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Slow relaxation in spin glasses*

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Abstract. The remarkable remanence effects observed in spin glasses are discussed. Some theoretical approaches and results are reviewed.

1. Introduction

In this talk I shall stress the remarkable properties of non-ergodicity exhibited by spin glasses and the theories that are being developed to understand them. Consider a spin glass (say $Au_{0\cdot96}$ $Fe_{0\cdot04}$) at a temperature below the freezing of spin glass temperature Tsg ($\simeq$ 16° K in this case). In equilibrium the bulk magnetisation is essentially zero, indeed neutron scattering studies show that all fourier components of m are zero. If we switch on a d.c. magnetic field, the response of the spin glass is as follows. A (reversible) component of magnetisation is developed immediately. On waiting for a little while, it is found that the magnetisation increases very slowly and saturates only after a very long time (a few hours). The slow response component is the remanent magnetisation and is found in all the known spin glasses. Detailed experiments (see Guy 1975) indicate that the remanence vanishes as Tsg is approached (from below) and hence is a characteristic of the frozen spin glass state. The decay of the magnetisation on switching off the external field is similar; a fast (reversible) component followed by a slow decay of the remanent magnetisation.

The non-exponential decay of the remanence (often fitted to a form ln t by experimentalists) has tempted several workers (e.g. Tholence and Tournier 1974) to adopt the heuristic Neel picture of large super-paramagnetic spin clusters being formed. These clusters are supposed to relax very slowly due to large potential barriers produced by anisotropies. It is hard, at least for me, to understand how this theory would yield a vanishing remanence at Tsg; indeed there is no sharp Tsg in this model.

In this talk, I will adopt the point of view that a sharp transition does exist and that a theory must have this fact built into it. Given this constraint, I am left with essentially two classes of theories to survey. The talk is organised as follows.

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I will give a brief account of the *statics* of the spin glass transition to fix notation. I will then discuss the Glauber Master Equation approach relevant to a random Ising model. I will finally speak about the time-dependent Ginzburg Landau (TDGL) approach. I will content myself with a qualitative description of the results rather than an account of the details.

2. A theoretician's idea of a spin glass

Let us think of the Ising model

$$\mathscr{H} = - \sum_{\langle ij \rangle} J_{ij} S_i S_j \;;\; s_i = \pm 1,$$

where the variables S_i represent say the manganese spins. Let the exchange interaction J_{ij} be a random variable, with some probability distribution $P(J_{ij})$. This can be an idealisation of experiments where Mn spins interact through RKKY interactions, randomness being introduced by random positions of Mn atoms brought about by quenching. The quenched condition implies that physically relevant quantities must be calculated by first computing the *free energy* for a given set of J's and then the configuration average over J's. Models where $P(J)$ is an even function of J lead to vanishing magnetic moment. Frequently used distributions are

$$P(J_{ij}) = a \exp(-J_{ij}^2/2J_0^2) \ \forall_{i,j} \qquad \text{(infinite ranged Gaussian)}$$

$$= c\delta(J_{ij} - J_0) + (1 - C)\delta(J_{ij} + J_0) \qquad (i \text{ nearest neighbour of } j)$$

(Frustration model).

The first model is due to Sherrington and Kirkpatrick (1975) and was solved by Thouless *et al* (1977). It exhibits a phase transition at $T = J_0/k_\beta$. The second is believed to exhibit a transition for sufficiently high dimensions. The transition is characterised by a cusp in the magnetic susceptibility. Edwards and Anderson (1975) introduced a novel order parameter q

$$q = \{\langle S_i \rangle^2\}, \tag{1}$$

where $\{\ \}$ represents a configuration average and $\langle\ \rangle$ the thermodynamic average. The order parameter is $= 0$ for $T > Tc$ and $\neq 0$ for $T < Tc$.

3. Dynamics: The Glauber model

Before launching into the details of the model I should mention that in the spin glass problem, dynamics has played a crucial role in identifying the order parameter equation (1). (We shall not enter the controvertial and "hot" problem of the correctness of equation (1) but I may mention the work of Bray and Moore (1978) and de Almeida and Thouless (1978) which questions the relevance of the above order parameter) Edwards and Anderson (1975) taking cue from μ meson stopping experiments realised that the transition is characterised by a long memory. The two-time autocorrelation function $\chi(t) = \{\langle S_i(t) S_i(o) \rangle\}$ for $t = \infty$ should be non-zero in the ordered phase. Hence they suggested $\chi(\infty)$ (which is clearly equation (1)) as the order parameter.

The Glauber equation for spin dynamics gives the following relaxational equations:

$$\left(1 + \tau \frac{d}{dt}\right) \langle S_i(t) \rangle = \langle \tanh \beta E_i(t) \rangle, \tag{2}$$

and $$\left(1 + \tau \frac{d}{dt}\right) \langle S_i(t) S_i(o) \rangle = \langle \tanh (\beta E_i(t) S_i(o) \rangle, \tag{3}$$

where $$E_i = \sum_j J_{ij} S_j. \tag{4}$$

(We have assumed a particular form for the transition probability; other similar equations are possible but have essentially same physical content). The Glauber model assumes that equilibrium is brought about by single spin flips only and contains τ as a phenomenological constant. Given these equations, one may proceed either by analytical or numerical methods. Kinzel and Fischer (1977) use mean-field theory to analyse these equations near T_c and find

$$\begin{aligned} q(t) &= \exp\left(- t \mid T - T_c \mid / T_c \tau\right) \text{ for } T > T_c \\ &= \frac{1}{Nt}\; T = T_c. \end{aligned} \tag{5}$$

This calculation thus shows a critical slowing down and shows that a transition takes place for $T = T_c$. The calculation for $T < T_c$ is not yet in a satisfactory shape: the difficulty being the combination of randomness (of J's) and nonlinearity (i.e. terms like $\langle S_i \rangle^3$).

