

reader is likely to be familiar with at some level, are dealt with in detail. The important and useful procedures of diagonalising symmetric/hermitian matrices using orthogonal/unitary matrices, the significance of eigenvalues and the correspondence of hermitian symmetric matrices with quadratic forms is discussed in fair detail and no background is assumed. Useful ways of partitioning a matrix into submatrices and using this to simplify problems are also discussed.

The subject of tensor algebra and tensor calculus is built up from scratch. Explicit details are provided so that the reader does not get scared. This is important as most students are daunted by tensors. The concepts of covariant and contravariant tensors are developed and applications to special relativity and the covariant formulation of electromagnetic theory are worked out in detail. A preliminary discussion on covariant differentiation, Christoffel symbols and

curvature prepares the reader interested in learning more about general relativity.

Most of the problems in the book are rather routine. It may be necessary to have a number of standard problems to reinforce basic ideas but it is equally essential to give thought provoking problems; this is one major shortcoming of the book. Some introduction to the coordinate independent approach may also have been useful. Elasticity could have been discussed while dealing with the applications of tensors.

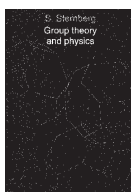
On the whole the book is written with a down to earth approach, giving useful material without getting lost in detailed proofs and at the same time maintaining the requisite mathematical rigour. It should serve as a useful text book/reference for B.Sc as well as M.Sc students. The price is very reasonable.

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## A Mathematician Looks at Physics

An Introduction to Group Theory and its Applications

**Vishwambhar Pati**



*Group Theory and Physics,*  
S Sternberg,  
Cambridge University Press, 1994  
pp. 427. £ 50

That geometry and physics are closely interlinked human endeavours harking back to antiquity is well known. What is not so well known is that group theory, in an implicit way, is also at least as old. For example, the notion of congruence in Euclidean geometry, in modern language would be called equivalence under the group  $E(2)$  of rigid motions of the Euclidean plane. Much later, when geometries other than the Euclidean were discovered, they spawned their own



groups of isometries (congruences), such as  $PSL(2,R)$  for the hyperbolic plane. At the turn of the last century, Felix Klein's Erlanger program sought to classify and understand geometries as a manifestation of their corresponding symmetry groups. Similarly in physics, for example, the Galilean invariance of classical mechanics, the Lorentz invariance of electromagnetism, or the more recent gauge invariance under various gauge groups have underscored the same leitmotiv, i.e. understand physical laws by their symmetries. This is what the book under review sets out to investigate, starting essentially from scratch.

After a few preliminaries, the author starts off with a purely group theoretic proof of the existence of only five regular (Platonic) solids in three space. It boils down to the classification of all finite subgroups of the three dimensional rotation group  $SO(3)$  which do not have a common invariant axis (i.e. are not planar rotation groups, which are also easily classified). The proof involves nothing more complicated than counting a set in two different ways, but already illustrates the power of group theory. The sections 1.9 and 1.10 contain a fascinating excursion into crystallography. Consider the following remarkable fact, which was first empirically observed. The symmetry groups of naturally occurring crystals (which are not Platonic solids in general) did not contain any rotations of orders other than  $k = 1, 2, 3, 4, 6$ . The explanation for this is to be found in section 1.9. Nature has arranged crystal shapes so that

their groups of symmetries also preserve three dimensional crystal lattices. This forces only those allowed rotations. Of course, we're still left with the question of what groups do occur as symmetry groups of crystals. Section 1.9 shows that this is basically the classification of finite subgroups of  $O(3)$  having only the allowed rotations above, and there are thirty two of them. All but two of these are symmetry groups of crystals occurring in nature, and their table with accompanying pictures occurs at the end of section 1.9. The related question of classifying the crystallographic groups is answered in Appendix A.

If this doesn't strike you as anything more than a curiosity, I would urge you to persist with the representation theory of finite groups in Chapter 2. It is a very clean and efficient account of all the basics, with plenty of illustrative examples and computations of character tables, explicit bases etc. For his or her perseverance, the reader will be amply rewarded in Chapter 3, which contains a detailed discussion of why selection rules occur in physics. The finite group representation theory comes into the study of selection rules for a molecule vibrating in an electromagnetic field, and Raman scattering.

