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Profit rate performance optimization for a generalized irreversible combined refrigeration cycle

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Abstract. Finite-time exergoeconomic performance of a Newtonian heat transfer law system generalized irreversible combined refrigeration cycle model with finite-rate heat transfer, heat leakage and internal irreversibility is presented in this paper. The operation of the generalized irreversible combined refrigeration cycle is viewed as a production process with exergy as its output. The performance optimization of the cycle is performed by taking profit as the objective. The optimal profit rate, optimal COP (coefficient of performance), as well as the relation between the optimal profit rate and COP of the cycle are derived. The focus of this paper is to obtain the compromise optimization between economics (profit rate) and the energy utilization factor (COP) for the cycle, by searching the optimum COP at maximum profit rate, which is termed as the finite time exergoeconomic performance bound. Moreover, the effects of various factors, including heat leakage, internal irreversibility and the price ratio, on the profit rate performance of the cycle are analysed by detailed numerical examples.

Keywords. Finite-time thermodynamics; generalized irreversible combined refrigeration cycle; exergoeconomic performance; generalized thermodynamic optimization.

1. Introduction

Since finite time thermodynamics and entropy generation minimization has been advanced (Andresen 1983; Bejan 1996; Berry *et al* 1999; Chen *et al* 1999a; Chen & Sun 2004; Chen 2005; Durmayaz *et al* 2004; Sieniutycz & Salamon 1990), the research into identifying the performance limits of thermodynamic processes and optimization of thermodynamic cycles has been made a tremendous progress by scientists and engineers. The objective functions in finite time thermodynamics are often pure thermodynamic parameters including power, efficiency, entropy production, effectiveness, cooling load, heating load, coefficient of performance (COP), loss of exergy, etc. Berry & Salamon (1978) carried out the economic optimization of the heat engine with the maximum profit rate as the objective function.

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Salamon & Nitzan (1981) viewed the operation of the endoreversible heat engine as a production process with work as its output.

A relatively new method that combines exergy with conventional concepts from long-run engineering economic optimization to evaluate and optimize the design and performance of energy systems is exergoeconomic (or thermoeconomic) analysis (Tsatsaronts 1993; El-Sayed 2003). Salamon & Nitzan (1981) combined the endoreversible model with exergoeconomic analysis. It was termed as finite time exergoeconomic analysis to distinguish it from the endoreversible analysis with pure thermodynamic objectives and the exergoeconomic analysis with long-run economic optimization (Chen *et al* 1991, 1996, 2001, 2004a; Wu *et al* 1996, 1998; Wu *et al* 2000; Zheng *et al* 2006). Similarly, the performance bound at maximum profit rate was termed as finite time exergoeconomic performance bound to distinguish it from the finite time thermodynamic performance bound at maximum thermodynamic output. A similar idea was provided by Ibrahim *et al* (1992), De Vos (1995, 1997) and Bejan (1993). De Vos (1995, 1997) used the basic idea of finite time thermodynamics into the thermoeconomics for heat engine in which the heat transfer between the working fluid and the heat reservoirs obeys Newtonian heat transfer law, derived the relation between the optimal efficiency and economic returns. Chen *et al* (2005) investigated the endoreversible thermoeconomic performance of heat engine with the linear phenomenological heat transfer law based on the work of De Vos (1995). Scholars provided a new thermoeconomic optimization criterion, thermodynamic output rates (power, cooling load or heating load for heat engine, refrigerator or heat pump) per unit total cost, investigated the performances of endoreversible heat engine (Sahin & Kodal 2001), refrigerator and heat pump (Sahin & Kodal 1999), combined cycle refrigerator (Sahin & Kodal 2002), combined cycle heat pump (Kodal *et al* 2000b), as well as irreversible heat engine (Kodal & Sahin 2003), refrigerator and heat pump (Kodal *et al* 2000a), combined cycle refrigerator (Sahin *et al* 2001), combined cycle heat pump (Kodal *et al* 2002), and three-heat- reservoir absorption refrigerator and heat pump (Kodal *et al* 2003). This method was also applied to the optimization of an endoreversible four-heat-reservoir absorption-refrigerator by Qin *et al* (2005).

Some authors have analysed the influences of heat resistance (Chen & Yan 1988; Chen *et al* 1995), heat resistance and internal irreversibility (Goktun 1996), and heat resistance and heat leakage (Chen *et al* 1997) on the performance of combined refrigeration cycles. Chen *et al* (1999b) established a generalized irreversible combined refrigeration cycle model with heat resistance, heat leakage and internal irreversibility, and derived the analytical formulae of optimal cooling load and COP of the refrigeration cycle model. A further step made in this paper is to study the finite time exergoeconomic performance of the generalized irreversible combined refrigeration cycle. The analytical formulae about optimal profit rate, optimal COP and relation between the optimal profit rate and COP of the cycle with Newtonian heat transfer law are derived. The aim of this paper is to obtain the compromise optimization between economics (profit rate) and the energy utilization factor (COP) for the cycle, by searching the optimum COP at maximum profit rate which is termed as the finite time exergoeconomic performance bound. Moreover, analysis and optimization of the model are carried out in order to investigate the effects of cycle parameters on the performance of the cycle using numerical examples.

