# Think It Over

***This section of* Resonance *is meant to raise thought-provoking, interesting, or just plain brain-teasing questions every month, and discuss answers a few months later. Readers are welcome to send in suggestions for such questions, solutions to questions already posed, comments on the solutions discussed in the journal, etc. to* Resonance, *Indian Academy of Sciences, Bangalore 560 080, with "Think It Over" written on the cover or card to help us sort the correspondence. Due to limitations of space, it may not be possible to use all the material received. However, the coordinators of this section (currently A Sitaram and R Nityananda) will try and select items which best illustrate various ideas and concepts, for inclusion in this section.***

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## 1 Capillarity

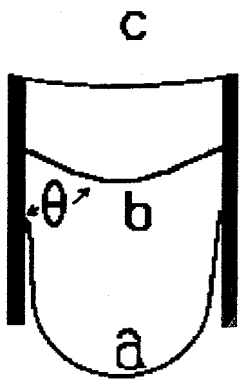
*Discussion of question raised in* ***Resonance****, Vol.1, No.5.*

1) What happens if the height of the tube is less than the value given by the formula above? Would the liquid squirt out? (Beware of perpetual motion!)

2) The formula contains the quantity $T$, which is a property of the liquid , but does not appear to contain any property of the material of which the capillary tube is made. This is surprising, because surely it is the attractive force between the material of the tube and the liquid which is responsible for the liquid rising against gravity! What is going on?

In spite of the warning against perpetual motion, some readers felt that the liquid would pour out of the top of the tube!

P Viswanath of Vellore pointed out that what matters is the radius of curvature of the 'meniscus' i.e the liquid surface. In its attempt to rise beyond the top, the liquid surface can take a spherical shape with a larger value $r'$ (marked **c** in the figure) than the radius $r$ of




**a.** Meniscus when the liquid wets the walls. Note the zero contact angle and the thin film rising above.
**b**. Shape of the meniscus for contact angle θ.
**c**. Even with zero contact angle, the meniscus adjusts its radius of curvature at the top of the tube.

the tube. $2T/r'$ can then balance $h\rho g$ for the height $h$ which is less than $2T/r$. Viswanath very properly points out that D S Mathur in his textbook on *Properties of Matter* gives this explanation. P Viswanath, Sulva Bhattacharyya (Jadavpur) and P K Thiruvikraman (Bangalore) all brought up the contact angle $\theta$. This is the angle between the tangent to the liquid surface and the tube wall (see the surface marked **b** in the figure). Our formula assumed that this was zero, and the more general formula is $h = 2T\cos\theta/r\rho g$. This is one way in which the material of the wall can influence the capillary rise.

But there is one more twist to the story. We say the liquid wets the surface when the contact angle is zero — for example water wets a range of different materials. In all these cases, $h$ has the same value! D Tabor, in his beautiful book *Gases, Liquids and Solids*, remarks that in this case, a thin film of the liquid covers the surface of the capillary above the meniscus (see the surface marked **a** in the figure). The attraction is now between water and water, and hence does not depend on the material of the wall, *once the wetting condition is met.*

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## 2 St. Petersburg Paradox

Consider the following gambling game. First you pay a fee, say Rs 10, to play this game. Then you go on flipping a *fair* coin until a tail first appears. Your reward will be Rs $2^n$ if you make the coin come up heads $n-1$ times before a tail appears. For example, if the outcome is *HHHT*, you win $2^4 = 16$ rupees. Your expected gain from playing this game is

$$\sum_{n=1}^{\infty} (2^n - 10)\, P(n-1 \text{ heads followed by a tail}) =$$

$$\sum_{n=1}^{\infty} (2^n - 10)\, 2^{-n} = \infty .$$

In fact, the expected gain is $\infty$ not just for the fee of Rs.10 (that you need to pay first) but for any fixed amount, however large. But most of us will not play this game for a large fee. Why?

**Fractals**