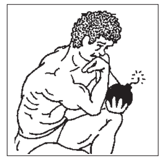
Think It Over



This section of *Resonance* presents thought-provoking questions, and discusses answers a few months later. Readers are invited to send new questions, solutions to old ones and comments, to 'Think It Over', *Resonance*, Indian Academy of Sciences, Bangalore 560 080. Items illustrating ideas and concepts will generally be chosen.

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Solution to Mileage from Push Pull Express

Problem: A train starts from rest at station A, with the engine exerting a constant pull f_1 per unit mass. After reaching a certain velocity, the brake is then applied, the resistive force being f_2 per unit mass. The train comes to rest at station B. Assume that the air resistance experienced by the train during motion is proportional to its velocity, the proportionality constant being k. If T is the time of travel between A and B, express the distance between the two stations in terms of f_1, f_2, k and T.

Solution: Let C be the point at which maximum velocity is reached. For the portion AC, let the distance covered and time taken be x_1 and t_1 respectively, and for the portion CB, let the corresponding values be x_2 and t_2.

For each portion, write x for the distance covered in time t, measured from the start of the trip, and v for the velocity attained at that time. For the portion AC, we have:

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = f_1 - kv, \tag{1}$$

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where $x = 0$ and $v = 0$ at $t = 0$. Solving the equation, we get

$$1 - \frac{kv}{f_1} = e^{-kt}, \qquad \frac{f_1}{k} \ln \frac{1}{1 - kv/f_1} - v = kx. \qquad (2)$$

These relations yield:

$$1 - \frac{kv_0}{f_1} = e^{-kt_1}, \qquad \frac{f_1}{k} \ln \frac{1}{1 - kv_0/f_1} - v_0 = kx_1. \qquad (3)$$

For the portion CB, we get similarly,

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = -\left(f_2 + kv\right), \qquad (4)$$

where $v = v_0$ and $x = 0$ at $t = 0$. These yield, on solution,

$$\frac{1 + kv_0/f_2}{1 + kv/f_2} = e^{kt}, \qquad \frac{f_2}{k} \ln \frac{f_2 + kv}{f_2 + kv_0} + v_0 - v = kx. \qquad (5)$$

These relations yield:

$$1 + \frac{kv_0}{f_2} = e^{kt_2}, \qquad \frac{f_2}{k} \ln \frac{1}{1 + kv_0/f_2} + v_0 = kx_2. \qquad (6)$$

From equations (3) and (6) we get, with $T = t_1 + t_2$ as the total time of travel,

$$\frac{1 + kv_0/f_2}{1 - kv_0/f_1} = e^{kT}. \;\therefore\; kv_0 = \left(e^{kt} - 1\right) \left(\frac{e^{kt}}{f_1} + \frac{1}{f_2}\right)^{-1}, \qquad (7)$$

and, finally,

$$X = x_1 + x_2 = \ln\left[\left(\frac{f_2 e^{kT} + f_1}{f_1 + f_2}\right)^{f_1/k^2} + \left(\frac{f_1 e^{-kT} + f_2}{f_1 + f_2}\right)^{f_2/k^2}\right]. \qquad (8)$$

Comment. If we let $k \to 0$, then we get

$$v_0 \;\to\; \frac{T}{1/f_1 + 1/f_2}, \qquad X \;\to\; \frac{T^2/2}{1/f_1 + 1/f_2}. \qquad (9)$$