Thermal convection induced by g -jitter in space environment

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Abstract. The article discusses thermal convection in an enclosure induced by spacecraft vibrations (g -jitter). Two theoretical investigations are described. The conclusions are that if the g -jitter is decomposed into a time-mean part and an oscillatory part, the mean part is more important than the oscillatory part, in determining the flow field and heat transfer rate. Under normal circumstances (no manoeuvres, no intentional spinning of the spacecraft) the g -jitter generates predominantly oscillatory velocity and temperature fields with zero time-mean values. The g -jitter can also generate secondary flows with non-zero mean but they are of much smaller order. Some implications of the g -jitter on materials processing in space are also discussed.

Keywords. g -jitter convection ; flows generated ; space processing,

1. Introduction

One of the primary objectives foreseen for processing materials in space is the reduction of natural convection associated with earth's gravity. However there have been some indications (Grodzka *et al* 1971 ; Bannister *et al* 1973) that spacecraft vibrations might cause appreciable thermal convection. Such convection may be important in fluids experiments and also affect the quality of crystals grown in space. Some indications of the effects of g -jitter on convection can be obtained by related research (Richardson 1967 ; Pak *et al* 1970 ; Gershuni *et al* 1970). It was found that vibrations can either substantially enhance or retard local heat transfer, significantly increase total heat transfer and drastically affect convection by altering the transitions from quiescent to laminar flow (critical Rayleigh number) and from laminar to turbulent flow. More detailed discussion of related work is presented in Spradley *et al* (1975).

This phenomenon merits special consideration not only because it represents a convection mode that is suppressed under normal gravitational conditions and which is significant at reduced gravity but also because it is inherent in all spacecraft environments and cannot, therefore, be easily controlled. Therefore to study the effectiveness of spacecraft vibrations in generating fluid flows, the present article discusses two theoretical investigations of a case in which a fluid-filled container with differentially heated walls is subjected to spacecraft vibrations. The article is divided into three parts.

- (i) Brief discussion of numerical analysis for various geometric configuration by Spradley *et al* (1975) and by Forbes (1968) to evaluate effects of g -jitter on fluid motion.
- (ii) Comprehensive theoretical study by the author and his co-workers (Kamotani *et al* 1980) of effects of g -jitter on a fluid filled container with differentially heated walls, in an orbiting spacecraft. The derivation in this part has been reproduced from that given in Kamotani *et al* (1980).
- (iii) Implications of g -jitter for materials processing in space.

2. Numerical analysis

In this work (Spradley *et al* 1975) specific consideration was given to models for evaluating g -jitter effects and they are summarised in this part. These are shown in figure 1 and are a rectangle heated from the side, one heated from below, and a cylinder heated from below. Mercury, helium, and water were chosen as the fluids to span the Prandtl number range from 10^{-2} to 10. Three representative types of g -jitter (see figure 2) were investigated. Numerical solutions of the boundary-value problem were obtained and the results are shown in figures 3 and 4. Isotherms are presented in figure 3 for three cases : (i) rectangular box of water heated from the side, (ii) rectangular box of water heated from below, and (iii) cylindrical container of mercury heated from below. The constant g solutions are for $10^{-3}g$ and the saw tooth g -jitter model is used with the minimum g level being 10^{-3} , the amplitude of g variations is 10^{-3} , and the period is 1 sec. The wall temperatures for the rectangular configurations were 95°C and 25°C with the other walls adiabatic. The 80°C isotherm in figure 3a is seen to be located farther into the water for the g -jitter case than for constant g and the basic shape is the same. The 80°C isotherm in figure 3b has penetrated somewhat farther for g -jitter case. Thus, the g -jitter is shown to influence the temperature distribution significantly.

Figure 1. Configurations used in computer models of g -jitter convection.

Figure 2. g-jitter models used for parametric sensitivity study.

Figure 3. Isotherms at $t = 3$ minutes for three configurations showing effect of g-jitter model.