2. Model of the combined refrigeration cycle (Chen *et al* 1999b)

A combined refrigeration cycle formed by two steady irreversible Carnot refrigeration cycles and its surrounding heat reservoirs are shown in figure 1. The combined cycle operates between

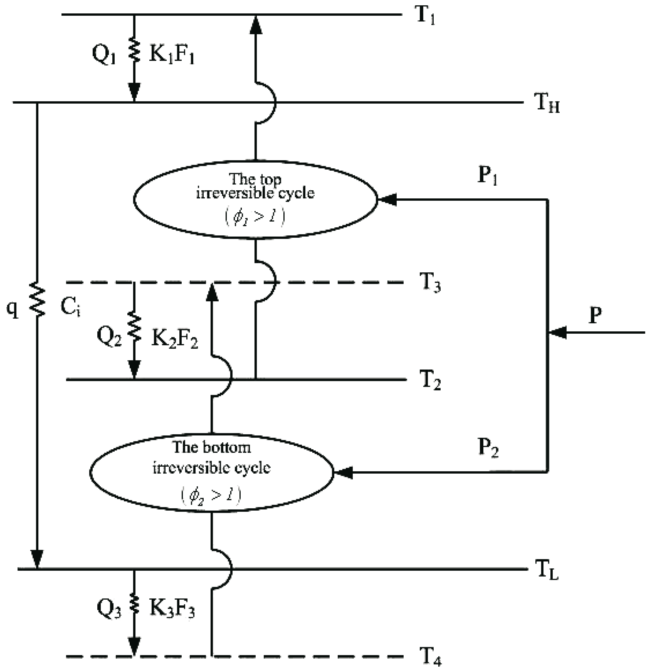

Figure 1. Generalized irreversible combined Carnot refrigeration cycle model.

two reservoirs at temperatures T_H and T_L $(T_H > T_L)$. The temperatures of the working fluid in the top irreversible cycle are T_1 and T_2, when heat leaves and enters the working fluid. In the bottom irreversible cycle, they are T_3 and T_4. These temperatures are related to one another in the following order:

$$T_1 > T_H > T_3 > T_2 > T_L > T_4. \tag{1}$$

Heat exchange between the two irreversible cycles is carried out directly through the same heat transfer surface area so that the quantity of heat absorbed by the working fluid in the top cycle is equal to that rejected by the working fluid in the bottom cycle. The irreversibilities are the constant rate of bypass heat leak q, defined by Bejan (1988, 1989), which flows from the heat sink at temperature T_H to the heat source at temperature T_L, the internal dissipation of the working fluids in the top and bottom cycles, and the three heat transfer losses caused by the three finite temperature differences $(T_1 - T_H)$, $(T_3 - T_2)$ and $(T_L - T_4)$ required by the heat rejection and absorption rates.

The first law of thermodynamics requires

$$Q_H = Q_1 - q,\ R = Q_L = Q_3 - q, \tag{2}$$

where Q_1 and Q_3 are the rates of heat flow from the top cycle to the heat sink in the high-temperature side heat exchanger and from the heat source to the bottom cycle in the low-temperature side heat exchanger of the combined cycle, respectively, Q_H and Q_L are the heat transfer rate to the heat sink and the heat removed rate from the heat source (i.e. cooling load, R, for convenience), respectively.

The internal dissipation including miscellaneous factors such as friction, turbulence and non-equilibrium activities inside the two cycles are characterized by two constant coefficients

(Chen *et al* 1999b):

$$\phi_1 = Q_1/Q'_1 \geq 1, \phi_2 = Q_2/Q'_2 \geq 1, \tag{3}$$

where Q_2 is the rate of heat exchange between the top and bottom irreversible cycles in the intermediate heat exchanger, Q'_2 is the rate of heat absorbed from the bottom endoreversible cycle to the top irreversible cycle in the intermediate heat exchanger, and Q'_1 is the rate of heat rejected from the top endoreversible cycle to the heat sink in the high-temperature side heat exchanger. Because the power input required by the irreversible refrigeration cycle is larger than that of the endoreversible one with the same cooling load, the rate of heat rejected from the warm working fluid to the heat sink for the irreversible cycle is larger than that for the endoreversible one. Thus, $Q_1 > Q'_1$ and $Q_2 > Q'_2$ hold. For the top endoreversible refrigeration cycle, the second law of thermodynamics requires that (no matter whether the bottom refrigeration cycle is endoreversible)

$$Q_2/Q'_1 = T_2/T_1. \tag{4}$$

For the bottom endoreversible refrigeration cycle, the second law of thermodynamics requires that (no matter whether the top refrigeration cycle is endoreversible)

$$Q_3/Q'_2 = T_4/T_3. \tag{5}$$

The power input of the irreversible combined refrigeration cycle is:

$$P = P_1 + P_2 = Q_H - Q_L = Q_1 - Q_3, \tag{6}$$

where P_1, P_2 and P are the power inputs to the top and bottom cycles and the total power input to the irreversible combined refrigeration cycle.

