***In this section of* Resonance*, we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.***

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# Diffraction at a Steel Scale in the Reflection Mode

**We use a steel scale and a broken piece of a long-playing record as plane reflection gratings. These are placed at grazing incidence with the incident laser beam. A beautiful diffraction pattern is seen on the opposite wall. This article describes the necessary theory, measurements and results.**

## Introduction

Diffraction from a plane grating is a familiar topic in undergraduate optics. Students study the theory in the classroom where they derive the condition for diffraction and how the incident beam splits into various orders of diffracted beams, each corresponding to path difference of different multiples of wavelengths. They also do the experiment in the laboratory where they measure the angles between the diffracted lines of various orders. (They use sodium lamp as a source, a spectrometer for angular measurements and a grating with 2000 to 6000 lines per cm.) A simple formula connects the wavelength $\lambda$, grating constant $d$, order $n$, and angle $\theta_n$, through which the $n^{\text{th}}$ order diffracted beam is deviated from the forward undiffracted beam.

We have observed that a good steel foot-scale serves as a grating in the reflection mode. Such a scale is commonly used by engineering students. The one which we have used has five different markings (least counts) etched on it, which are 1.0 mm, 0.5 mm, 1/16 inch, 1/32 inch and 1/64 inch. We have also used a broken piece of an old long-playing gramophone record and determined the separation between successive groves. We have used a He-Ne laser as a source of monochromatic radiation which has negligible divergence over the length of the hall.

These plane gratings are placed at a grazing incidence with the laser beam. A beautiful pattern of spots is observed on the opposite wall. When this phenomenon was shown to some persons, doubts were raised as to whether it is diffraction or simply reflection from different markings on the scale. It was also pointed out that the least count ($d$-spacing) on the scale was about 1000 times the wavelength used, and that diffraction could not possibly be seen in such a case. But when the laser beam was allowed to fall on different positions on the scale, it was observed that the separation between spots on the wall is inversely proportional to the $d$-spacing on the scale, which is characteristic of diffraction. A bit of theory was worked out and numerical estimations made which finally confirmed that it was indeed diffraction from the plane grating in the reflection mode. This article describes the theory of diffraction from a plane grating in the reflection mode, the experiment, measurements, and concludes with some remarks.

## Theory

Consider a laser beam, which is essentially a plane monochromatic wave, incident on a steel scale at a grazing incidence, the grazing angle $\theta$ being about 5°. The markings on the scale should be perpendicular to the incident beam. This is schematically shown in *Figure* 1. The beam thus forms an elongated spot on the scale, covering almost 1cm on it. Consider two successive markings on the scale separated by a distance $d$ (the least count where the beam is incident). Consider two rays in the beam which fall on these two successive marks on the scale. At each mark, they generate secondary spherical waves in

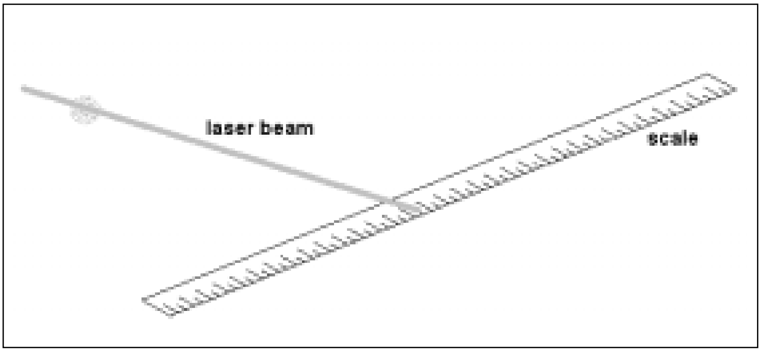



*Figure 1.*

accordance with Huygen's principle. Let us analyze the two rays in a direction making a grazing angle *f*, which may be different from $\theta$. In *Figure* 2, PB and QA are two rays falling on two successive marks on the scale. AS and BR are two parallel rays emerging at a grazing angle $\phi$. AC is normal to the ray PB and BD is normal to ray AS. The path difference between the two rays PBR and QAS is CB – AD, which is seen to be $d(\cos\theta - \cos\phi)$. A strong diffracted beam will appear in a direction $\phi$ if

$$d\,(\cos\theta - \cos\phi) = n\lambda, \tag{1}$$

where $\lambda$ is the wavelength of incident light and $n$ is any integer (the order of diffraction). Note that if $\phi = q$, there is no path difference between the emergent rays. This case corresponds to specular reflection ($n = 0$), which can occur for any $\theta$.