Figure 4. Streamlines at $t = 3$ minutes showing effect of g-jitter on flow patterns.

For the mercury-filled cylinder heated from below the wall temperatures are 200°C and 25°C with the side wall adiabatic. The effect shown in figure 3c is more drastic. The 200°C isotherm for g -jitter has a different shape. This behaviour is due to the change in the flow from a single cell to a double cell (see figure 4). The streamline patterns (figure 4) give another indication of g -jitter effects. The pattern with g -jitter and the rectangle heated from the side is skewed towards the hot side and the shape is altered. For heating the cylinder with mercury below the effect is even more pronounced as was mentioned above. Thus the flow patterns are also significantly influenced by g -jitter. The examples presented above are representative of all the cases investigated.

Spradley *et al* (1975) also found that if the g -jitter is decomposed into a time-mean part and an oscillatory part, the mean part is more important than the oscillatory part in determining the flow field and heat transfer rate. A somewhat related work has been done by Forbes (1968) where the effect of sinusoidal vibrations on natural convective heat transfer in a rectangular enclosure is studied experimentally as well as numerically under 1- g conditions. The results indicate that the vibrations have very little effect on the heat transfer when the flow is laminar.

3. Effect of spacecraft vibration on thermal convection in a fluid-filled container with differentially heated walls

The theoretical investigation by Kamotani *et al* (1980) is more comprehensive than the one reported by Spradley *et al* (1975). The former is meant to give a better physical insight into the g -jitter in space.

3.1. Formulation of the problem

g-jitter : Consider an experimental set-up which is in a spaceship orbiting the earth (figure 5). The co-ordinates (X_0, Y_0, Z_0) are fixed at the centre of the earth with origin O (fixed frame of reference), and the coordinates (X', Y', Z') are attached to the spacecraft with origin O' at its centre of mass. The coordinates (x, y, z) are attached to the experiment. We suppose that the coordinates (X', Y', Z') are rotating with angular velocity $\vec{\Omega}$ about the origin O' . Then the absolute acceleration of a moving fluid element in the experiment is given by

$$\begin{aligned} \vec{a} = & \frac{D\vec{u}}{Dt} + 2\vec{\Omega} \times \vec{u} + \frac{d\vec{\Omega}}{dt} \times (\vec{R}' + \vec{r}) \\ & + \vec{\Omega} \times [\vec{\Omega} \times (\vec{R}' + \vec{r})] + \vec{g} + \vec{a}_0 + \vec{F}/M, \end{aligned} \quad (1)$$

where $\vec{u}$ and $\vec{r}$ are the velocity and location, respectively, of the element with respect to (x, y, z) , $\vec{a}_0$ is the acceleration of O' with respect to (X_0, Y_0, Z_0) caused by centrifugal and aerodynamic forces on the spacecraft, $\vec{g}$ is the gravitational acceleration which acts towards O and $\vec{F}$ denotes forces applied to the spacecraft whose mass is M .

Figure 5. Coordinate systems.

When $\vec{u}$, $\vec{\Omega}$ and $\vec{F}$ are zero, then the force per unit mass acting on the fluid element is given as

$$\Delta\vec{g} = \vec{g} + \vec{a}_0,$$

$|\Delta\vec{g}|$ is called the reduced-gravity level whose value varies depending on the position in the spacecraft. Because of such variation of $\Delta\vec{g}$ inside the spacecraft, the spacecraft generally spins slowly around its centre of mass, and thruster engines are used to correct its altitude. Other possible sources of external forces are crew motions and on-board machine vibrations. When forces act on the spacecraft, the moving fluid element experiences a body force per unit mass according to (1)

$$\vec{f} = 2\vec{\Omega} \times \vec{u} + \frac{d\vec{\Omega}}{dt} \times (\vec{R}' + \vec{r}) + \vec{\Omega} \times [\vec{\Omega} \times (\vec{R}' + \vec{r})] + \vec{g} + \vec{a}_0 + \vec{F}/M, \quad (2)$$

in addition to the real acceleration $D\vec{u}/Dt$ in the moving frame. $|\vec{f}|$ is termed the g-jitter level.

