Some fundamental considerations to precede image analysis

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Abstract. This paper explains the different types of specimens which are of a deterministic or stochastic nature. Especially for stochastic specimens it is pointed out that, before any measurement is done, the problem has to be analyzed as to why we measure. As a size per se does not exist it is shown that particles must be analyzed by size and shape. For this, two methods are indicated, while it is stressed that in materials science the Delfiner approach for description of particle sizes is the most practicable.

Based on Delfiner's concept of size the classification of particles according to their size, shape and orientation is explained. For measuring use of the image transformations *erosion*, *ouverture* is proposed. In addition, a new transformation called "*Template matching*" is explained. This new transformation avoids the disadvantages of erosion and ouverture in case of concave particles. Besides, template matching, in contrast to erosion or ouverture, is able to give true values of area and circumference. The concept of classification is explained by simple examples.

Keywords. Image analysis; erosion; ouverture; template matching.

1. Introduction

Many instruments for image analysis are presently on the market. Each of these instruments has many characteristic features. With the steady increase of computer performance the image analysers offer a growing number of possibilities. The user of these instruments is confused. It is the aim of this paper to give some hints on how to begin measurements with an image analyser. These considerations are made from the standpoint of a materials scientist. Possibly in life sciences the questions are different and likewise the solutions; but, maybe, the basic ideas are similar.

2. Categories of problems

Before a measurement starts the underlying problem must first be defined. This may seem a trivial condition, and it is without any significance as long as there are only limited possibilities of measurement. It becomes relevant only as a result of the fact that the theoretical and technical advances in measurement have made it possible to find answers to a broader range of questions.

When selecting the measuring technique, first of all the character of the sample material must be studied. In this respect, two categories of problems must be distinguished:

- Deterministic problems. They refer to sample material, each detail of which must be analyzed and classified. A typical problem in this category is, e.g., automatic reading of texts in which each and every letter must be identified and classified. Another

- example is the automatic quality control of workpieces for perfection. Also in this case, each individual workpiece must be analyzed and faulty workpieces sorted out.
- Stochastic problems arise if parts of a sample are measured which satisfy a distribution function and the sample satisfies conditions of stationarity. In this sample material it is not important to measure each individual element of the particles of interest. It is sufficient to measure a statistically significant fraction of the sample. However, care must be taken to avoid a bias to be introduced into the statistics by the choice of the fraction of the sample measured. Typical applications of this technique are measurements of segregations in alloys or counts of pore size distributions. The automatic examination for cracks in workpieces does not fall under this category of problems, because in this case even a single crack in a workpiece may mean that the respective item must be scrapped. This is a typical deterministic problem, because every potential crack must be found.

Also the measuring techniques to be applied depend on the problem at hand. In the case of deterministic problems, the individual objects must be singularized before measurement; in general, methods of pattern recognition are applied subsequently. In the case of stochastic samples we are not interested in readings obtained from individual particles. In this case we measure field quantities (Nawrath and Serra 1979), it may even be necessary to carry out transformation before measuring within the image. These techniques are based on Minkowski's operations of sets (Hadwiger 1950) or on correlation functions (Vollath 1981). In this connection, especially the field of mathematical morphology has advanced quite far (Matheron 1967; Haas *et al* 1967a,b; Serra 1969). These are the fundamental papers, which have produced an immense amount of literature.

Occasionally, however, also sample material of a stochastic nature may cause problems which cannot be solved by the familiar techniques applied to analysis of such sample materials. In this case, a roundabout way will have to be found, which involves individual analysis of the particles of the sample and further processing by the rules of statistics of the results of such deterministic analysis.

Table 1 is a summary of these problems and the corresponding methods of evaluation. This paper will be limited to the problems of processing samples of a

Table 1. Problems and methods of evaluation used in image analysis

	Character of sample	
	Deterministic	Stochastic
Structure of sample	Group of significant shapes	Randomly distributed particles
Application	Pattern recognition, sorting out faulty workpieces	Materials analysis, characterization of powders
Methods	All techniques of pattern recognition	Minkowski operations of sets (mathematical morphology), correlation function
Results	Significance of a symbol, yes-no information about the quality of workpieces	Distribution function with respect to one or more parameters

stochastic nature, although a considerable part of the treatment also applies to samples of a deterministic analysis.