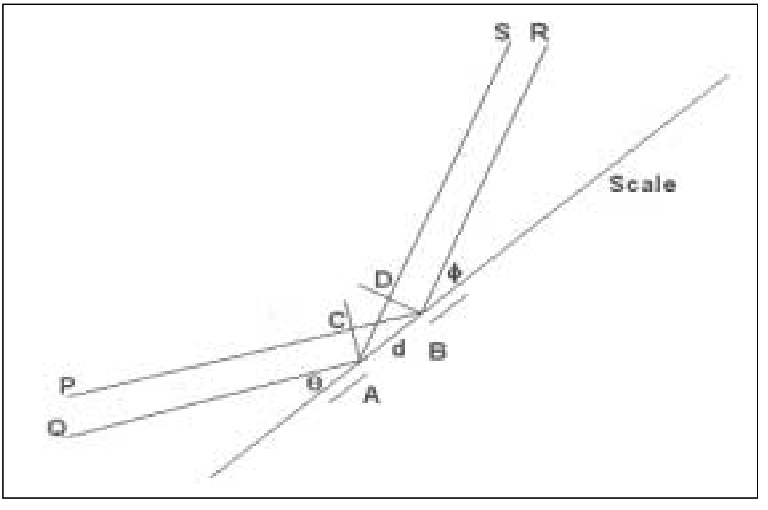



*Figure 2.*

Lets us now assume that $\phi$ is only slightly different from $\theta$, i.e.,

$$\phi = \theta + d\theta. \tag{2}$$

Then, up to first order in $d\theta$, we have

$$\cos\phi = \cos(\theta + d\theta) = \cos\theta - \sin\theta\, d\theta. \tag{3}$$

Using this in (1) we get the condition of diffraction as

$$d \sin\theta\, d\theta = n\lambda$$
$$\Rightarrow d\theta_n = n\lambda / (d \sin\theta), \tag{4}$$

where, in the last step, we have replaced $d\theta$ by $d\theta_n$ which will correspond to the value of $n$ on the right hand side. We indeed see quite a few diffracted spots on the wall above and below the specularly reflected bright spot. Note that it is $d\theta$, and not $\theta$, that takes specific discrete values. We shall comment on the difference between Bragg reflection/diffraction and the present case later in the article.

Even before making any measurements, we can make some numerical estimates. For He-Ne red laser, $\lambda = 633$ nm. If we use $d = 0.5$ mm $= 500$ μm and $\theta = 5°$, we get for $n = 1$,

$$d\theta_1 = 0.0145 \text{ radian} = 0.832°. \tag{5}$$

This will produce a separation of the order of 1 or 2 cm between the $n = 0$ and $n = 1$ spots on a wall about 1 to 2 m away. This is exactly what is observed.

## Experiment and Observations

A laser beam APO was directed at a spot O on the opposite wall, taking care that the beam is normal to the wall. The position O was marked on the wall; see *Figure* 3. Then a steel scale (or any other object which can serve as a plane grating) was inserted in the path of the beam at some grazing angle of incidence. The beam is incident on the scale at a point P which can be chosen to lie in a desired part of the scale. A diffraction pattern is seen on the opposite wall, which consists of a bright spot S and other spots which have been marked as $S_1$, $S_2$, $S_{-1}$, $S_{-2}$, etc. in *Figure* 3.