For the spacecraft to remain in orbit (no manoeuvres) it is necessary that the time-average of the forces $\vec{F}$ must vanish. In the present study the situation in which the spacecraft is intentionally spinning is not considered so that the time-average of $\vec{\Omega}$ is also taken to be zero herein. The third term on the right of (2) represents the centrifugal force due to spacecraft oscillations and it has been shown in (Fichtl and Holland 1978) to be negligible under normal circumstances.

In view of all the above, contributions to the body force, (2) except for the reduced gravity terms must be of an oscillatory nature. Since the reduced gravity level is usually between 10^{-7} to $10^{-9} g_0$, where g_0 is the gravitational acceleration on the surface of the earth, and at any instant of time the oscillatory part of the g -jitter can be as great as $10^{-3} g_0$ (Bannister *et al* 1973), the latter contributes most to the g -jitter. In reality the g -jitter vector $\vec{f}$ changes randomly with time. However, in the present work the g -jitter level is simulated by a sinusoidal function of time superposed on a constant reduced gravity level.

Basic equations : Consider a rectangular container filled with fluid (figure 6). Plate 1 is maintained at temperature θ_1 and plate 2 at θ_2 , and the other walls are thermally insulated. The basic equations with respect to the coordinate system (x, y, z) are as follows.

Continuity equation ;

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0. \quad (3)$$

Momentum equation ;

$$\rho \frac{D\vec{u}}{Dt} = -\nabla p + \mu \nabla^2 \vec{u} + \frac{\mu}{3} \nabla (\nabla \cdot \vec{u}) - \rho \vec{f}. \quad (4)$$

Energy equation ;

$$\rho C_p \frac{D\theta}{Dt} = k \nabla^2 \theta. \quad (5)$$

In general the body force due to g -jitter can act in any direction. However, at first, it is assumed that the g -jitter acts only in the x -direction, normal to the imposed temperature gradient, and thus the flow is considered to be uniform in the z -direction (two-dimensional flow). We consider the g -jitter which is expressed as

$$f_x = A' F(t) + F_m, f_y = f_z = 0,$$

where A' is the amplitude of the oscillatory part and F_m is the mean g -jitter level. If the oscillatory part is caused by a periodic vibration of amplitude a , frequency ω , then $A' = a\omega^2$.

Figure 6. Experiment-fixed coordinate system.

The boundary conditions are the no-slip conditions on the wall surfaces for velocities and specified temperatures at plates 1 and 2 and insulated conditions on the other wall.

Equations (3) to (5) are non-dimensionalized by introducing the following:

$$X = x/L_x, \quad Y = y/L_y, \quad \tau = \omega t, \quad U = u/a\omega, \\ V = \frac{v}{a\omega} \frac{L_x}{L_y}, \quad P = \frac{p}{\rho(a\omega)^2} \frac{a}{L_x}, \quad T = \frac{\theta - \theta_1}{\theta_2 - \theta_1} = \frac{\theta - \theta_1}{\Delta T}.$$

The characteristic velocity $a\omega$ in the x -direction is obtained by equating the dominant body force term and the unsteady term in the x -direction momentum equation, since those are two dominant terms in the equation as long as the Reynolds number is much larger than unity as shown below.

If the pressure in the container is constant (by means of bleed openings), variation of density with temperature is expressed as

$$\rho = \rho_r [1 - \beta(\theta - \theta_r)] \\ = \rho_r [(1 - \beta\Delta T)(T - T_r)], \quad (6)$$

where θ_r is a reference temperature. The coefficient of volumetric expansion β is, in general, of the order of 10^{-2} to 10^{-4} ($1/^\circ\text{C}$), and thus $\beta\Delta T$ is considered to be small. As in natural convection problems (Ostrach 1964), the dependent variables in (3) to (5) are expanded in terms of $\epsilon = \beta\Delta T$, namely

$$\left. \begin{aligned} U &= U_0 + \epsilon U_1, & V &= V_0 + \epsilon V_1, \\ T &= T_0 + \epsilon \frac{a}{L_x} T_1, & P &= P_0 + \epsilon P_1 \end{aligned} \right\}. \quad (7)$$

Substitution of (7) into (3) to (5), followed by the formal process of equating terms of like power in ϵ yields a set of differential equations for each power in ϵ .