The COP of the irreversible combined refrigeration cycle is:

$$\varepsilon = Q_L/P = (1 - q/Q_3)/(Q_1/Q_3 - 1) \tag{7}$$

Assuming that the rate of heat flow in the heat exchangers follows the Newtonian law, one has

$$Q_1 = (T_1 - T_H)K_1F_1 = (T_1 - T_H)U_1 \tag{8}$$

$$Q_2 = (T_3 - T_2)K_2F_2 = (T_3 - T_2)U_2 \tag{9}$$

$$Q_3 = (T_L - T_4)K_3F_3 = (T_L - T_4)U_3, \tag{10}$$

where K_1 is the overall heat transfer coefficient and F_1 is the heat transfer surface area of the high-temperature (hot) side heat exchanger between the top cycle and the heat sink, K_2 is the overall heat transfer coefficient and F_2 is the heat transfer surface area of the intermediate heat exchanger between the top and bottom cycles, and K_3 is the overall heat transfer coefficient and F_3 is the heat transfer surface area of the low-temperature (cold) side heat exchanger between the bottom cycle and the heat source. U is the heat exchanger inventory: $U_1 = K_1F_1$, $U_2 = K_2F_2$, and $U_3 = K_3F_3$.

The total heat transfer surface area (F_T) of the combined cycle is assumed to be a constant

$$F_1 + F_2 + F_3 = F_T. \tag{11}$$

3. Analysis and optimization

Combining Eqs. (2)–(10), one has

$$Q_1 = \frac{\phi_T T_H (R+q)}{T_L - (R+q)[(K_1F_1/\phi_T)^{-1} + (K_2F_2/\phi_2)^{-1} + (K_3F_3)^{-1}]}, \tag{12}$$

where ϕ_T is the total degree of internal dissipation in the combined system,

$$\phi_T = \phi_1\phi_2. \tag{13}$$

Substituting Eqs. (8), (10) and (12) into Eq. (7) yields (Chen *et al* 1999b)

$$\varepsilon = \frac{R}{R+q}\left\{\frac{\phi_T T_H}{T_L - (R+q)[(K_1F_1/\phi_T)^{-1} + (K_2F_2/\phi_2)^{-1} + (K_3F_3)^{-1}]} - 1\right\}^{-1}. \tag{14}$$

Eq. (14) provides a general relation between the COP and the cooling load of the irreversible combined refrigeration cycle.

Assuming the environmental temperature is T_0, the rate of exergy input of the refrigeration cycle is:

$$A_{\text{rev}} = Q_L(T_0/T_L - 1) - Q_H(T_0/T_H - 1) = Q_L\eta_2 - Q_H\eta_1, \tag{15}$$

where η_i is the Carnot coefficient of the heat reservoir i, and defined as $\eta_1 = T_0/T_H - 1$ and $\eta_2 = T_0/T_L - 1$.

Assuming that the prices of exergy output rate and the power input be ψ_1 and ψ_2, the profit rate of the refrigeration cycle is:

$$\pi = \psi_1 A_{\text{rev}} - \psi_2 P. \tag{16}$$

Substituting Eqs. (2), (6), (9), (14) and (15) into Eq. (16) yields

$$\pi = \kappa_2 R + \kappa_1 q - \frac{\kappa_1\phi_T T_H (R+q)}{T_L - (R+q)[(K_1F_1/\phi_T)^{-1} + (K_2F_2/\phi_2)^{-1} + (K_3F_3)^{-1}]}, \tag{17}$$

where $\kappa_1 = (\psi_1\eta_1 + \psi_2)$ and $\kappa_2 = (\psi_1\eta_2 + \psi_2)$.

Obviously, both the COP (ε) and profit rate (π) of the combined refrigeration cycle are functions of F_1, F_2 and F_3 for fixed T_H, T_L, ψ_1, ψ_2, η_1, η_2, ϕ_1, ϕ_2, K_1, K_2, K_3, q and R.

