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Computer simulation of serrated yielding

K NEELAKANTAN
Indira Gandhi Centre for Atomic Research, Kalpakkam 603 102, India

Abstract. A model for serrated yielding based on the negative resistance characteristics of materials is discussed. An analog computer based on this model is described. The simulated curves show oscillations which are regular and uniform in amplitude. To simulate more realistic tensile test curves, a refined model which includes the effects of fluctuations in dislocation density and velocity is described. Some simulated curves using this refined model are presented. These results are close to observed tensile test curves.

Keywords. Serrated yielding; machine equations; electronic analog; noise.

1. Introduction

The phenomenon of serrated yielding during tensile test experiments has been widely observed (Keh *et al* 1968; Brindley and Worthington 1970; Hall 1970) and is discussed in the literature under various names such as the Portevin-Le Chatelier effect, jerky flow etc. Experiments have shown that serrated yielding is affected by temperature, strain rate and composition. On the theoretical side the basic idea has been that of dynamic strain ageing going back, in a sense, to Cottrell and Bilby (1949).

While the strain ageing concept no doubt gives an insight into the physical origin of serrated yielding, it does not by itself provide a description of the temporal behaviour observed during experiments. It is interesting to examine the stress oscillations in terms of the interplay between the machine kinematics and the defect kinetics. The crucial point is that the dislocation drag mechanism has a negative resistance characteristic (Cottrell 1953).

In this paper, the temporal behaviour of the stress oscillations is examined in terms of the Cottrell model (which incorporates a negative resistance feature). A simple electronic analog computer (Neelakantan and Venkataraman 1983) to simulate the behaviour of the material during a constant strain rate test is described. It will be seen that the serrations are regular and uniform in amplitude unlike the ones in real life tensile test curves. However the circuit describes the observed behaviour in other respects. A refined model (Neelakantan and Venkataraman 1986) which takes into account the fluctuations in dislocation density and velocity is then described. Results from numerical simulation using this refined model are presented.

2. The machine equations

The starting point is the machine equation which quantifies the material response during tensile testing. The machine schematic is shown in figure 1a. Figures 1b and c show the mechanical and electrical equivalent (to be discussed in §3). The cross

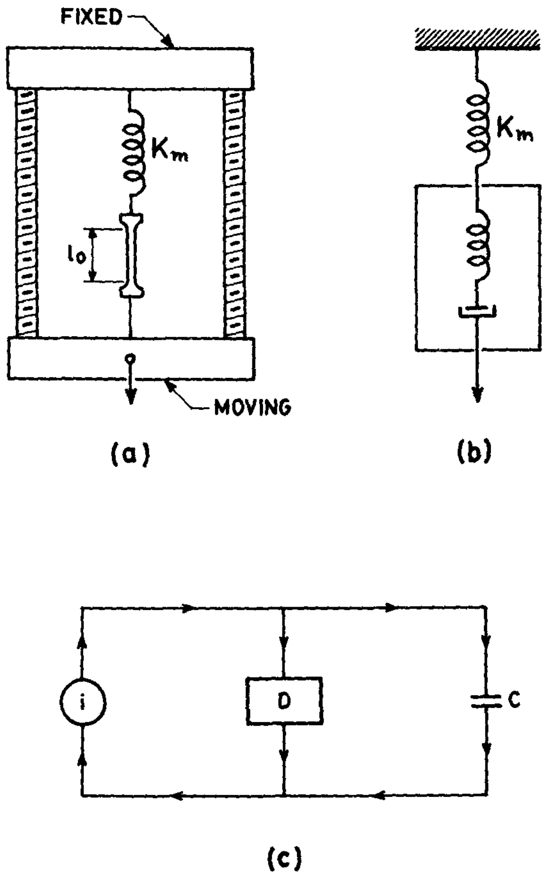

Figure 1. (a) Machine schematic for tensile testing (b) Mechanical equivalent. (c) Electrical equivalent.

head moves with a constant speed S. If the machine stiffness is K_m then at time t

$$St = P/K_m + (Pl_0/AE) + \varepsilon_p l_0, \tag{1}$$

where A is the area of cross-section of the specimen, E its elastic modulus and ε_p the plastic strain. The load P is given by $A\sigma$ where σ is the stress developed. Defining an effective stiffness K_{eff} as

$$1/K_{\text{eff}} = A/l_o K_m + 1/E, \tag{2}$$

we get

$$\dot{\varepsilon} = \dot{\sigma}/K_{\text{eff}} + \dot{\varepsilon}_p, \tag{3}$$

where $\dot{\varepsilon}$ is the applied strain rate.