It can be shown (Prasad and Ostrach 1978) that the zeroth-order solution is simply a hydrostatic balance, namely $U_0 = V_0 = 0$. This means that the whole fluid moves as a bulk with the container to this order of approximation. The fact that there is no fluid motion in this order of approximation was shown in the drop-tower tests reported by Prasad and Ostrach (1978). The zeroth-order temperature distribution is described by conduction.

$$\frac{\partial T_0}{\partial \tau} = \frac{\nu}{\omega L_x^2} \frac{1}{\text{Pr}} \left[\frac{\partial^2 T_0}{\partial X^2} + \left(\frac{L_x}{L_y} \right)^2 \frac{\partial^2 T_0}{\partial Y^2} \right], \quad (8)$$

where Pr is the Prandtl number. Under steady conditions T_0 is simply expressed as $T_0 = Y$.

Therefore, as in natural convection problems the most important equations are of the order of ϵ . The first-order equations are as follows

$$\frac{\partial U_1}{\partial X} + \frac{\partial V_1}{\partial Y} = 0, \quad (9)$$

$$\frac{DU_1}{D\tau} = -\frac{\partial P_1}{\partial X} + \frac{\nu}{\omega L_x^2} \nabla^2 U_1 + \left(T_0 + \epsilon \frac{a}{L_x} T_1 \right) \left(F(\tau) + \frac{F_m}{a\omega^2} \right), \quad (10)$$

$$\frac{DV_1}{D\tau} = - \left(\frac{L_x}{L_y} \right)^2 \frac{\partial P_1}{\partial Y} + \frac{\nu}{\omega L_x^2} \nabla^2 V_1, \quad (11)$$

$$\frac{DT_1}{D\tau} = \frac{\nu}{\omega L_x^2} \frac{1}{Pr} \nabla^2 T_1 - V_1 \frac{\partial T_0}{\partial Y}, \quad (12)$$

where $D/D\tau = \partial/\partial\tau + \epsilon a/L_x (U_1 \partial/\partial X + V_1 \partial/\partial Y)$,

and $\nabla^2 = \partial^2/\partial X^2 + (L_x/L_y)^2 \partial^2/\partial Y^2$.

The boundary and initial conditions are ;

$$U_1(X, 0, \tau) = U_1(X, 1, \tau) = U_1(0, Y, \tau) = U_1(1, Y, \tau) = 0,$$

$$V_1(X, 0, \tau) = V_1(X, 1, \tau) = V_1(0, Y, \tau) = V_1(1, Y, \tau) = 0,$$

$$T_1(X, 0, \tau) = T_1(X, 1, \tau) = 0,$$

$$\partial T_1(0, Y, \tau)/\partial X = \partial T_1(1, Y, \tau)/\partial X = 0,$$

$$U_1(X, Y, 0) = V_1(X, Y, 0) = T_1(X, Y, 0) = 0.$$

The following significant dimensionless parameters in the problem appear in (10) to (12).

$\omega L_x^2/\nu = \text{Re}$, Reynolds number, ratio of viscous diffusion time to period of oscillation.

Pr , Prandtl number.

$L_x/L_y = A$, aspect ratio.

$\epsilon a/L_x = \epsilon A$ relative amplitude of fluid oscillation.

$F_m/a\omega^2$ ratio of mean g -jitter level to oscillatory level.