With the constraint of Eq. (11), one may obtain, when

$$\left(\frac{F_1}{F_T}\right)_{\text{opt}} = \left(\frac{\phi_T}{K_1}\right)^{\frac{1}{2}}\left[\left(\frac{1}{K_3}\right)^{\frac{1}{2}} + \left(\frac{\phi_2}{K_2}\right)^{\frac{1}{2}} + \left(\frac{\phi_T}{K_1}\right)^{\frac{1}{2}}\right]^{-1} \tag{18}$$

$$\left(\frac{F_2}{F_T}\right)_{\text{opt}} = \left(\frac{\phi_2}{K_2}\right)^{\frac{1}{2}}\left[\left(\frac{1}{K_3}\right)^{\frac{1}{2}} + \left(\frac{\phi_2}{K_2}\right)^{\frac{1}{2}} + \left(\frac{\phi_T}{K_1}\right)^{\frac{1}{2}}\right]^{-1} \tag{19}$$

$$\left(\frac{F_3}{F_T}\right)_{\text{opt}} = \left(\frac{1}{K_3}\right)^{\frac{1}{2}}\left[\left(\frac{1}{K_3}\right)^{\frac{1}{2}} + \left(\frac{\phi_2}{K_2}\right)^{\frac{1}{2}} + \left(\frac{\phi_T}{K_1}\right)^{\frac{1}{2}}\right]^{-1} \tag{20}$$

the optimal COP and profit rate of the combined refrigeration cycle are in the following forms, respectively

$$\varepsilon = \frac{(T_L - qB^*)R - B^*R^2}{B^*R^2 + (\phi_T T_H - T_L + 2qB^*)R + (\phi_T T_H - T_L + qB^*)q} \tag{21}$$

$$\pi = \psi_1 q(\eta_1 - \eta_2) + \frac{-\kappa_2 B^* R^2 + (\kappa_2 T_L - 2\kappa_2 B^* q - \kappa_1 \phi_T T_H)R + (\kappa_2 T_L - \kappa_2 B^* q - \kappa_1 \phi_T T_H)q}{T_L - (R+q)B^*} \tag{22}$$

where $B^* = [(1/K_3)^{1/2} + (\phi_2/K_2)^{1/2} + (\phi_T/K_1)^{1/2}]^2/F_T$.

Combining Eqs. (21) with (22), by eliminating the cooling load R, one can obtain the relation between the optimal COP and profit rate of the combined cycle in the following forms

$$\{[\varepsilon(\kappa_2 - \kappa_1) - \kappa_1]T_L - [\pi\varepsilon + q\varepsilon(\kappa_2 - \kappa_1) - q\kappa_1]B^*\}[\pi\varepsilon + q\varepsilon(\kappa_2 - \kappa_1) - q\kappa_1 + \pi] - \phi_T T_H[\pi\varepsilon + q\varepsilon(\kappa_2 - \kappa_1) - q\kappa_1][\varepsilon(\kappa_2 - \kappa_1) - \kappa_1] = 0. \tag{23}$$

From Eq. (22), one may obtain that $\pi = \psi_1 q(\eta_1 - \eta_2)$ when $R = [\kappa_1\phi_T T_H - \kappa_2(T_L - qB^*)]/\kappa_2 B^*$ and $R = -q$. It can be proved using Rolle's Theorem that there exists a maximum profit rate point. To find the maximum profit rate, taking the derivation of π with respect to R and setting it equal to zero ($d\pi/dR = 0$), one can obtain the maximum profit rate ($\pi_{\max}$) and the corresponding cooling load R_π as follows.

$$\pi_{\max} = \psi_1 q(\eta_1 - \eta_2) + \frac{[T_L - (\kappa_1\phi_T T_H T_L/\kappa_2)^{\frac{1}{2}}][(\kappa_1\kappa_2\phi_T T_H T_L)^{\frac{1}{2}} - \kappa_1\phi_T T_H]}{B^*(\kappa_1\phi_T T_H T_L/\kappa_2)^{\frac{1}{2}}} \tag{24}$$

$$R_\pi = \frac{T_L - (\kappa_1\phi_T T_H T_L/\kappa_2)^{\frac{1}{2}}}{B^*} - q. \tag{25}$$

Substituting Eq. (25) into Eq. (21) gives ε_π, which is the finite-time exergoeconomic bound of the generalized irreversible combined refrigeration cycle:

$$\varepsilon_\pi = \frac{(\kappa_1\phi_T T_H T_L/\kappa_2)^{\frac{1}{2}}[T_L - qB^* - (\kappa_1\phi_T T_H T_L/\kappa_2)^{\frac{1}{2}}]}{[T_L - (\kappa_1\phi_T T_H T_L/\kappa_2)^{\frac{1}{2}}][\phi_T T_H - (\kappa_1\phi_T T_H T_L/\kappa_2)^{\frac{1}{2}}]}. \tag{26}$$