Now $\dot{\varepsilon}_p$ is described by Orowan's equation

$$\dot{\varepsilon}_p = gb\rho u, \tag{4}$$

where g is a parameter, b is the Burger's vector, u the velocity and ρ the density of mobile dislocations. The dislocation density can be modelled as (Bergstrom and Roberts 1973)

$$\rho = \rho_0 \exp(-\Omega\varepsilon_p) + \rho_f (1 - \exp(-\Omega\varepsilon_p)), \tag{5}$$

where ρ_0 and ρ_f are the initial and final dislocation densities and Ω is a parameter related to the remobilization rate. For low values of plastic strain we can write (5) as

$$\rho = \rho_0 + B\varepsilon_p. \tag{6}$$

Now the only thing that remains to be specified is the dislocation velocity. This is a highly nonlinear function of the stress. Choosing the Cottrell model which is schematically shown in figure 2 and taking into account the work hardening effect one can write (Nabarro *et al* 1964)

$$u = f(\sigma - \alpha\varepsilon_p^m), \tag{7}$$

where α is the work hardening coefficient and m is a parameter.

3. The electronic analog

Going back to figure 1c which shows the electrical equivalent of the tensile test, one sees the following correspondences

$$\dot{\varepsilon} \leftrightarrow i \text{ (current) and } \sigma \leftrightarrow v \text{ (voltage)}.$$

In the circuit shown, the capacitor represents the compliance and the device D simulates the complex nonlinear behaviour of the plastically deforming material. Maintaining a constant strain rate is equivalent to driving the circuit with a constant current source.

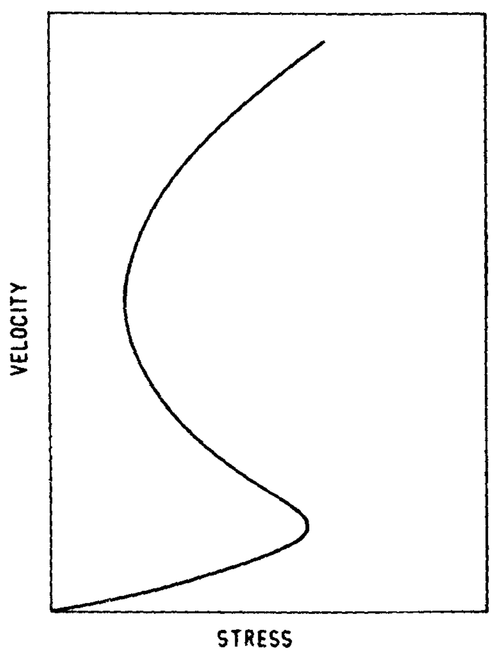

Figure 2. Schematic of dislocation velocity versus stress.

Initially, when the circuit is switched on, both i and the voltage v will be zero. After switching on, the system will evolve towards an operating condition where the load line (applied current) intersects the device characteristics. The device characteristics will however, evolve with the plastic charge (plastic strain) as shown in figure 3 which shows that initially the load line intersects the device characteristics in a region where the device exhibits a positive slope (curve a). Hence the circuit will be stable and no oscillations will be seen. As some 'plastic current' flows through the device D, its characteristics evolve through curves b and c. The load line then intersects curve c in a negative slope region and hence the circuit is unstable and shows oscillations. Later, when the characteristics evolve to curve d, the oscillations will cease.

In order to simulate such behaviour, a tunnel diode was used to mimic the negative slope characteristics of the material. The circuit schematic is shown in figure 4. The capacitor C represents the compliance of the specimen. The voltage across the capacitor is buffered by OPAMP A1 and suitably amplified by A2. The output of A2 is applied to a tunnel diode TD. The current drawn by the tunnel diode is sensed by A3 and the output of A3 represents the dislocation velocity. OPAMP A4 acts as an integrator and its output represents the plastic strain. The output of A6 represents the dislocation density. The outputs of A3 and A6 are multiplied by the multiplier M whose output controls the current ('plastic current') through the transistor T. The work hardening effect is incorporated through A5 whose output substracts from the voltage across the capacitor.