Binder and Schröder (1976) have performed Monte-Carlo simulations on the short-ranged version of the Gaussian model. Kirkpatrick and Sherrington (1978) have also used the Monte-Carlo method to study the infinite ranged model. These studies give a slow, non-exponential decay of $q(t)$ and also the magnetisation ($m = \{\langle S_i(t) \rangle\}$) for all $T < T_c$. This is strongly suggestive of a line of critical points, such as is found in the $x - y$ model and its cousins (the interface roughening model, neutral coulomb gas, etc.). The decays are found to have the form

$$m(t) = \tau^{1/2} m_0 / (t + \tau)^{1/2}. \tag{6}$$

In summary numerical studies on the random Glauber model indicate a line of critical points for $T < T_c$ rather than an isolated critical point. Mean field-like treatments, however, fail to yield this picture and give an isolated critical point. Thus phenomena such as remanence cannot be understood in a meanfield theory.

4. Time-dependent Ginzburg-Landau theory

The TDGL equations are similar to master equations but work out at a somewhat coarse-grained (semi-macroscopic) level. Such equations have been used in the context of spin glasses by Ma and Rudnick (1978) with a random axis model. This model is believed to have a spin glass transition but the physics is somewhat unclear, at least to me. Hence I will not go into the details, except to say that their final result for $q(t)$ is similar to that of the Monte-Carlo simulations.

Quite recently, a new approach was found by Kumar and Barma (1978) and Shastry and Shenoy (1978). The approach rests heavily on the work of Thouless *et al* (1977). Thouless *et al* showed that the infinite ranged Gaussian model has a very strange free energy: one has a point of inflexion rather than a minima at equilibrium. They showed that

$$\phi_{TAP} = \tfrac{1}{2}\,\epsilon^2 q^2 - 2/3\,\epsilon\, q^3 + \tfrac{1}{4}\, q^4\ ;\ q \geqslant \epsilon \tag{7}$$

where $\epsilon = (\text{Tsg} - \text{T})/\text{Tsg}$. Thus $\delta\phi/\delta q = \delta^2\phi/\delta q^2 = 0$ for $q = \epsilon$ the equilibrium value. (The constraint $q \geqslant \epsilon$ rules out the spurious $q = 0$ solution for $T < \text{Tsg}$.) The second derivative being zero is unusual and suggests weak restoring forces and hence slow relaxation. The formalism necessary to implement this idea into dynamics is precisely the TDGL equations. Here we postulate

$$\tau_q \dot{q} = -\frac{\partial\phi}{\partial q}(q, m) + \zeta_q(t), \tag{8}$$

$$\tau_m \dot{m} = -\frac{\partial\phi}{\partial m}(q, m) + \zeta_m(t), \tag{9}$$

where ϕ is a generalisation of the free energy ϕ_{TAP} to include magnetisation m (equation (8) with $M = 0$ is due to Kumar and Barma and equation (9) to us). The random forces ζ_q and ζ_m can be dropped in the present problem (q and m being macrovariables these forces are $O(N^{-1/2})$). τ_q and τ_m are phenomenological relaxation times.

The function ϕ must satisfy several requirements. Symmetry arguments give us the form:

$$\phi(q, m) = \phi_{TAP}(q) + bqm^2 + cq^2 m^2 - mh, \tag{10}$$

where b and c are constants. Using (10) in (8) and (9) we find coupled differential equations. The solutions are indicated below. We get for all $T \leqslant \text{Tsg}$

$$\begin{aligned} q(t) &\propto \frac{1}{(t+\tau)^{1/2}},\quad t_1 > t > 0,\\ &\propto \frac{1}{t+\tau'},\quad t_2 > t > t_1,\\ &\propto \exp(-t/\tau''),\quad t > t_2, \end{aligned} \tag{11}$$

and $\quad m(t) = m(\text{fast}) + m(\text{slow}) \qquad (12)$

where $\quad m(\text{slow}) \propto q(t) \qquad (13)$

and $\quad m(\text{fast}) \propto \exp(-t/\tau). \qquad (14)$

This model makes detailed predictions about the field dependance of the remanence which can be tested.

5. Conclusions and comments

We have seen that Monte-Carlo studies on theoretical models have features which are similar to real spin glasses. The Thouless–Anderson–Palmer free energy of a spin glass exhibits an inflexion point which could be the key to the observed behaviour.

I should like to mention a new method that Girish Agarwal and myself are trying out at Hyderabad. Here the basic observation is that in a distribution function (unlike in a partition function), the random variables (Js) can be averaged over, using standard (projection operator) techniques. Thus a master equation (Fokker–Planck) is considered for a continuous spin model, random Js averaged out and a non-markovian master equation for the averaged distribution function obtained. Thus hopefully, a purely dynamical approach to spin glasses may be taken (avoiding n 0 and other tricks).

In conclusion I should say that the problems in spin glasses are quite unique in that even meanfield theory is non-trivial. Thus our quantitative understanding of these systems may saturate non-exponentially in time ($\sim$ a few years).

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