Think It Over

This section of* Resonance *is meant to raise thought-provoking, interesting, or just plain brain-teasing questions every month, and discuss answers a few months later. Readers are welcome to send in suggestions for such questions, solutions to questions already posed, comments on the solutions discussed in the journal, etc. to* Resonance, *Indian Academy of Sciences, Bangalore 560 080, with "Think It Over" written on the cover or card to help us sort the correspondence. Due to limitations of space, it may not be possible to use all the material received. However, the coordinators of this section (currently A Sitaram and R Nityananda) will try and select items which best illustrate various ideas and concepts, for inclusion in this section.

1 Capillarity

From Rajaram Nityananda, Raman Research Institute, Bangalore.

We are all taught the phenomenon of capillarity, in which a liquid rises in a narrow tube, when it 'wets' the material of the tube. The formula for the height is $h = 2T / (r \rho g)$. T is the surface tension of the liquid, r the radius of the capillary tube, g the acceleration due to gravity, and ρ the density of the liquid. Here are two questions relating to this everyday phenomenon.

1) What happens if the height of the tube is less than the value given by the formula above? Would the liquid squirt out? (Beware of perpetual motion!)

2) The formula contains the quantity T, which is a property of the liquid, but does not appear to contain any property of the material of which the capillary tube is made. This is surprising, because surely it is the attractive force between the material of the tube and the liquid which is responsible for the liquid rising against gravity! What is going on?

From Rajeeva L Karandikar, Indian Statistical Institute, New Delhi.

2 To Switch or Not to Switch

You are a winner in the preliminary round of a TV game show and the host gives you a chance to win the super prize: a fancy car. You are shown three doors numbered 1, 2 and 3. Behind one of them is the car. You are asked to choose a door. If the chosen door is the one hiding the car, you win the prize.

You choose, say, door number 2. The host of the show then says: "First, let us see what is behind door number 1?" He opens it and you see that the car is not there. Now he asks you: " Do you want to stay with your initial choice (number 2), or would you like to switch to door number 3?" What would you do?

Does this have a familiar ring to it? May be the 'Prisoner's dilemma' has the same logic.

Mohan Delampady, Indian Statistical Institute, Bangalore.

3 Prisoner's Dilemma

Discussion of question raised in **Resonance** *Vol. 1, No.2.*

Three prisoners, A,B and C are each held in solitary confinement. A knows that two of them will be hanged, but one will go free. However, he does not know who will go free. He thus reasons that there is a 1/3 chance of his survival. Anxious to know his fate, he asks his guard. But the guard will not tell A his fate. A thinks and puts the following proposal to the guard: "If two of us must die, then I know that either B or C must die and possibly both. If you tell me the name of just one of them who is certain to die, then I learn nothing about my fate; and since we are kept apart, I cannot inform them of theirs. So tell me which one of B or C is to die?" The guard accepts the logic and tells A that C is to die. A now reasons that either he or B will live. Thus A now has a 1/2 chance of survival. Is A's reasoning correct?

A's argument is incorrect. He hasn't considered what the guard might say if both B and C are to die. Conditional probability argument is needed to analyze this puzzle. Let AB be the event that A and B are to die and define BC and AC similarly. Let D be the event that the guard says 'C is to die'. Then

$$\begin{aligned} P\,(A\text{lives}\,|\,D) &= P\,(BC\,|\,D) \\ &= \frac{P\,(BC \text{ and } D)}{P\,(D)} = \frac{P\,(D\,|\,BC\,)P(BC\,)}{P\,(D)} \end{aligned}$$

Note that

$$P(D) = P(D|AB)P(AB) + P(D|BC)P(BC) + P(D|AC)P(AC)$$
$$= P(D|BC)P(BC) + P(AC)$$

since $P(D|AB) = 0$ and $P(D|AC) = 1$. Therefore,

$$P(\text{Alives}|D) = \frac{P(D|BC)P(BC)}{P(D|BC)P(BC) + P(AC)} .$$

Since A initially believes that they all have an equal chance of surviving, it follows that $P(AB) = P(AC) = P(BC) = 1/3$. If he further assumes that the guard is equally likely to say 'B is to die' and 'C is to die' if BC is to occur, then $P(D|BC) = 1/2$. Therefore,

$$P(BC|D) = \frac{(1/2) \times (1/3)}{(1/2) \times (1/3) + (1/3)} = \frac{1}{3} .$$

That is, his probability of survival should be 1/3 still. Note, however, that he might not hold $P(D|BC) = 1/2$. Then other values of $P(BC|D)$ are possible. When does he obtain $P(BC|D) = 1/2$?

Count Rumford burns his tongue ... Water was considered a good conductor of heat until, in the last years of the eighteenth century, Count Rumford burnt his mouth when eating apple pie; he then decided to do some experiments 'to show that water, and probably all other liquids, are non-conductors of heat'. (From *Stories from Science IV* by Sutcliffe and Sutcliffe)