Editorial

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Some three and a half centuries ago, Galileo Galilei declared that the book of nature was written in the language of mathematics. Isaac Newton following after him set up a comprehensive scheme for the description of physical phenomena, and aptly called it "Mathematical Principles of Natural Philosophy". For a couple of centuries thereafter physics and mathematics advanced largely in tandem, the needs of the one spurring ideas and advances in the other. In that era, some of the greatest contributors to mathematics were also outstanding physicists and conversely – to name but a few: Euler, Lagrange, Laplace, Gauss, Hamilton and Jacobi. Over the past century or so, however, there has been some parting of ways; one has come to acknowledge that each endeavour has its own character and spirit.

I refer to these matters now because in recent issues we have had articles both on mathematical topics *per se*, and on useful viewpoints and calculational techniques for the user of mathematics. Sometimes this has led to interesting discussions among us editors; there is a need to acknowledge that mathematics can be explained, understood and used at more than one level. No chance to recount an amusing anecdote should be lost, so here is one.

"But it (the book of nature) cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language" — Galileo Galilei

Physicists of an earlier generation would remember the names of Arthur S Wightman, Marvin L Goldberger and Geoffrey F Chew. It appears these three theorists were once discussing the way mathematics should be used in physics. Wightman felt that at any given point in time the best available resources and methods of mathematics must be used and the same demands of rigour should apply. Goldberger however differed, and said there should be room for physical intuition, the "sixth sense", to

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offset precision and rigour — "proof by persuasion" as he called it. And finally Chew asked "What is *proof*?".

Apocryphal stories apart, one has to allow for these variations in style – chemists and physicists rely upon their "feel" of a problem and translate it into expectations of the mathematical expression of a solution. These are as real as anything else and cannot be denied or wished away. Here is a stunning instance from the history of modern physics. When Werner Heisenberg invented matrix mechanics in June 1925, he did not know what matrices were. Yet his deep physical intuition led him to represent physical position and momentum by arrays of complex numbers; and he then invented a law of multiplication for these arrays *based on the Ritz Combination Law of spectroscopy*. Later his teacher Max Born realized this was just the law of matrix multiplication, something *he* had learnt as a student. And in all honesty one can say – had the concept of matrices not been invented in the last century, it would have come into being through the laws of spectroscopy and Heisenberg's genius!

Two other striking examples come from Paul Dirac's work in quantum mechanics – his invention of the delta function, only much later accepted, formalised and christened as "distribution" in mathematics; and his rediscovery of spinors while constructing a relativistic wave equation for the electron.

Where then do we stand? Each of us must recall and admit what Hamlet said to his dear friend – "There are more things in heaven and earth, Horatio, Than are dreamt of in your philosophy". And ponder over this from Chen Ning Yang from more recent times: "It would be wrong, however, to think that the disciplines of mathematics and physics overlap very much; they do not. And they have their separate aims and tastes. They have distinctly different value judgements, and they have different traditions. At the fundamental conceptual level they amazingly share some concepts, but even there, the life force of each discipline runs along its own veins".