The Story of Fermat's Last Theorem

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Fermat's Last Theorem
Unlocking the Secret of an Ancient Mathematical Problem
Amir D Aczel
(Viking, 1996)

Fermat's Enigma
The Epic Quest to Solve the World's Greatest Mathematical Problem
Simon Singh
(Walker and Company, 1997)

How does one go about writing a book about the 'World's Greatest Mathematical Problem"? Presumably one must start at the very beginning. But where do the beginnings lie? How far back in time does one have to go?

Problems, declared Hilbert in a famous address (1900), are the life blood of mathematics; and following this declaration he presented a list of 23 problems that was to have considerable influence on the development of mathematics in the decades to come. To back up Hilbert's claim with case studies is an easy task. The history of mathematics shows that problems have an extraordinary way of firing the imagination – of the student as well as the established mathematician. As such, they often inspire new and rich areas of mathematics. For instance, much of algebraic number theory was born from attempts to solve two great problems – Fermat's last theorem (or FLT, as it is widely known), and the problem of generalizing the law of quadratic reciprocity. Non-Euclidean geometry provides another such example; it was born from the attempt to prove the parallel postulate. This means that problems, in addition to possessing intrinsic interest, also provide the historian with a ready vehicle for writing about the history of mathematics.

But in the case of FLT the difficulties of writing a complete historical account would seem much greater; it becomes necessary to reach far back into history. Consider the following list of mathematicians whose work in some way lies 'behind' the proof in 1993/4 of FLT by Andrew Wiles : Pythagoras, Euclid, Archimedes, Diophantus, Fibonacci, Pacioli, Tartaglia, Cardano, Descartes, Pascal, Fermat (of course!), Euler, Lagrange, Galois, Abel, Cauchy, Fourier, Gauss, Dirichlet, Dedekind, Kummer, Cantor, Riemann, Lobachevsky, Bolyai, Hilbert, Poincare, Mordell, the Bourbakis, Hecke, Taniyama, Shimura, Weil, Lang, Grothendieck, Deligne, Langlands, Serre, Faltings, Frey, Ribet, Mazur, Taylor, Wiles. Whew! – a long list! In what way have all these individuals been connected with FLT? The beginnings of number theory go back to the Greeks. It is in the Pythagorean school that one first sees the human fascination with number *per se* and the belief that 'number is all'; from Euclid come the first instances of proofs that are fully modern in conception – 'proof by contradiction' – and from Diophantus comes the first study of equations in integers, the topic now referred to as 'Diophantine equations'. [1] To Fibonacci goes the credit of importing algebra, trigonometry and the decimal numeration system from India and Arabia into Europe. Then comes the story of how cubics and quartics came to be solved by the Italians – Tartaglia, Cardano and Ferrar. Shortly later the link between geometry and algebra is forged by Descartes, Pascal and Fermat, and the beginnings of algebraic geometry are laid; and from Fermat come new ideas and an entirely new level of sophistication in dealing with problems in number theory – the idea of descent. Then comes Euler, to whom so much in general is due(!); group theory, from Lagrange and Abel; Galois theory; the integration, by Gauss, of complex numbers into number theory and into the mainstream of mathematics; shortly later the birth of algebraic number theory (through the work of Gauss, Kummer, Dedekind,...); the pioneering use of complex variables in number theory (Dirichlet, Riemann), which culminates in the proof by Hadamard and de la Vallee Poussin of the prime number theorem, towards the end of the 19th century. Meanwhile non-Euclidean geometry is born (Bolyai, Lobachevsky), the study of elliptic functions is initiated (Gauss, Jacobi, Abel), and the idea of countable infinities (Cantor). What a lot of rich drama lies behind these developments! Then comes Poincaré with his amazing outpouring of ideas in diverse fields, including that of addition of points on an elliptic curve (giving rise to a finitely generated abelian group). This later inspires the famous conjecture by Mordell which then has to wait a half-century for its proof by Faltings. Then comes the conjecture of Taniyama and Shimura, connecting modular forms with elliptic curves. This is shown in the 1980's to imply FLT (Frey, Ribet). Finally, the conjecture is proved for a certain class of elliptic curves (Wiles, with the help of Taylor), and this is enough to prove FLT. What a rich and tangled story! Alongside the main line of the story are numerous human stories: prizes for a correct proof of FLT; the Wolfskehl prize, worth 100000 marks, whose value later is reduced temporarily to nil thanks to the runaway inflation of the 1920's; innumerable false proofs – 621 false proofs received in 1908 alone! Added to this is the