3. What to measure?

Once the type of sample has been defined, a technique of measurement can be found. Any measurement must be based on a clear-cut technical or physical problem. In order to allow the measured result to be interpreted in the sense of such a problem, it is necessary to derive from the problem a behavioural model and develop a measuring technique compatible with it.

If we have understood what we wish to measure the next intuitive question will be: What is the size of the particles? If we look at the particles drawn in figure 1 we will realize that a size per se is not existing. If we wish to make a statement about the size of these particles, we have to give additional information about these shapes. We must realize that any information about size of a particle in principle supposes a dimension adequate to the shape of the particle to be evaluated. Consequently, the new demand for taking into account the shape in classifying particles has much more far reaching consequences. It incorporates the need to break up into subsets a group of particles, the descriptors of these subsets being different shapes simple to describe in terms of geometry. The desired result is not a size distribution function, but several distribution functions, which can be assigned to different shapes describable in geometric terms. Each of these distribution functions describes only that fraction of the particles which can be described by the corresponding shape.

When performing an actual analysis we are usually not solely interested in obtaining just data. So we will never ask, e.g., a question such as "how large are the pores in a sintered body with respect to a certain shape". Instead, the question must be, "how large are the pores with respect to a certain property (e.g., thermal conductivity)." Asking for size per se is not meaningful, because any definition of size will also incorporate the shape, and the shapes by which the pores should be analyzed are a function of the model underlying the description of a property. If, e.g., in our example the property is the thermal conductivity of the sintered material the shapes suited for analyzing this property are more frequently ellipses.

Some examples of such correspondence between a physical problem and its description in a model or by image analysis will be given below. For instance, if the technique of image analysis is to be applied to simulate the measuring process in mercury porosimetry when characterizing sintered specimens, this is best done by using an approximately circular structuring element used for the "ouverture" image transformation (Müller 1973; Vollath 1979b). If area measurements are conducted after

Figure 1. How can we describe these shapes?

these image transformations, comparison with the results of mercury porosimetry will furnish very interesting insights into pore structures (Vollath 1979).

Figure 2 shows a comparison of the principles of mercury porosimetry and image analysis. The fundamental difference that in case of mercury porosimetry pores are filled from outside and in image analysis the pores are filled in a cut plane leads to significant differences of the results giving interesting information of the pore network.

The question about the "size" of particles rich in plutonium in $\text{UO}_2\text{-PuO}_2$ nuclear fuels can be answered if it is known that the size of these particles is limited because of potential overheating in cases of rapidly rising reactor power. Hence, in a first approximation, the maximum heat conduction pathway is the dimension characteristic of size in this case. The maximum heat conduction pathway is indicated in a first approximation by the radius of the largest spheres inscribable in the particles (Vollath 1979a). Refinement of this model leads to the description of particles by ellipsoids (Vollath 1982).

Depending on the method of fabrication adopted sintered bodies sometimes have pores approximately tetrahedral in shape besides those approximately spherical.

Figure 3 depicts the situation observed in a ceramographically prepared specimen.

Figure 2. Comparison of measurement: Hg-intrusion porosimetry vs image analysis.

Figure 3. Typical microstructure of a sintered material made of granulated powder.

Some of the physical properties, such as thermal conductivity, strongly depend on the different fractions of these two pore shapes. If the sections of rather spherical pores are approximated by squares and those of more tetrahedral ones by triangles, the fractions of these two types of pores can be estimated (Vollath 1982).

4. How to measure?

We have stated now the type of results we would like to obtain and we have shown a few problems and results of measurements. But we have not considered any method of measurement.

For determining shapes of particles we have in general two philosophies which could be applied. The first is to singularize the particles of an image and to determine shape factors for each of these particles. Having done this we collect all the measures for size and shape factors and calculate a new distribution function.

