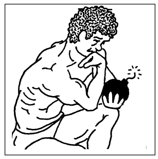
Think It Over



This section of* Resonance *is meant to raise thought-provoking, interesting, or just plain brain-teasing questions every month, and discuss answers a few months later. Readers are welcome to send in suggestions for such questions, solutions to questions already posed, comments on the solutions discussed in the journal, etc. to* Resonance *Indian Academy of Sciences, Bangalore 560 080, with "Think It Over" written on the cover or card to help us sort the correspondence. Due to limitations of space, it may not be possible to use all the material received. However, the coordinators of this section (currently R Nityananda and C S Yogananda) will try and select items which best illustrate various ideas and concepts, for inclusion in this section.

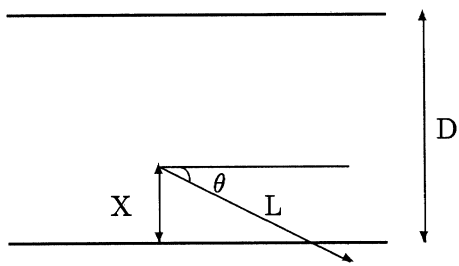
1. Buffon's Needle Problem

Discussion of question raised in *Resonance*. Vol.2, No.5.

Consider a plane ruled with parallel lines that are a distance D apart. Take m needles of length L each, $L \leq D$. Drop them all randomly on this plane. Count the number of needles that come to rest crossing any one of the lines. Repeat this n times. Let $x_1, x_2, \ldots, x_n$ be the number of needles that cross lines in these n trials. If you are told that, for large values of m and n, $\frac{2L}{D}\frac{mn}{\sum_{i=1}^{n} x_i}$ will be close to π what will be your reaction?

Correct answers have been received from Bikramjit Banerjee and Soubhik Chakraborty. A sketch of the solution follows.

Consider just two parallel lines and just one needle as shown in the figure. Let X denote the distance of the point of rest of the needle from the bottom line. Then X has the uniform distribution on the interval $(0, D)$. Let θ denote the angle of rest. Then θ has the uniform distribution on $(0, 2\pi)$. The probability that the needle comes to rest crossing one of these two lines is precisely $P(L\sin(\theta) > X, 0 < \theta \leq \pi/2) + P(L\sin(\pi - \theta) > X, \pi/2 < \theta \leq \pi) + P(L\sin(\theta - \pi) > D - X, \pi < \theta \leq 3\pi/2) + P(L\sin(2\pi - \theta) > D - X, 3\pi/2 < \theta \leq 2\pi)$. Now,



$$P(L\sin(\theta) > X, 0 < \theta \leq \pi/2)$$

$$= \frac{1}{2\pi D}\int_0^{\pi/2}\int_0^D I_{\{L\sin(\theta)>x\}}\,d\theta\,dx$$

$$= \frac{1}{2\pi D}\int_0^{\pi/2} L\sin(\theta)\,d\theta = \frac{L}{2\pi D},$$

and, in fact, the other three probabilities are also equal to the same quantity, $\frac{L}{2\pi D}$. Therefore, the probability that any needle dropped randomly on this plane comes to rest crossing any of the lines is $\frac{2L}{\pi D}$.

Next, recall the *Law of Large Numbers* (see the article by R L Karandikar, 'On Randomness and Probability', *Resonance*, Vol.1. No.2. pp.55-68, February 1996). Note that, if x_i denotes the number of needles (out of m dropped) that come to rest crossing lines in the ith trial, then $\sum_{i=1}^{n} x_i$ is the total number of needles that come to rest crossing lines out of mn needles which are dropped randomly and independently on the plane. Therefore, for large values of mn, $\sum_{i=1}^{n} x_i/mn$ will be close to $\frac{2L}{\pi D}$, which is the probability that any needle dropped randomly on this plane comes to rest crossing any of the lines.

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2. How Many Functional Molecules?

Discussion of question raised in *Resonance* Vol.2, No.7.

The biochemist is correct in the first part of the answer. Three types of molecules will be produced. We cannot differentiate between D-N and N-D. After that, however, he makes a mistake which is too common among non-mathematicians. The fact that there are n possible events is not sufficient to infer that the probability of each is $1/n$. The presence of three types of molecules does not necessarily mean that all are equally abundant.

The mathematician starts with the assumption that the association of two polypeptides is random. If you pick up one polypeptide randomly, let the probability that it is normal be p and defective be q. Here, since the rates of synthesis of the normal and the defective polypeptide are assumed to be equal, we can say that $p = q = 0.5$. Now we can ask the question, what is the probability that two consecutively picked up polypetides turn out to be normal ? It will be $p \times p$ or p^2. It turns out to be 0.25 or one-fourth, and for that of both being defective it will be $q^2 = 0.25$. The probability of picking up a N followed by D will be $p \times q$. Likewise, for D followed by N, it will be $q \times p$. Since we are unable to differentiate whether it was N-D or D-N, both being possible, we will add these two to get $2 \times p \times q = 0.5$. Thus half of the proteins will have one normal and one defective polypeptide. Write the three in sequence — $p^2 + 2pq + q^2$. You will immediately realise that it is $(p + q)^2$. This binomial expansion is extremely important in both population and molecular genetics.

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> Science consists in grouping facts so that general laws or conclusions may be drawn from them.
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> *Charles Darwin*