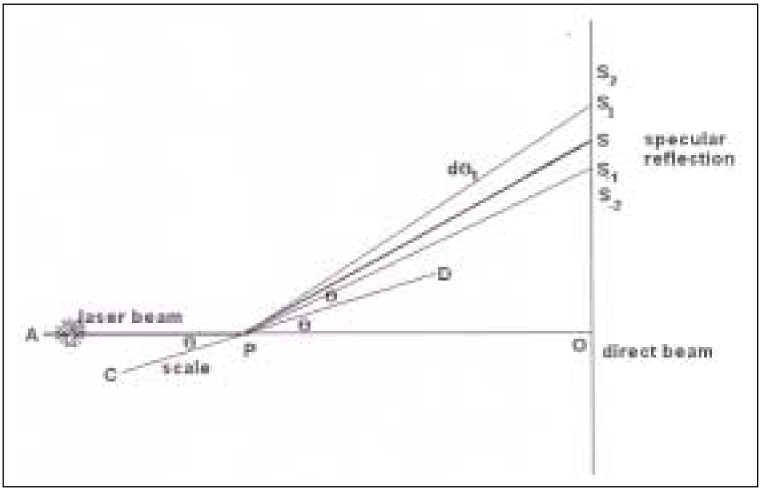



***Figure 3.***

The subscript on S corresponds to the value of $n$ in (4). Various distances are measured.

The angle of grazing incidence can be calculated from

$$2\theta = \tan^{-1}(OS/PO). \quad (6)$$

Note that $\theta$ is not pre-decided. The scale is fixed at a certain inclination and $\theta$ is obtained by measuring OS and PO.

The angle $d\theta_n$ can be calculated, as shown in *Figure* 3, from

$$2\theta + d\theta_n = \tan^{-1}(OS_n / PO). \quad (7)$$

We have taken observations by changing the position of P on the scale, which means using two different values of $d$, $d$ = 1mm and $d$ = 0.5 mm, and also two different grazing angles $q$. We have also varied the distance PO between the scale and the wall in each case in order to get a clear pattern. Although several orders of diffraction are seen on the wall we found that there is a large amount of scatter and structure around each spot, which increases with $|n|$. Therefore in our measurements, we have used only two spots above and two below the specular spot S expecting that at least these would be reliable. This gives us $d\theta_1$, $d\theta_2$, $d\theta_{-1}$, $d\theta_{-2}$. The average of $d\theta_1$, $d\theta_2/2$, $d\theta_{-1}$, $d\theta_{-2}/2$ is taken as $d\theta_1$ with $n$ = 1 in (4).

| No. | $D$ (mm) | OP (cm) | OS (cm) | $\theta$ (rad) | Average $d\theta$ (rad) | Observed $\lambda$ (nm) |
|---|---|---|---|---|---|---|
| 1 | 1 | 70.5 | 18.9 | 0.134 | $4.79 \times 10^{-3}$ | 640 |
| 2 | 0.5 | 51.5 | 10.2 | 0.0978 | $1.33 \times 10^{-2}$ | 649 |
| 3 | 1 | 108.0 | 20.7 | 0.0947 | $6.698 \times 10^{-3}$ | 633 |
| 4 | 0.5 | 49.5 | 11.5 | 0.114 | $1.15 \times 10^{-2}$ | 654 |

***Table 1. Values of d, OP, OS, and average dθ.***

We show in *Table* 1 the essential results, using two values of $d$ and two different inclinations of the scale. Not all measurements and calculations are shown; only the average $d\theta$ resulting from each set of ($d$, $\theta$) values is listed in the sixth column. We can use this in (4) (where it is in radians), with $n = 1$, to find the observed wavelength $\lambda$. The average wavelength thus obtained comes out to be

$$\lambda = 644 \text{ nm.} \tag{8}$$

This compares well with the known wavelength of He-Ne laser to within 2%.