[1] It is not out of place here to mention the strong interest in India in such equations. The problem of finding solutions in integers to the linear equation $ax + by = c$ was treated in detail by Āryabhata (5th century) and Bhāskar I (7th century). The equation which later came to be known as 'Pell's equation' (incorrectly, as it is now known) received considerable attention from Brahmagupta (7th century) and Bh skar II (12th century). This is the problem of finding positive integers x for which Nx^2+1 is a square, N being a given integer. Bh skar found an ingenious and beautiful general solution to the problem. For $N = 61$ the smallest solution is given by him as $x = 226153980$(!). The interested reader should refer to the text by Varadarajan (see suggested reading).

strange story of Wiles and his solitary and extraordinary seven-year battle with FLT, following a style reminiscent of medieval times and quite at variance with the way mathematics tends to be done today. It is hard to capture or to describe in adequate terms the magnitude of Wiles' achievement. In the foreword to Simon Singh's book, John Lynch writes, 'To Andrew Wiles [FLT] was ··· nothing less than his life's ambition. When he first revealed a proof in that summer of 1993, it came at the end of seven years of dedicated work on the problem, a degree of focus and determination that is hard to imagine. Many of the techniques he used had not been created when he began ··· . In a sense, reflected [Mazur], it turned out that everyone was working on Fermat, but separately and without having it as a goal ··· . What [Wiles] had done was to tie together once again areas of mathematics that had seemed far apart. His work therefore seemed a justification for all the diversification that mathematics had undergone since the problem had been stated.' It may be mentioned here that Wiles was awarded the 'IMU Silver Plaque' at the International Congress of Mathematicians in August 1998. This is a 'custom-made' tribute which the International Mathematics Union made specially for Wiles.

Amir Aczel has undertaken such a historical study in his book. In each section of the book he presents a historical episode together with the mathematical concepts involved. Most of the individuals whose names feature above find mention in the book, which therefore spans a huge interval in time. The book has clearly been written with the lay reader in mind, and the author certainly succeeds in conveying the vastness and depth of the story. 'A delightfully simple and brief book, part detective chase and part mathematical popularization'; 'Readable and enjoyable', claim the blurbs at the back of the book; and indeed this is so.

The only part of the book which may jar on the reader (it did have that effect on me) is one of the 'human interest' stories, titled *The Lie* by Aczel. This deals with the genealogy and naming of the Taniyama–Shimura conjecture and involves no less a mathematical personage than André Weil. However the case built up so dramatically by Aczel does not seem quite as clear and unambiguous as he makes it out to be. It might have been wiser for the author to have underplayed the role of this controversy and to have been more tentative. However this does not detract from the essential value of the book. To the serious student the mathematical content will seem disappointing; but this is inevitable, given the difficult nature of the subject matter.

It is curious that *two* books on FLT should appear in the market at nearly the same time, and that they should be written in so similar a style. Many of the comments noted above apply to Simon Singh's book as much as they do to Aczel's book. However Singh does a *far* better job than Aczel. The descriptions are more full and satisfying (Singh's book is twice as thick as Aczel's), and this is true for the historical accounts as well as the descriptions of mathematical concepts. Also, Singh (wisely, in my opinion) avoids any mention of the controversy referred to above. Overall, both books succeed in conveying a sense of why mathematicians 'do' mathematics, and why the subject is so very exciting to them; but while Aczel's book is good, Singh's book is excellent, and is *very* warmly recommended to those who have an interest in mathematics and its history. (It may not be out of place to mention here that Simon Singh and John Lynch of the BBC have produced a television film on the story of FLT. The film is just as good as the book.) I immensely enjoyed reading the two books and consider myself fortunate in having acquired copies (reviewer copies!) of them.

The interested reader should also refer to the article by C S Yogananda, 'Fermat's Last Theorem – a Theorem at last!' in *Resonance*, Volume 1, Number 1, 1996.

Suggested Reading

V S Varadarajan. *Algebra in Ancient and Modern Times*, TRIM series, Hindustan Book Agency, 1997.

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