4. Discussions

If $\phi_1 = \phi_2 = 1$, Eqs. (21), (22), (25), (24) and (26) become

$$\varepsilon = \frac{(T_L - qB_1^*)R - B_1^*R^2}{B_1^*R^2 + (T_H - T_L + 2qB_1^*)R + (T_H - T_L + qB_1^*)q} \tag{27}$$

$$\pi = \psi_1 q(\eta_1 - \eta_2) + \frac{(R+q)\{\kappa_2[T_L - (R+q)B_1^*] - \kappa_1 T_H\}}{T_L - (R+q)B_1^*} \tag{28}$$

$$R_\pi = \frac{T_L - (\kappa_1 T_H T_L/\kappa_2)^{\frac{1}{2}}}{B_1^*} - q \tag{29}$$

$$\pi_{\max} = \psi_1 q(\eta_1 - \eta_2) + \frac{[T_L - (\kappa_1 T_H T_L/\kappa_2)^{\frac{1}{2}}][(\kappa_1\kappa_2 T_H T_L)^{\frac{1}{2}} - \kappa_1 T_H]}{B_1^*(\kappa_1 T_H T_L/\kappa_2)^{\frac{1}{2}}} \tag{30}$$

$$\varepsilon_\pi = \frac{(\kappa_1 T_H T_L/\kappa_2)^{\frac{1}{2}}[T_L - qB_1^* - (\kappa_1 T_H T_L/\kappa_2)^{\frac{1}{2}}]}{[T_L - (\kappa_1 T_H T_L/\kappa_2)^{\frac{1}{2}}][T_H - (\kappa_1 T_H T_L/\kappa_2)^{\frac{1}{2}}]}, \tag{31}$$

where $B_1^* = [(1/K_1)^{1/2} + (1/K_2)^{1/2} + (1/K_3)^{1/2}]^2/F_T$.

Eq. (31) is the finite-time exergoeconomic performance bound of the irreversible combined refrigeration cycle with losses of heat resistance and heat leakage.

If $\phi_T > 1$ and $q = 0$, Eqs. (25), (24) and (26) becomes

$$R_\pi = \frac{T_L - (\kappa_1 \phi_T T_H T_L/\kappa_2)^{\frac{1}{2}}}{B^*} \tag{32}$$

$$\pi_{\max} = \frac{[T_L - (\kappa_1 \phi_T T_H T_L/\kappa_2)^{\frac{1}{2}}][(\kappa_1\kappa_2 \phi_T T_H T_L)^{\frac{1}{2}} - \kappa_1 \phi_T T_H]}{B^*(\kappa_1 \phi_T T_H T_L/\kappa_2)^{\frac{1}{2}}} \tag{33}$$

$$\varepsilon_\pi = \frac{(\kappa_1 \phi_T T_H T_L/\kappa_2)^{\frac{1}{2}}}{\phi_T T_H - (\kappa_1 \phi_T T_H T_L/\kappa_2)^{\frac{1}{2}}}. \tag{34}$$

Eq. (34) is the finite-time exergoeconomic performance bound of the irreversible combined refrigeration cycle with losses of heat resistance and internal irreversibility.

If $\phi_T = 1$ and $q = 0$, Eqs. (23), (25), (24) and (26) become

$$\{[\varepsilon(\kappa_2 - \kappa_1) - \kappa_1]T_L - \pi\varepsilon B_1^*\}(\pi\varepsilon + \pi) - T_H\pi\varepsilon[\varepsilon(\kappa_2 - \kappa_1) - \kappa_1] = 0 \tag{35}$$

$$R_\pi = \frac{T_L - (\kappa_1 T_H T_L/\kappa_2)^{\frac{1}{2}}}{B_1^*} \tag{36}$$

$$\pi_{\max} = \frac{[T_L - (\kappa_1 T_H T_L/\kappa_2)^{\frac{1}{2}}][(\kappa_1\kappa_2 T_H T_L)^{\frac{1}{2}} - \kappa_1 T_H]}{B_1^*(\kappa_1 T_H T_L/\kappa_2)^{\frac{1}{2}}} \tag{37}$$

$$\varepsilon_\pi = \frac{(\kappa_1 T_H T_L/\kappa_2)^{\frac{1}{2}}}{T_H - (\kappa_1 T_H T_L/\kappa_2)^{\frac{1}{2}}}, \tag{38}$$

where $B_1^* = [(1/K_1)^{1/2} + (1/K_2)^{1/2} + (1/K_3)^{1/2}]^2/F_T$.

Eqs. (38) is the finite-time exergoeconomic performance bound of the endoreversible combined refrigeration cycle.

The finite-time exergoeconomic performance bound at the maximum profit rate is different from the classical reversible bound and the finite-time thermodynamic bound at the maximum cooling load. It is dependent on T_H, T_L, T_0, ϕ_1, ϕ_2 and ψ_2/ψ_1. Note that for the process to be potential profitable, the following relationship must exist: $0 < \psi_2/\psi_1 < 1$, because one unit of power input must give rise to at least one unit of exergy output rate.

As the price of exergy output rate becomes very large compared with that of the power input, i.e. $\psi_2/\psi_1 \to 0$, Eq. (22) becomes

$$\pi = \psi_1(\eta_1 q + \eta_2 R) - \frac{\psi_1 \eta_1 \phi_T T_H}{T_L/(R + q) - B^*}. \tag{39}$$

When $T_H \to T_o$, Eq. (39) becomes $\pi = \psi_1 \eta_2 R$. It indicates that the profit rate maximization approaches the cooling load maximization (Chen *et al* 1999b).