Information about shape factors does not always give the true relationships. For example, if we consider the shape factor given by the ratio of area to circumference of a particle it is maximum for a circular figure. As the circle is deformed to form an ellipse the shape factor decreases as the axis ratio of the ellipse increases. It is thus possible with this shape factor to obtain information about the shape of the particles. This information is not, however, unequivocal. If a circular particle has a wavy or even a frayed border, it is certainly possible to take measures in this particular case to obtain usable results but it must be remembered that in some special cases these measures will induce inaccuracies.

Even if there are many disadvantages resulting from the use of shape factors, in the case of complicated patterns such as concave particles there is even now no choice of the method.

If we wish to analyze more or less convex particles and if we are satisfied with describing these particles by simple patterns the concept of size proposed by Delfiner (1972) is unequalled today. Delfiner defines the size of a particle P by the largest template T ("structuring element") which can be inscribed.

The data obtained by this method can be evaluated by mathematical morphology pioneered by Serra (1969) and Matheron (1967). If we analyze our specimens with templets of different shapes we can obtain a classification of particles by size and shape.

5. Definition of a particle size

In particles of simple shapes, such as circles, particle size can be defined unequivocally, e.g., through the diameter of the circle. In a similar way it is also possible to describe in simple terms the size of linear particles. However, in most cases much more complicated particle shapes are encountered. In those cases, description by way of maximum chord lengths or maximum dimensions (Feret diameters) are inadequate.

In the light of this problem Delfiner (1972) proposed a generalized concept of particle size. Delfiner assumes a basic pattern of any given shape and establishes a monotonic sequence of elements which are homothetic of the basic pattern selected. The size of a particle is then given by the largest element of this sequence which can be inscribed in this particle.

Let us consider sets (particles) G which are to be compared with other sets T_i (templates). The sets T_i are subsets of the ordered set T . All subsets of T are supposed to be homothetic.

$$T_i \subset T_{i+j} \quad j > 0 \quad i, j \in \mathbb{N}.$$

The size of G with respect to the elements of τ is given by

$$G =: G_T \Leftrightarrow T_i \subset * . \wedge . T_{i+j} \not\subset * G \quad i, j \in \mathbb{N} \quad (1)$$

(Vollath 1982) with

$$T_i \subset * G: \exists (X(x) \oplus T_i) \subset G.$$

The set $X(x)$ shifts T_i by x as a result of a connection. $\oplus$ stands for Minkowski addition (Hadwiger 1950; Matheron 1967; Serra 1982).

If the criterion (1) is met, the particle G has the size i relative to the sequence of structuring elements given by T . Classification in accordance with (1) can be clearly conceived as involving the use of templates of different sizes of the shape T . Out of the inventory of such templates one selects the largest template particle G will fit. As a coefficient of size measurement, the particle is assigned the number i of the largest fitting template.

If the possible sets T are restricted to the class of convex sets, the size can be determined by means of a Minkowski set operation. The elements of the sequence T can be determined from a convex unit element T_1 as partial sums of a Minkowski series (Meschkowski and Ahrens 1974),

$$T_i = T_0 \oplus \bigoplus_{k=1}^i T_1 \quad i, k \in \mathbb{N}, \quad (2)$$

where the set T_0 generally only consists of the origin. T_i can also be represented by means of the recursion formula (Vollath 1982)

$$T_i = T_{i-1} \oplus T_1 \quad i \in \mathbb{N}.$$

The classification of sets G in accordance with elements of T now can be arranged by means of Minkowski subtractions in analogy with (1). G can be assigned to the class i of a sequence T (Vollath 1982) if

$$G = G_T \Leftrightarrow G \ominus T_i \neq \phi . \wedge . G \ominus T_{i+j} = \phi \quad i, j \in \mathbb{N} \quad (3)$$

The particle G to be classified is the subset of an image B . In processing an image one proceeds by eroding the image by Minkowski subtractions. A sequence of deformed images is obtained,

$$B_{T_i} = B \ominus T_i.$$

According to Matheron (1967), the two Minkowski set connections can be linked so as to form another operation suitable for classification, the ouverture. A sequence of images is obtained, which were deformed by the ouverture transformation

$$B'_{T_i} = (B \ominus T_i) \oplus T_i. \quad (4)$$

$\oplus$ stands for the Minkowski addition (Hadwiger 1950; Matheron 1967; Serra 1982).