The values of Pr for liquid metals are on the order of 10^{-2} , and for water Pr is about 10. This is the range of Pr studied herein. In space flight ω is typically 0.1–10 rad/sec (Fichtl and Holland 1978) and the values of ν for liquids, gases and liquid metals are of the order of 10^{-6} – 10^{-7} m²/sec. Therefore Re is considered to be larger than unity unless one has a very small scale set-up ($L_x, L_y < 1$ cm). The aspect ratio is considered to be unity in the following analysis, but other aspect ratios are also studied in some cases. Since in the present analysis the g -jitter is considered to be predominantly oscillatory, the ratio $F_m/a\omega^2$ is very small. The parameter ϵA is considered to be much smaller than unity. When $A (= a/L_x) < 1$, then $\epsilon A < \epsilon$ so that the terms of order ϵA in (10) to (12) are at most one order magnitude smaller than other terms, and thus those terms are considered to be negligible in the present analysis. On the other hand when $A > 1$, then $\epsilon < \epsilon A \ll 1$ so that the terms of order ϵA are retained as correction terms.

2.2. Analysis

Steady-state analysis: Steady-state solutions are studied first. It is convenient to decompose the velocities, temperature and pressure into the oscillatory parts and the mean parts as follows:

$$\left. \begin{aligned} U_1 &= U'_1 + \epsilon A \bar{U}_1 \\ V_1 &= V'_1 + \epsilon A \bar{V}_1 \\ T_1 &= T'_1 + \epsilon A \bar{T}_1 \\ P_1 &= P'_1 + \epsilon A \bar{P}_1 \end{aligned} \right\}. \quad (13)$$

The over-bar denotes time averages. The equations for the fluctuating and mean flow fields are obtained by substituting (13) into (9) to (12), and following the procedures used to study turbulent flows. The following equations are obtained by noting that under steady-state conditions $T_0 = Y$, and neglecting small order terms.

For fluctuating flow field,

$$\frac{\partial U'_1}{\partial X} + \frac{\partial V'_1}{\partial Y} = 0, \quad (14)$$

$$\frac{\partial U'_1}{\partial \tau} = -\frac{\partial P'_1}{\partial X} + \frac{1}{\text{Re}} \nabla^2 U'_1 + YF(\tau), \quad (15)$$

$$\frac{\partial V'_1}{\partial \tau} = -A_r^2 \frac{\partial P'_1}{\partial Y} + \frac{1}{\text{Re}} \nabla^2 V'_1, \quad (16)$$

$$\frac{\partial T'_1}{\partial \tau} = \frac{1}{\text{Pe}} \nabla^2 T'_1 - V'_1, \quad (17)$$

where $\text{Pe} = \text{RePr}$.

For the mean flow field,

$$\frac{\partial \bar{U}_1}{\partial X} + \frac{\partial \bar{V}_1}{\partial Y} = 0, \quad (18)$$

$$\frac{1}{\text{Re}} \nabla^2 \bar{U}_1 - \frac{\partial \bar{P}_1}{\partial X} = \overline{U'_1 \frac{\partial U'_1}{\partial X}} + \overline{V'_1 \frac{\partial U'_1}{\partial Y}} - \overline{T'_1 F(\tau)} - \frac{1}{\epsilon A} \frac{F_m}{a\omega^2}, \quad (19)$$

$$\frac{1}{\text{Re}} \nabla^2 \bar{V}_1 - A_r^2 \frac{\partial \bar{P}_1}{\partial Y} = \overline{U'_1 \frac{\partial V'_1}{\partial X}} + \overline{V'_1 \frac{\partial V'_1}{\partial Y}}, \quad (20)$$

$$\frac{1}{\text{Pe}} \nabla^2 \bar{T}_1 = \overline{U'_1 \frac{\partial T'_1}{\partial X}} + \overline{V'_1 \frac{\partial T'_1}{\partial Y}} + \bar{V}_1. \quad (21)$$