The curve in figure 5 was obtained using the circuit described above. It can be seen that the features of a tensile test curve namely the initial yield drop, the delayed onset of serrations and the serrations themselves are all manifested in the curve. Several different biasing conditions were tried and the results showed that the simulation was faithful

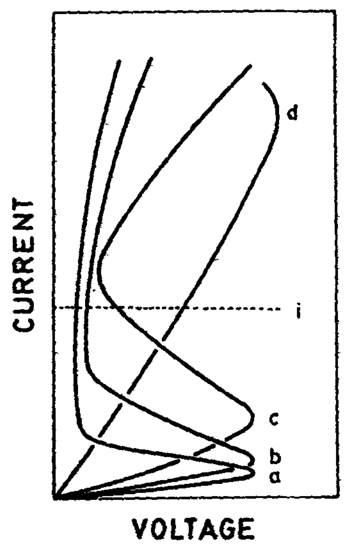

Figure 3. Evolution of the material characteristics with plastic strain. The curves a, b, c and d represent the material characteristics as the specimen is strained. The dotted line is the applied strain rate.

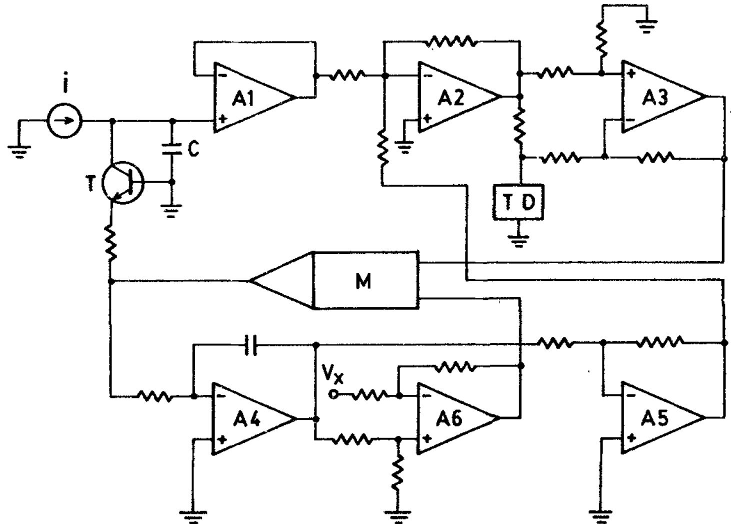

Figure 4. Circuit schematic for the analog computer.

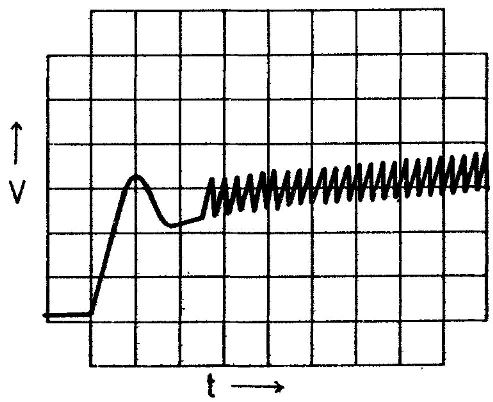

Figure 5. Output of the analog computer when the circuit is biased in the negative resistance region.

except for details of pulse shape, frequency and amplitude. These details cannot be reproduced by a circuit in which some necessary simplifications are made. Two major simplifications are the assumption that the dislocation density follows (6) with a relatively small value of B (compared to reality) and the assumption that m is unity in (7).

One major drawback of the above model is that it ignores the effects of fluctuations in the dislocation density and velocity. This is considered in the next section.

4. Role of noise

The few models available for serrated yielding (Malygin 1973; Kubin *et al* 1982; Neelakantan and Venkataraman 1983) all predict regular serrations whereas the observed ones are usually irregular and occur in various types (Pink and Grinberg 1981). In this section, a realistic simulation of the various types of serrations is described.

We start with equations (3), (4), (5) and (7). Using a three-piece linear approximation to the S-shaped σ-u curve, the system of equations describing the phenomenon was solved numerically. However, during the straining, dislocations are not only born and annihilated but also temporarily immobilized as well as unlocked. The dislocation velocity is also a random variable as it depends on thermal interactions with the lattice etc. To take into account the fluctuations in these quantities, the dislocation density and velocity are made random according to

$$u(\sigma) = \langle u(\sigma) \rangle + \beta(\sigma)\eta, \tag{8}$$

and

$$\rho(\varepsilon_p) = \langle \rho(\varepsilon_p) \rangle + \gamma(\varepsilon_p)\eta, \tag{9}$$

where η is a fluctuating quantity.