$\ominus$ stands for the Minkowski subtraction (Hadwiger 1950; Matheron 1967; Serra 1982).

In the transformed images B_{T_i} it is now possible to determine areas, perimeters and particle numbers as coefficients of measure.

The quantities measurable in the images transformed by erosion or ouverture will not always generate the required information. Thus, e.g., dumb-bell shaped particles may disintegrate into two separate particles after erosion or ouverture. This may have a disturbing effect on particle counting. There are also measuring problems when particles are to be classified in accordance with given patterns, but also the coefficients of measure are to be determined in the unchanged particles. In such cases, classification must be carried out by means of a template matching procedure. For the case of a sequence T of convex templates the following method can be used for image transformation in which, first of all, the particles in the image are classified by erosion,

$$B_{T_i} = B \ominus T_i$$

and the original shape of the particles is then reconstructed by the recursion formula

$$B_{T_i}^*(k+1) = \bigcup_{j=1}^8 (B_{T_i}^*(k) \oplus S_j) \cap B. \quad (5)$$

The first element B_{T_i} of the sequence of sets produced is

$$B_{T_i}^*(1) = B_{T_i}.$$

The set S_j always consists of the origin and one of its eight neighbourhood points. The process of iteration must be continued until the condition

$$B_{T_i}^*(k+1) = B_{T_i}^*(k)$$

is met.

In the following sections, which will deal with classification by shape and size, it will be assumed that classification has already been carried out by one of the three methods of classification discussed (erosion, ouverture, or template matching).

6. Classification of particles in accordance with several sequences of patterns

Let us now consider the case of particles of a sample being classified in accordance with M different sequences of convex template T^l . Each set T^l consists of N^l elements of the size j , $T_j^l \in T^l$. The subset T_j^l is used to determine in the images such measures as particle numbers or areas. For the sake of simplicity we assume that these measures were determined by means of the template matching technique of classification. When measuring with the templates T_j^l the cumulative frequencies, H^l are determined. These cumulative frequencies, result from the class frequencies h_j^l through the sum of

$$H_j^l = \sum_{k=j}^{N^l} h_k^l \quad l \in \{1, \dots, M\}. \quad (6)$$

From the matrix of measured values (H_j^l) a matrix (h_j^{*l}) is now to be determined of the class frequencies at which also separation by shapes will be performed. Such separation may be carried out on the basis of two different principles.

6.1 Separation by inclusions

For all elements of the sequences of patterns T^m it holds that

$$T_i^m \subset * T_{i+j}^m \quad i, j \in \mathbb{N}. \quad (7)$$

If we compare two sequences of patterns, we always find pairs of elements obeying the following relationship:

$$T_i^m \subset^* T_{l(i, m, n) + j} \quad i, j, l, m, n \in \mathbb{N}. \quad (8)$$

The element $T_{lm}^n(i, m, n) + 1$ is the smallest element of the sequence T^n containing the element T_i^m as a subset. Obviously, in determining the class frequencies h_j^{*m} , corrected by shapes, the numbers and areas, respectively, of all patterns must be subtracted in which T_i^m is fully contained.

$$h_j^{*m} = H_j^m - \sum_{k=1}^M \sum_{i=l(j, m, k) + 1}^N h_i^{*k} \quad (9)$$

Consequently, calculating the class frequencies, h_j^{*m} presupposes knowledge of the class frequencies h_i^{*k} of the elements T_i^k in which T_j^m is contained. Evaluation of the formula (9) is started with the element T_i^m for which no further element in the series of measurements contains T_i^k .

As a simple example, figure 4 shows two ordered sequences of patterns. The sequences T^1 consists of squares, the sequence T^2 consists of right angled isosceles triangles. Below the images those elements are indicated which contain elements T_i^k .