Since ϵA is considered to be smaller than unity, the oscillatory flow is dominant over the mean flow field according to (13) so that the former is studied first. Analytical solutions to (14) to (17) are difficult to obtain. Since Re is large, the inviscid solution to (14) to (16) may give us some important information on the

velocity field. Introducing the stream function ψ'_1 such that $U'_1 = \partial\psi'_1/\partial Y$, $V'_1 = -\partial\psi'_1/\partial X$ and setting $F(\tau) = \cos \tau$, the inviscid solution is given as

$$\psi'_1 = A_r^2 \psi(X, Y) \sin \tau, \quad (22)$$

$$\text{where } \psi(X, Y) = 1/2 X \left(\frac{1}{8} - X \right) - \frac{4}{\pi^3} \sum_{n=0}^{\infty} \left\{ \frac{(-1)^n \cos [(2n+1)\pi(X-1/2)] \cosh [(2n+1)\pi(Y-1/2)/A_r]}{(2n+1)^3 \cosh [(2n+1)\pi/2A_r]} \right\}.$$

According to (22) ψ'_1 is characterized by $\psi(X, Y)$ spacewise, and by $\sin \tau$ time-wise. The function ψ is plotted in figure 7 for $A_r = 1$. As seen in the figure, the fluid motion is unicellular spacewise.

To obtain the complete solutions the set of (14) to (17) were solved numerically using the ADI method (Roache 1972). The problem was studied in the ranges $Re \leq 1000$, $0.01 \leq Pr \leq 10$ and $A_r = 1$. $F(\tau)$ was taken as $\cos \tau$. After some computation a 11×11 grid system for $Re \leq 200$ and 16×16 for $Re > 200$ were found to be adequate. As for the time step size 32 steps per cycle were found to give stable solutions for all the ranges of the parameters studied herein. Using the numerical results several aspects of the problem are discussed below. The magnitudes of the oscillatory fluid velocities U'_1 and V'_1 , streamfunction ψ'_1 and temperature T'_1 are represented herein by the maximum values in the container, namely $|U'_1, \max|$, $|V'_1, \max|$, $|\psi'_1, \max|$ and $|T'_1, \max|$.

Figure 8 shows the values of $|U'_1, \max|$ and $|\psi'_1, \max|$ for various values of Re and $A_r = 1$. Both increase with Re , and approach those for the above inviscid solution. Beyond $Re = 1000$ those values do not change appreciably. Figure 9 shows the values of $|U'_1, \max|$, $|V'_1, \max|$ and $|\psi'_1, \max|$ for various values of A_r , calculated from the inviscid solution. All the values increase with A_r .

Figure 7. Distribution of inviscid function.

Figure 8. Velocity oscillation level for $Ar = 1$.

Figure 9. Inviscid velocity oscillation levels as function of Ar .

One important effect of viscosity is that it gives rise to phase shifts in the velocity oscillations. Figure 10 shows the velocity changes with time at different Y locations at $X = 0.5$. As seen in the figure the fluid particles at different Y locations are not oscillating synchronously. The phase shift becomes more noticeable towards the wall.

The temperature distribution depends on Pr as well as Re . Figure 11 shows the values of $|T_1', \max|$ for various values of Pr and Re . As seen in (17) when

Pe is very large, $|T_1| \approx |V_1|$. According to figure 7 that happens in the region $Pe \geq 500$ for $A_r = 1$. Below $Pe = 500$ the temperature fluctuation level becomes smaller as Pr decreases for a given value of Re .