On the other hand, as the price of exergy output rate approaches the price of the power input, i.e. $\psi_2/\psi_1 \to 1$, equation (22) becomes

$$\pi = -\psi_1 T_o \left\{ \frac{\phi_T T_H (R+q) - q[T_L - (R+q)B^*]}{T_H [T_L - (R+q)B^*]} - \frac{R}{T_L} \right\} = -\psi_1 T_o \sigma, \qquad (40)$$

where σ is entropy production rate of the combined refrigeration cycle. That is the profit rate maximization approaches the entropy production rate minimization, in other word, the minimum exergy loss. Equation (40) indicates that the combined refrigeration cycle is not profitable regardless of the COP at which the combined refrigeration cycle is operating. Only the combined refrigeration cycle is operating reversibly ($\varepsilon_\pi = \varepsilon_C = T_L/(T_H - T_L)$) will the revenue equal to the cost, and then the maximum profit rate will be equal to zero. The corresponding entropy production rate is also zero.

Therefore, for any intermediate values of ψ_2/ψ_1, the finite-time exergoeconomic performance bound (ε_π) lies between the finite-time thermodynamic performance bound and the reversible performance bound. ε_π is related to the latter two through the price ratio ψ_2/ψ_1.

For a combined refrigeration cycle, which is formed by several irreversible Carnot refrigeration cycles, with losses of heat resistance, bypass heat leakage and internal irreversibilities, the optimal performance characteristics have the same expressions as equations (21)–(26), simply replace B^* by B_2^*.

$$B_2^* = \left\{ \frac{1}{F_T} \sum_{i=1}^{n+1} \left(\prod_{j=i}^{n+1} \phi_j / K_i \right)^{1/2} \right\}^2, \qquad (41)$$

where n is the number of stages of the combined refrigeration cycle, K_i is the heat transfer coefficient for the ith stage heat exchanger between working fluids or between working fluid and the reservoirs, and ϕ_j is the internal dissipation for the jth stage refrigeration cycle and $\phi_{n+1} = 1$ is taken.

For the combined cycle, the optimization problem can be extended by distributing the total heat exchanger inventory U_T (Bejan 1988, 1989; Chen *et al* 2004b), which is assumed to be a constant $U_T = U_1 + U_2 + U_3$.

Using the above constraint, the optimal distribution of the heat exchanger inventory is obtained in the following expressions:

$$(U_1/U_T)_{\text{opt}} = \phi_T^{1/2}/(1 + \phi_2^{1/2} + \phi_T^{1/2}) \qquad (42)$$

$$(U_2/U_T)_{\text{opt}} = \phi_2^{1/2}/(1 + \phi_2^{1/2} + \phi_T^{1/2}) \qquad (43)$$

$$(U_3/U_T)_{\text{opt}} = 1/(1 + \phi_2^{1/2} + \phi_T^{1/2}). \qquad (44)$$

The optimal performance characteristics have the same expressions as equations (21)–(26), again, simply replace B^* by B_3^*.

$$B_3^* = (1 + \phi_2^{1/2} + \phi_T^{1/2})/U_T. \qquad (45)$$

For a combined cycle formed by n stages of irreversible refrigeration cycles, B^* should be replaced by B_4^*.

$$B_4^* = \frac{1}{U_T}\left\{\sum_{i=1}^{n+1}\left[\prod_{j=i}^{n+1}\phi_j\right]^{1/2}\right\}^2. \tag{46}$$

5. Numerical examples

To illustrate the preceding analysis, numerical examples are provided. In the calculations, it is set that $T_H = 300\,\text{K}$, $T_L = 260\,\text{K}$, $T_0 = 290\,\text{K}$, $K_1 = K_2 = K_3$, $K_1F_T = 10\,\text{kW}/K$, $\psi_1 = 1000\,\text{yuan/kW}$, $\phi_1 = \phi_2 = 1$ or $1{\cdot}1$, and $q = C_i(T_H - T_L)$ (same as the work of Bejan (1988, 1989)) where $C_i = 0$ or $0{\cdot}04\,\text{kW/K}$, C_i is the thermal conductance inside the combined refrigeration cycle.

Figure 2 shows the effects of ψ_2/ψ_1 on the optimal profit rate π versus the cooling load R characteristic with $C_i = 0{\cdot}04$ kW/K and $\phi_1 = \phi_2 = 1{\cdot}1$. It illustrates that the profit rate π is a monotonic increasing function of cooling load R with $0 \le \psi_2/\psi_1 \le 0{\cdot}033$, which indicates that the refrigerator is profitable regardless of the cooling load is at which the refrigerator is operating. When $0{\cdot}033 < \psi_2/\psi_1 < 0{\cdot}393$, the curves of $\pi - R$ are parabolic-like ones, which means there exists the maximum profit rate point. The optimal profit rate π is a monotonic decreasing function of cooling load R when $\psi_2/\psi_1 \ge 0{\cdot}393$, which indicates that the refrigerator is not profitable regardless of the cooling load is at which the refrigerator is operating.