6.2 Separating shapes by given measures

Instead of the inclusions (7) and (8), also a measure, $\mu(T_i^m)$, to be assigned to the pattern T_i^m , can be used to distinguish by different shapes. The only measure to be used for this purpose are areas. We then obtain the following relationships as the basis on which to

Figure 4. Separating particle shapes by inclusions.

assign particles to the different patterns:

$$\mu(T_i^m) < \mu(T_{i+j}^m) \quad i, j \in \mathbb{N}. \quad (10)$$

Since we take into account only positive and monotonic measures this condition is identical with (7). For assignment to different sequences of patterns one uses

$$\mu(T_i^m) < \mu(T_{l(i, m, n) + j}^n) \quad i, j, l \in \mathbb{N}.$$

The element $T_{l(i, m, n) + 1}^n$ is the smallest element in the sequences T^n whose measure is larger than that of T_i^m .

The class frequencies corrected by shapes are determined in accordance with (9) also in this case.

For the example shown in figure 4 it is outlined in figure 5 which elements have larger areas than an element T_i^k . The example of classification by squares and triangles also indicates another major characteristic of classification by shapes. While for a square of the given size there is only one orientation, the triangle used in principle can be at any of four orientations. Consequently, if it is not known in advance that the triangles in the sample can have only one orientation, all four possible congruent triangles must be used as templates. In the given example this would imply the need to analyze not for two, but for five sequences of patterns. However, if one is not interested in the additional discrimination of triangles of different orientations, the measured results obtained from the four orientations can be combined in the evaluation (Vollath 1982).

Figure 5. Separating particle shapes by given measures.

7. Classification of particles by multi-parameter sequences of patterns

So far we have looked at sequences of patterns T whose elements depended on only one parameter describing the size. Let us now consider sequences T whose elements obey the law of formation

$$T_{i,j} = T_{0,0} + T_{i,0} \oplus T_{0,j}. \quad (11)$$

With

$$T_{i,0} = T_{0,0} \oplus \bigoplus_{m=1}^i T_{1,0} \quad i, j, m, n \in N_0$$

$$T_{0,j} = T_{0,0} \oplus \bigoplus_{m=1}^j T_{0,1}.$$

As in (2) also in this case the set $T_{0,0}$ consists of only one point, normally the origin. The sets $T_{0,1}$ and $T_{1,0}$ are the two basic patterns from which the two-parameter sequence of sets considered is built up. Naturally, also multi-parameter sequences of sets can be built up in accordance with scheme (11) or, as in §6, several multi-parameter sequences of sets can be combined with each other. However, it should be avoided that, as a result of the introduction of too many classes, insufficient, i.e., insignificant, coverage in the individual classes is produced.

Figure 6 shows a simple two-parameter sequence of patterns built up of the unit vectors, e_1 as $T_{1,0}$ and e_2 as $T_{0,1}$. In this example one clearly sees the typical structure of such a two-parameter family of structuring elements. Adding the two diagonal vectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$, one obtains a system of templates which in a first approximation, come close to the size, the axial relationships and the orientations of ellipses approximated by octagons. In a similar way, in a hexagonal grid, one obtains by continued Minkowski additions of the three basic vectors, a system of hexagons to approximate ellipses.

Figure 6. Example of a two-parameter sequence of structural elements.

On the basis of the law of formation of patterns $T_{i,j}$ it is seen that

$$T_{i,j} \subset^* T_{i+m,j+n} \Leftrightarrow \mu(T_{i,j}) < \mu(T_{i+m,j+n})$$

$$i, j \in N, m, n \in N$$

$$m+n > 0.$$

Hence, unlike in the case of classification by several single parameter sequences of patterns, one should not distinguish between two criteria of classification. Since in this case the measurement proceeds analogously to that described above, one obtains, by using the same argument for the corrected class frequencies,

$$h_{i,j} = H_{i,j} - \sum_{m=i+\delta_1} \sum_{n=j+\delta_2} h_{m,n}^* \quad i, j, m, n \in N$$

$$\delta_i \in \{0, 1\} \quad \delta_1 + \delta_2 > 0.$$

Also in this case, calculation of $h_{i,j}^*$ must be started with the largest element. To explain this evaluation in more detail, a simple image (figure 7) has been constructed as an example so that the "particles" it contains can be described by the elements shown in figure 6. Table 2 shows the measured frequencies $H_{i,j}$, from which the class frequencies $h_{i,j}^*$, corrected for shapes and directions were calculated. The $h_{i,j}^*$ values are summarized in table 3.