As explained above if $\epsilon < \epsilon A \ll 1$ or if the reduced gravity level is important, there are corrections to the above velocity field. Those corrections result in the mean velocity field. The mean flow field is discussed using the numerical results. If the reduced gravity level F_m is on the order of 10^{-7} – $10^{-9} g_0$, $a\omega^2 10^{-3}$ – $10^{-4} g_0$ and ϵA about 0.1, then $(1/\epsilon A) F_m/a\omega^2 < 10^{-2}$ so that the term was considered to be negligible in the numerical analysis. According to (19) the mean flow field is caused by the inertia terms of the fluctuating velocity field and by the body force associated with the fluctuating temperature field. As discussed above when Pr is much less than unity, the temperature oscillation is much smaller than the velocity oscillation so that in that case the mean flow field is mainly caused by the inertia terms. This kind of mean fluid motion is generally called streaming motion (Schlichting 1960). A typical streaming motion in the present problem is presented in figure 12, where $\bar{\psi}_1$ is defined such that $\bar{U}_1 = \partial\bar{\psi}_1/\partial Y$, $\bar{V}_1 = -\partial\bar{\psi}_1/\partial X$. The flow pattern is such that the fluid is pushed out from the corners along the walls, and for $A_r = 1$, as seen in figure 12, a pair of counter-rotating eddies in one gradient are symmetric. For later use we denote $\bar{\psi}_{1,1}$ and $\bar{\psi}_{1,2}$ for the values of $\bar{\psi}_1$ at the centre of the eddies (figure 12). As Pr becomes large, the temperature oscillations become important. It was found that as Pr increases the mean flow field changes in such a way that the value of $\bar{\psi}_{1,1}$ becomes larger than $\bar{\psi}_{1,2}$ and eventually the eddies associated with $\bar{\psi}_{1,2}$ disappear. A typical example is shown in figure 13. The values of $\bar{\psi}_{1,1}$ and $\bar{\psi}_{1,2}$ for various values of Re and Pr are plotted in figure 14. For $Pr = 0.01$ the asymmetry appears around $Re = 300$, and for $Pr = 1$ $\bar{\psi}_{1,2}$ disappears around $Re = 100$.

Figure 10. Phase shifts in velocity oscillations.

Figure 11. Temperature oscillation level for $Ar = 1$.

As for the mean temperature field the ratio of $\bar{T}_1$ to T_0 is $(\epsilon A)^2$ so that $\bar{T}_1$ has to be very large to modify T_0 and thus to change the heat transfer rates at the walls. An inspection of (21) reveals that the value of $\bar{T}_1$ increases with Pr and Re . According to the present numerical calculations the heat transfer rate changes only about 2% from that of conduction at $Pr = 1$ and $Re = 500$. Therefore for the ranges of the parameters studied herein the effect of $\bar{T}_1$ and T_0 is considered to be very small.

Other aspects of the problem: In the above analysis both the temperature distribution (T_0) and g-jitter (f_g) were assumed to be in steady-states. The situations where either one is in a transient state are discussed below.

Figure 12. Symmetrical secondary flow patterns.

Figure 13. Asymmetrical secondary flow patterns.

In the case where the heating starts at $t = 0$ we set $T_0 = 0$ everywhere for $t < 0$, and $T_0 = 1$ at $Y = 1$ for $t \geq 0$. Then the solution to (8) using t instead of τ is given as

$$T_0 = \sum_{n=0}^{\infty} \operatorname{erfc} \frac{2n+1-Y}{2\sqrt{\alpha_t/L_y}} - \sum_{n=0}^{\infty} \operatorname{erfc} \frac{2n+1+Y}{2\sqrt{\alpha_t/L_y}},$$

where α is thermal diffusivity and $\alpha t/L_y^2$ is dimensionless temperature diffusion time. Figure 15 shows the changes of $|U_1, \max|$, $|T_1, \max|$ and $|\psi_1, \max|$ with time

Figure 14. Magnitude of secondary flow.

Figure 15. Velocity and temperature oscillation levels during transient period.

as T_0 develops. The profiles of T_0 at several times are also shown. As seen in the figure the stream function increases monotonically with time indicating that the flow as a whole increases its strength with time. The changes of U_1 and T_1 with time indicate that locally the velocity and temperature oscillation levels go

slightly above the steady-state levels. The same trends were found for other values of Pr and Re .