Figure 3 shows the relation between the optimal profit rate π and COP ε of the combined refrigeration cycle with heat resistances and heat leakage losses. It can be seen that the curves of $\pi - \varepsilon$ change quantitatively and qualitatively due to the change of ψ_2/ψ_1. The qualitative differences are as following: when $0 \le \psi_2/\psi_1 \le 0{\cdot}033$, the curves of $\pi - \varepsilon$ are hyperbola-like ones, and the refrigerator is profitable regardless of the cooling load is at

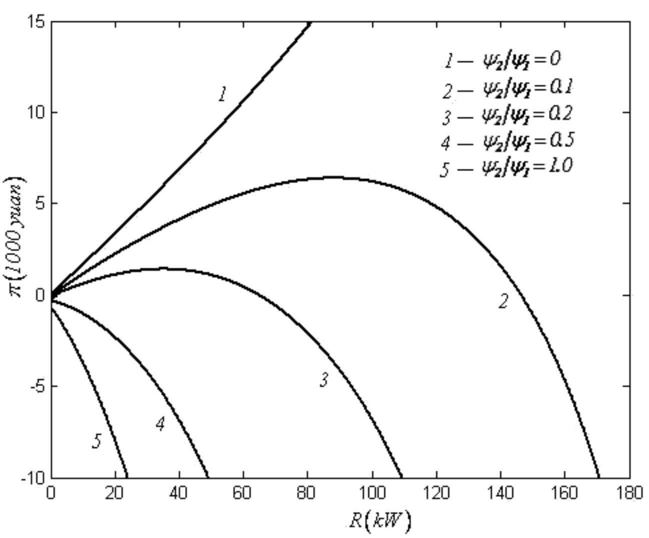

Figure 2. Effect of price ratio ψ_2/ψ_1 on π–R characteristic.

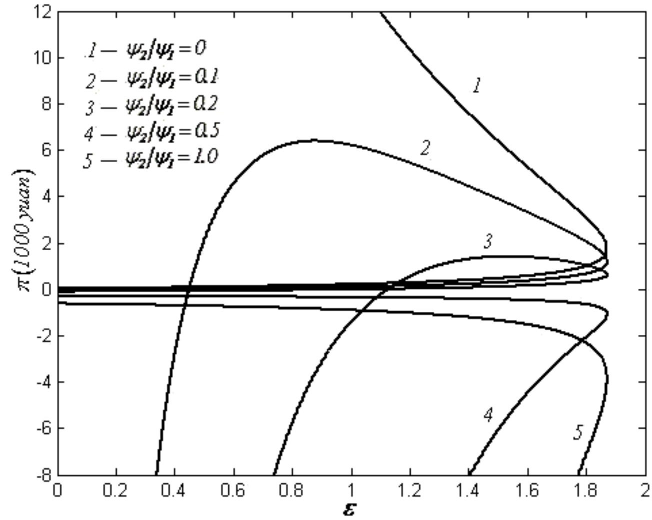

Figure 3. Effect of price ratio ψ_2/ψ_1 on $\pi - \varepsilon$ characteristic.

which the refrigerator is operating. When $0.033 < \psi_2/.\psi_1 \leq 0.393$, the curves of $\pi - \varepsilon$ are loop-shaped ones, which means that there exist a maximum optimal profit rate point and a maximum COP point. In addition, the areas enclosed by curves of $\pi - \varepsilon$ decreases with the increase of ψ_2/ψ_1. When $\psi_2/\psi_1 > 0{\cdot}393$, the curves of $\pi - \varepsilon$ are hyperbola-like ones, and the refrigerator is not profitable regardless of the cooling load is at which the refrigerator is operating.

Figure 4 shows the relation between the optimal profit rate π and COP ε of the combined refrigeration cycle with different loss items and $\psi_2/\psi_1 = 0{\cdot}1$. It illustrates that the curves of $\pi - \varepsilon$ are parabolic-like ones in the case of $q = 0$, while the curves are loop-shaped ones in the case of $q \neq 0$. It shows that the internal irreversibilities only have quantitative

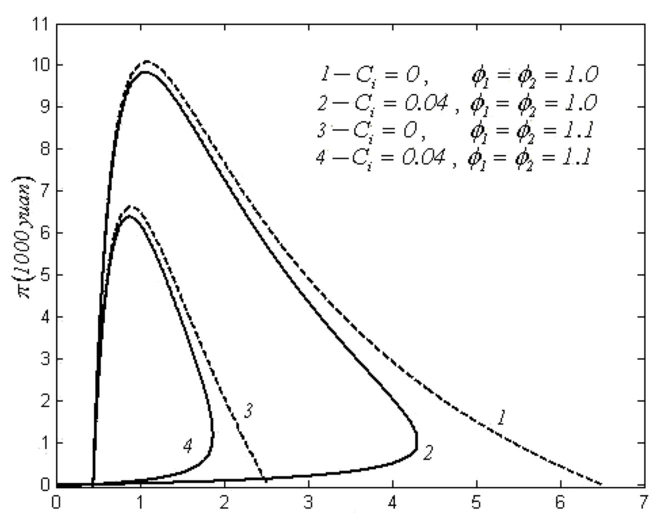

Figure 4. Effects of internal irreversibility and heat leakage on $\pi - \varepsilon$ characteristic.