Section 3.8 contains some basics about irreducible representations of semidirect

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products via the use of induced representations, and this is immediately deployed to achieve Wigner's classification of the irreducible unitary representations of the universal cover of the Poincaré group. In the physically relevant cases, these turn out to be parametrised by two parameters:  $m$  (rest mass) which has any non-negative value, and  $s$  (spin), which is allowed to have only half-integer values. The program of Wigner, a kind of Erlanger program for physics, to understand all elementary particles by looking at irreducible representations of the Poincaré group could not explain why only certain masses were found, or how the conserved quantity of charge was to be explained. On the other hand, all of present day physics accepts the validity of this philosophy, and seeks only to find the correct replacement for the Poincaré group. The chapter ends with a group theoretic discussion of parity.

Chapter 4 launches the representation theory of compact groups, which is a very beautiful and complete area of mathematics, and has the flavour of the finite group representation theory discussed in Chapter 2. Some of the proofs (such as the elegant proof of the existence of Haar measure using the Mean Ergodic theorem, or of the Peter-Weyl package for the regular representation) are relegated to the Appendices, and quite

wisely so. This is because the immediate application of  $SO(3)$  representation theory to the explanation of the various quantum numbers of orbital electrons in an atom is a very beautiful and intellectually satisfying exercise which begins in Section 4.5. In fact, if you were baffled by the numbers  $2(2l+1)$  of electrons in a given  $L$ -subshell while studying school chemistry, the answer lies in the representation theory of  $SO(3)$ , as you will discover in this section. Then the story moves on to the Clebsch-Gordan decomposition of two irreducible  $SU(2)$  representations, and its physical implications, e.g. isospin.

Chapter 5 moves to the classification of the irreducible finite dimensional representations of  $SU(n)$ ,  $SL(n, C)$  and  $GL(n, C)$ . This is done via the decomposition of the  $r$ th tensor power of  $C^n$  into its irreducible components under the natural representation of the symmetric group  $S_r$ . The remarkable application of the adjoint representation of  $SU(3)$  to the 'Eight-Fold Way' of Gell-mann and Neeman and its consequent prediction of the  $\Omega$  particle is discussed in 5.10. Section 5.13 goes into gauge theory, but not in too much detail.

Sternberg has done a masterly job of organising the material so that the book is readable from practically any point on. The proofs are always the slickest possible, and there are ample examples along the way so that a general reader does not lose himself or herself in abstraction. (Note the beautiful introduction of a vector bundle via molecular vibrations in 3.2, or the neat way of explicitly constructing



It is a very clean and efficient account of all the basics, with plenty of illustrative examples.

a Haar measure on a linear Lie group in 4.1.) The only prerequisites are a sound understanding of undergraduate analysis and linear algebra. While this book could be considered too sophisticated for the undergraduate level in an Indian university, an imaginative teacher could fashion several interesting courses out of this book at the M.Sc. level.

On the downside, the index is woefully inadequate for a book of such encyclopaedic reach. In fact, since Sternberg eschews the usual definition-theorem-corollary format of mathematics books, the index needs to be exhaustive. For example, *Poincaré Group*, *spin*

and *angular momentum* do not exist as entries in the index! Another inadequacy is the absence of exercises, which could have been used to provide the sometimes very elementary demonstrations that Sternberg goes through in painstaking detail. There are also more typos than one would expect from such a reputed publisher.

Finally, I hope the author will write a sequel to this book where the Cartan-Weyl theory and classification of groups such as  $Spin$  and  $Spin_c$  and their representations, which are of great interest to physicists, and finally gauge theory are dealt with in greater detail. Meanwhile, happy reading, or for that matter, even browsing!

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## The Strength of Materials

Without Stress or Strain

*Gangan Prathap*



*The New Science of Strong Materials or  
Why you don't fall through the floor*  
J E Gordon  
Reprinted in Penguin Books, 1991.  
pp.287, Rs.195.

Stress and strain are words used to describe the mental condition of human beings. In this sense, the words are used interchangeably to mean the same thing. In science and

engineering, these words are given distinct meanings and the entire science underlying structural engineering rests on these basic distinctions. The structural engineer's craft rests on two pillars: form (or how shape and size and manner of arrangement of structural material provides efficient design) and substance (the nature of material(s) out of which the structure is fashioned). An earlier book by Gordon, 'Structures or Why Things Don't Fall Down' (also available in Penguin edition), dealt with the former aspect; the book reviewed here deals with the latter issue.

"Why do things break?" is the substantial question addressed in this book. "How is this