Another transient case is when T_0 is steady but f_x starts at $t = 0$. An accurate treatment of this case is rather complex because in general f_x does not necessarily start as a well-defined sinusoidal function. The profiles of f_x during the adjustment period depend on how the external forces are applied to the spacecraft. However, barring some large peaks in f_x during the period the velocity and temperature oscillation levels are expected to be of the same orders as those for the steady state.

Another aspect of the problem is the effect of the g -jitter acting parallel to the imposed temperature gradient. Suppose f_y is expressed as $f_y = ay\omega^2 F(\tau)$ [both ω and $F(\tau)$ are considered to be approximately the same as those for f_x], then eliminating the pressure terms from (10) and (11), the body force term in the resulting vorticity equation is given as (F_m is considered to be negligible as before)

$$A^2 \left(\frac{\partial T_0}{\partial Y} + \epsilon A \frac{\partial T_1}{\partial Y} - \epsilon \frac{a_y}{L_x} \frac{\partial T_1}{\partial X} \right) F(\tau).$$

The last term in the parentheses is the contribution from the parallel component. As in the case of $\epsilon a/L_x$, $\epsilon a_y/L_y$ is in general considered to be much less than unity. Therefore, the main body force is still $(\partial T_0/\partial Y) F(\tau)$, that is, the oscillatory field is unaffected by the parallel component. On the other hand the mean flow field is affected by it unless the ratio $a_y L_y/aL_x$ is small.

Probably a more important effect of the parallel component is that it may cause a thermally unstable condition (due to heating from below). Although the critical condition for the onset of thermal instability is known for constant gravity, the critical conditions for oscillatory body forces have not been studied except for the work by Gresho and Sani (1970) in which they studied the critical conditions for constant gravity superimposed by a small oscillatory body force field. The results of the work are not applicable to the present work in which the g -jitter is assumed to be predominantly oscillatory. The stability aspect is left for future work.

4. Implications for materials processing in space

Although the effects of the g -jitter on materials processing in space depend on the specific nature of the process, some general implications are discussed below.

It is important to note that thermal convection in an enclosure associated with spacecraft vibration is predominantly oscillatory. The magnitude of fluid particle displacement is ϵa and thus $\epsilon a/L_x$ is the magnitude relative to the dimension of the container. Since in general $\epsilon a/L_x$ is much less than unity, the fluid particles oscillate in a relatively small region. This and the fact that for materials processing the orders of Pr and Schmidt number are less than ten result in the situation where heat and mass transfer are predominantly governed by conduction and diffusion, respectively, as explained in the previous section. One thing which may be important in materials processing is the temperature oscillation in the container. The order of temperature oscillation is $\epsilon A \Delta T$ or relative to the imposed temperature difference is ϵA . Although this value is small, it may

influence the dynamics of the growth front. The same is true for concentration oscillations if impurities are present.

5. Conclusions

The effects of g-jitter on laminar fluid motions in an enclosure whose two opposing walls are at unequal temperatures were investigated. The following conclusions can be drawn :

- (i) The time-mean component of g-jitter is more important than the oscillatory component in determining the flow-field and heat transfer rate,
- (ii) the important dimensionless parameters in the problems are Re , Pr , A_r , ϵA and $F_m/a\omega^2$,
- (iii) the component of g-jitter normal to the imposed temperature gradient is most important in generating thermal convection. It produces an unicellular oscillatory fluid motion. The oscillatory velocity level increases with Re for given A_r , and is of the order of $\epsilon a\omega$. The temperature oscillation level increases with Re and Pr and is of the order of $\epsilon A \Delta T$,
- (iv) if $\epsilon \triangleleft \epsilon A \ll 1$, the oscillating g-jitter generates a mean secondary flow which is superposed on the oscillatory flow. The secondary flow velocity is of the order of $\epsilon^2 A a\omega$,
- (v) during transient periods both the velocity and temperature oscillation levels are in general of the same orders as those for the steady state, and
- (vi) the g-jitter component parallel to the imposed temperature gradient does not cause appreciable changes in the oscillatory flow field except possibly when the fluid becomes thermally unstable.

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