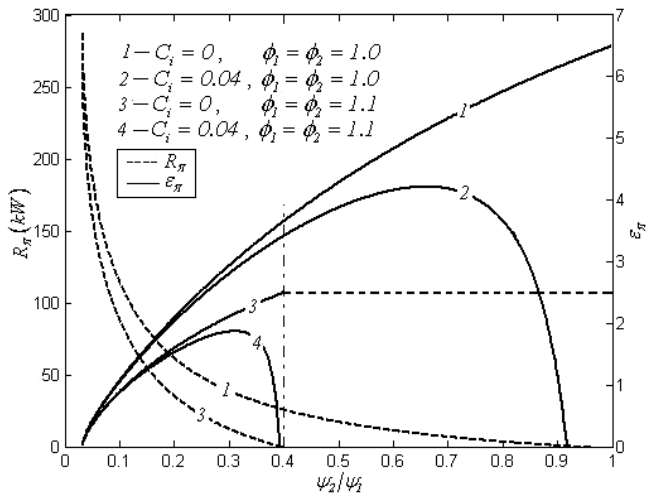

Figure 5. ε_π and R_π versus ψ_2/ψ_1 characteristic.

effects on the curves of $\pi - \varepsilon$, while the heat leakage has not only quantitative effects but also qualitative effects on the curves of $\pi - \varepsilon$. In addition, because of the existence of heat leakage and internal irreversibilities, both the maximum profit rate ($\pi_{\max}$) and the finite-time exergoeconomic performance bound (ε_π) decrease.

Figures 5 and 6 show the relations between the finite-time exergoeconomic performance bound (ε_π) and ψ_2/ψ_1, and between the maximum profit rate ($\pi_{\max}$) and ψ_2/ψ_1 with different loss items. Figure 5 shows that ε_π is a monotonic increasing function of ψ_2/ψ_1 without heat leakage, while the curves of $\varepsilon_\pi - \psi_2/\psi_1$ are parabolic-like ones with heat leakage. Figure 6 shows that $\pi_{\max}$ is a monotonic decreasing function of ψ_2/ψ_1. For the endoreversible combined refrigeration cycle, $\pi_{\max}$ is zero only when $\psi_2/\psi_1 = 1$. However, $\pi_{\max}$ is zero when $\psi_2/\psi_1 < 1$ with internal irreversibilities.

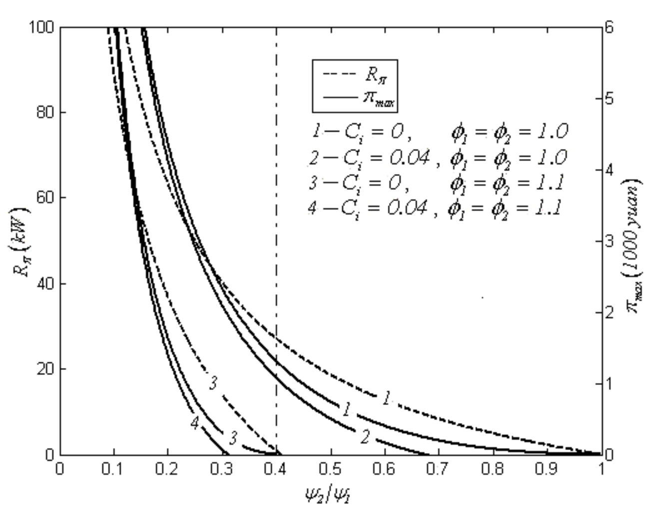

Figure 6. $\pi_{\max}$ and R_π versus ψ_2/ψ_1 characteristic.

6. Conclusion

Economics plays a major role in the refrigeration industry. This paper analyses the exergoeconomic performance of the generalized irreversible combined refrigeration cycle model. One seeks the economic optimization objective function instead of pure thermodynamic parameters by viewing the refrigerator as a production process. It is shown that the economic and thermodynamic optimization converged in the limits $\psi_2/\psi_1 \to 0$ and $\psi_2/\psi_1 \to 1$. When the profit rate for exergy conversion is small, the maximum profit operation is near the minimum exergy loss operation, while when the work input is very cheap compared to the price of exergy output, the maximum profit operation is near the maximum cooling load operation. The heat leakage and the internal irreversibility affect the finite time exergoeconomic performance of the combined refrigeration cycle obviously. It is necessary to investigate the optimal performance of a generalized irreversible combined refrigeration cycle.

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