will die down with ε as given by eq (21).

In tension when $\lambda>0$ i.e. $h<\sigma$ inhomogeneity in strain, $\delta\varepsilon$, will grow with further strain in regions I and III marked in figure 1. However, $\delta\varepsilon$ will eventually vanish in region II with further application of strain. The deformation behaviour in region III can be shown to satisfy the well-known necking criterion of Hart i.e.

$$(h/\sigma)+(1/m)<1. \tag{23}$$

4. Conclusions

The quantitative description of stress-strain behaviour through appropriate models for structural evolution of materials has been fairly successful in explaining both the monotonic and transient stress-strain behaviour of materials. Further exploration of this approach may lead to detailed understanding of evolution parameters and the influence of metallurgical variables. Thus, a quantitative method for alloy design for the purpose of structural applications at ambient and elevated temperature may emerge.

References

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