Another example is depicted in figures 8 and 9 (Vollath 1982). In this example the analysis of pores into more tetrahedral and more spherical shapes is shown. In figure 8 we see the results obtained for the selected templates (squares and triangles) and figure 9 depicts these results after separation by inclusion according to (9) (see also figures 3, 4 and 5).

Figure 7. Example of a sample for classifying particles by shape, size and orientation.

Table 2. Particle numbers obtained in figure 7 after application of the structural elements in a measurement (e.g., erosion) as represented in figure 6. The subscripts i and j have the same meaning as in figure 6.

j	i	1	2	3	4	5	6	7	8
1		36	29	17	11	6	5	3	3
2		25	22	11	8	6	5	3	3
3		15	13	7	3	2	1		
4		11	9	3	2	1			
5		6	6	2	1	1			
6		5	5	1					
7		1	1	1					
8		1	1	1					
9		1	1	1					

Table 3. Particle numbers of figure 7 corrected for size, shape and orientation. The subscripts i and j have the same meaning as in figure 6.

j	i	1	2	3	4	5	6	7	8
1		4	2	2	3				
2		1	4		1		1		3
3				3			1		
4		2	2		2				
5						1			
6			4						
7									
8									
9				1

Figure 8. Separation of pores in a sintered material due to their shapes (separation by inclusion).

Figure 9. Separation of pores in a sintered material due to their shapes (separation by areas).

8. Conclusions

In this paper a few basic considerations have been explained which are necessary for beginning in image analysis. The major point stressed was that before starting any measurement the physical problem imposing an analysis, e.g., particle sizes, must be examined. From this examination we have to derive a behavioural model and develop a measuring technique compatible with it.

It has been shown in this study how to extract measured values from an image, which characterize the individual elements of the image in terms of size, shape and orientation. The basis used throughout was a description of the image and of its elements by means of the methods of the theory of sets. Based on Delfiner's concept of shapes a method of particle classification was developed using erosion, ouverture or template matching as image transformation for classification. A new transformation, "template matching", for particles is explained.

This transformation avoids the disadvantages of erosion or ouverture in case of concave particles. Nevertheless, template matching is also restricted to application of convex templates.

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References

- Delfiner P 1972 *J. Microsc.* **95** 203
- Haas A, Matheron G and Serra J 1967a *Ann. Mines* **XI** 736
- Haas A, Matheron G and Serra J 1967b *Ann. Mines* **XII** 767

- Hadwiger H 1950 *Math. Z.* **53** 210
- Matheron G 1967 *Elements pour une Theorie des Milieux Poreux* (Paris: Masson)
- Müller W 1973 *Leitz Mittlg. Suppl.* **1** 101
- Meschkowski H and Ahrens I 1974 *Theorie der Punktmengen*, Bi-Wissenschaftsverlag, Mannheim
- Nawrath R and Serra J 1979 *Microsc. Acta* **82** 101
- Serra J 1969 *Introduction a la Morphologie Mathematique Cahier du Centre de Morphologie, Mathematique ENSMP No. 3*
- Serra J 1982 *Image analysis and mathematical morphology* (New York: Academic Press)
- Vollath D 1979a *Leitz-Mitt. Wiss. Tech.* **7** 129
- Vollath D 1979b *J. Nucl. Mater.* **81** 115
- Vollath D 1981 *J. Microsc.* **122** 35
- Vollath D 1982 *J. Nucl. Mat.* **10** 71
- Vollath D 1982 *Prakt. Metallogr.* **19** 7, 94