A wide variety of tensile test curves have been simulated and figures 6 and 7 offer a selection of the results. The various types of serrations reported in the literature have

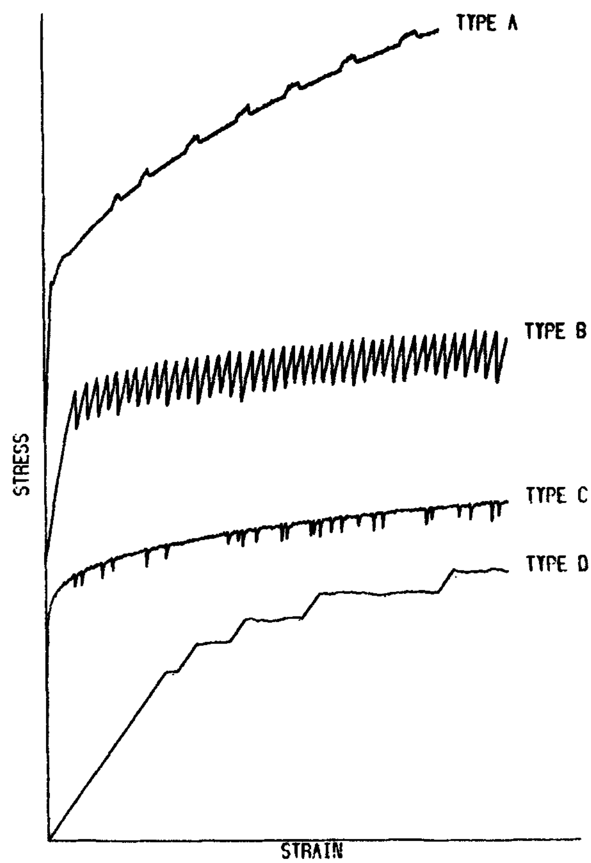

Figure 6. Various types of serrations obtained by numerical simulation.

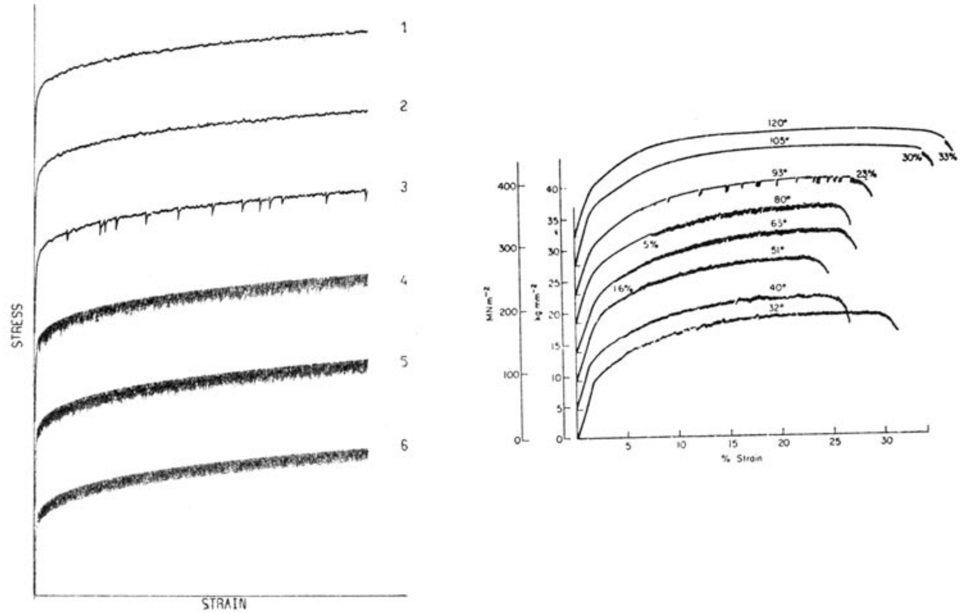

Figure 7. On the left are simulated curves. Curve 6 is for the same conditions as curve 4 but without noise. On the right are the observed curves for an Al-Mg alloy (Hall 1970).

been simulated in figure 6. The shapes depend upon the point of intersection of the load line with the material characteristics (among other factors). If the bias is near the threshold of oscillations, noise triggers random repeated yields (type C in figure 6). Figure 7 shows a family of curves corresponding to a situation where the strain rate is held constant but the temperature is varied. The similarity to observed patterns is striking.

5. Conclusions

A model for serrated yielding behaviour has been presented. Simulations on an analog computer using this model results in serrations which are regular. However, the inclusion of fluctuations in dislocation density and velocity refines the model to the extent that the simulated curves are very close to actual observed tensile test curves.

More than mimicking reality, the message is that the random aspects of serrations need to be studied and understood. Attention must be paid to spatial inhomogeneity also.

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