Now any plane grating with unknown $d$ spacing can be placed in the path of the laser beam. We have used a broken piece of an old long-playing gramophone record. Again it was set at two different inclinations and similar measurements were made, which are reported in *Table* 2. The average $d\theta$ from these measurements is shown in the fifth column, and the resulting $d$ is shown in the last column. The average separation between the successive grooves on the record is found to be

$$d = 0.253 \text{ mm.} \tag{9}$$

| No | OP (cm) | OS (cm) | $\theta$ (rad) | Average $d\theta$ (rad) | Observed $d$ (mm) |
|---|---|---|---|---|---|
| 1 | 40 | 13.5 | 0.1627 | 0.016 | 0.244 |
| 2 | 47.5 | 16.6 | 0.1618 | 0.015 | 0.251 |
| 3 | 345 | 120 | 0.1678 | 0.0146 | 0.259 |
| 4 | 345 | 90 | 0.1276 | 0.0194 | 0.258 |

***Table 2. Measurement for the LP record. (The fixed parameter is λ = 633 nm.)***

We have tried to use a stack of staple pins and a CD as plane gratings. However, these result in a large amount of scatter and there are no discernible diffraction spots.

## Discussion

This phenomenon was noticed in our Physics Education Laboratory last year. It was found that the same has been discussed in a few books on optics [1, 2] though the simple theory described in (1)-(4) was not seen elsewhere. As mentioned earlier, it was shown to several persons in various universities and colleges. This led to several (frequently asked) questions, discussion, comparison with other similar phenomena, advantages and disadvantages, etc.

One of the questions was related to grazing incidence and the large $d/\lambda$ ratio. Indeed the largest ratio in the case of $d$ = 1mm is of $d/\lambda$ = 1580. Shouldn't we have $d$ comparable with l for diffraction? The answer is *yes* if a conventional source and spectrometer are used, and *not necessarily* if a laser is used. Having $d/\lambda$ = 1580 makes $d\theta$ of (4), with $n$ = 1, very small, i.e. of the order of $6.3 \times 10^{-4}$ rad $= 0.036^o$ with $\theta = 90^o$, and equal to $7.3 \times 10^{-3}$ rad = $0.42°$ with $\theta = 5^o$. At any closer distance from the diffracting object, as would be necessary with a sodium lamp, the spots would not be resolved. It is seen that as $\theta$ increases, the diffracted spots tend to merge into each other. The grazing incidence helps to reduce the effective value of $d$, though even this would not work with a sodium lamp. The factor $\sin\theta$ in (4) plays a crucial role in reducing the effective $d$-spacing to $d \sin\theta$. The use of a laser and the large distance from the wall produces a good separation between diffracted spots.

The other question was regarding comparison between Bragg reflection/ diffraction and the present situation. Bragg diffraction [3, 4] requires three dimensional grating. It takes place because of constructive interference between rays which are reflected from successive planes of a 3-D regular lattice. Also in this case, we decide beforehand that we are going to observe the outgoing beam in a direction making the same grazing angle as that of the incoming beam. The (grazing) angle of incidence and that of reflection are kept equal to each other and monitored until we get constructive interference. The result is that only specific discrete values of $\theta$, known as Bragg angles, are allowed. On the other hand in the present case, all the rays of incident beam are reflected from the same plane, on which a regular 1-D grating has been etched. Therefore specular reflection can be observed for any $\theta$. It is $\phi - \theta = d\theta$ which takes specific allowed values as shown in *Figure* 2 due to conditions of constructive interference.

## Acknowledgements

Two of us (Umapati Pattar and F I Surve) would like to thank the UGC for the grant of leave under its Teacher Fellowship programme, for doing MPhil and PhD, respectively.

## Suggested Reading

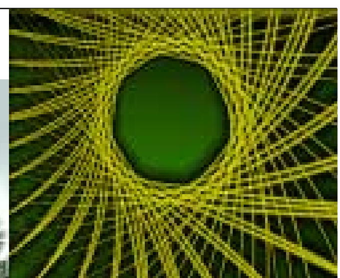
**[1] M Born and E Wolf, *Principles of Optics*, Cambridge University Press, Cambridge, VI Edn., pp. 407-412, 1997.**

**[2] R S Sirohi, *A Course of Experiments with He-Ne Laser*, Wiley Eastern Ltd., New Delhi, II Edn., pp. 47-50, 1991.**

**[3] C Kittel, *Introduction to Solid State Physics*, 5th Edition, Wiley Eastern Ltd, 1993.**

**[4] N W Ashcroft and N D Mermin, *Solid State Physics*, Holt, Rinehart and Winston, 1976.**

Fractals



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